CALCULATIONS ON FACE AND VERTEX REGULAR POLYHEDRA AND APPLICATION TO FINITE ELEMENT ANALYSIS

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DECLARATION

Work included in this thesis in part or whole, has not been submitted for any other academic qualification at any institution

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ACKNOLEDGEMENT

This is a result of a subject I developed for the last eight years. I introduced the word "Face and Vertex Regular Polyhedra" and I was able to publish a paper in Mathematical Gazette in March 2005, on the topic "Calculations on Face and Vertex Regular Polyhedra". Many people helped and encouraged me during the past years. Firstly I would like to thank my supervisors Prof.G.T.F. de Silva and Prof.M.Indralingam for their valuable advice. I would also like to thank all my friends and teachers who encouraged me during that time. My special thanks goes to Mr. Hema Nalin Karunarathne who gave me the chance to display my findings in 9.05 Rupavahini program in 1999. I would like to thank Prof. G.T.F. de Silva again for organizing a departmental seminar to present my findings to the staff of the Department of Mathematics, university of Moratuwa in 2001.

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LIST OF SYMBOLS

A_a -structure formed by bringing together a number of objects of type A

$A_a B_b ....$ -structure formed by bringing together a number of objects of type A ,

b number of objects of type B , ...

n_i -number of sides of the i th type polygon or the polygon of n_i number of sides

r_i -radius of the escribed circle radius of ith type polygon

M_i -number of i th type polygons meet at a vertex

R_i -radius of the escribed sphere radius of i th type polygon

R - radius of the escribed sphere radius of polyhedra

k -constant of the polyhedon

a -length of an edge

F -number of faces

E -number of edges

V -number of vertices

2D-two dimensional

3D- three dimensional

$V(x, y)$ -two variable Lagrange polynomial

$V(x, y, z)$ -three variable Lagrange polynomial

^nC_r -number of non repetitive combinations of n objects with r at a time

^nH_r -number of repetitive combinations of n objects with r at a time

B^{-1} -shape matrix

P -coordinate set of nodes with respect to X, Y coordinates

P' -coordinate set of nodes with respect to x, y coordinates

$f(X, Y)$ -raw vector of terms of the piecewise polynomial of two variables

$f(P)$ -matrix formed by substituting coordinate set of nodes to the terms of the piecewise polynomial

A -column vector of coefficients

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ABSTRACT

Polyhedron is a solid figure bounded by plane faces. Face and vertex regular polyhedra are the polyhedra whose faces are regular polygons and the arrangement of polygons around each vertex is identical. Here general equations to calculate the properties of the face and vertex regular polyhedra are developed. This includes equations for radius of the escribed sphere and internal solid angle of a vertex. Using these equations the radius of the escribed sphere of face and vertex regular polyhedrda are found including that of Snub Cube and Snub Dodecahedron. It is also shown that sphere is a limiting case of a polyhedron.

As application to finite element analysis, approximating the boundary by the sides of the finite elements is proposed. Also a method of defining the Lagrange interpolating polynomial is proposed. 2D tessellations are filling of infinite plane using polygons and 3D tessellations are filling of infinite space using polyhedra. With the piecewise polynomial selected in the above manner it is shown that the only possible regular tessellations that can be used in finite elements are Equilateral Triangle and Square in 2D and Triangular Regular Prism and Cube in 3D. It is shown in general that "any polygon having two axis of symmetry with nodes are selected at vertices cannot be used as a finite element if its Lagrange polynomial contains the complete polynomial of degree two" and "any polyhedron having a polygonal face with two axis of symmetry and having six or more number of vertices with the nodes are selected at vertices cannot be used as a finite element if its Lagrange polynomial contains a two variable complete polynomial of degree two".