

ESTIMATING PARAMETERS OF MAKEHAM'S LAW OF MORTALITY

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(07/8524)

Dissertation submitted in partial fulfillment of the degree of Master of Science in



University of Moratuwa, Sri Lanka.
Financial Mathematics
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DECLARATION

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ACKNOWLEDGEMENT

I would like to make this opportunity to express my indebtedness to my supervisor Mr. R.A.Dissanayake, senior lecturer of the Department of Mathematics, University of Moratuwa for his patience, and his precious time that he imparted with subject knowledge and generous guidance which steered me throughout this study.

It is with pleasure that I also thank Dr. M. D. T. Attygalle, Head of the Department of Statistics, university of Colombo, for her substantial assistance, in making this effort a success.

I must also thank Mr. T.M.J.A Cooray, Dr.T.S.G. Peiris and Mr. U.C.Jayathilake for their support.

I would like to offer my heartfelt thanks without any hesitation to the staff of the Department of Mathematics for strengthening me with knowledge, without which I would have not been able to accomplish this feat.

I would also thank my family for all the support extended to me in enduring this task.



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ABSTRACT

Actuaries and demographers have a long tradition of using collateral data to improve mortality estimates. Two main approaches have been used to accomplish the improvement, mortality laws and model life tables.

The most common mortality law that used to illustrate the life tables is Makeham's law. In Sri Lanka, Census and Statistics Department presents the life table and calculation of life table had been done using software package Mortpak-lite developed by united nations and the most insurance companies use UK standard life tables to construct insurance premiums. So my purpose is to construct the model life table for Sri Lanka, estimates the constants of Makeham's law of mortality that used to construct life table, and introduces a regression model that incorporates the early stages (below 20) and also wants to determine whether there's big difference for males and females.

The Makeham's estimates are based upon the equation $\mu(x) = A + BC^x$. Two methods are used to estimates the parameters, namely least square method and using the definition of age specific death rate.

The model is demonstrated the mortality data (year 2001) of Sri Lanka.



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TABLE OF CONTENTS

Declaration of the candidate	i
Declaration of the supervisor	ii
Acknowledgements	iii
Abstract	iv
Table of contents	v
List of Figures	viii
List of Tables	ix
List of abbreviations	x
Chapter 1: Introduction	1
1.1 Background	1
1.2 Focus of the research	5
1.3 Objectives	5
1.4 Overview of chapters	5
1.5 Summary	6
Chapter 2: Literature Review	7
2.1 Introduction	7
2.2 Mortality laws	7
2.3 Life tables	10
2.4 Summary	22
Chapter 3: Methodology	23
3.1 Introduction	23
3.2 Definitions	23
3.2.1 Force of mortality	23
3.2.2 Sex ratio	24
3.2.3 Mathematical expression of age specific death rate	24
3.2.4 Makeham's law	25
3.3 Direct method	25
3.3.1 Deriving a formula for age specific death rate	25
3.4 Least squares method	28
3.4.1 Linear least squares method	28
3.4.2 Application of linear least squares method	28
3.4.3 Residual analysis	30
3.4.3.1 Mean of residuals	31
3.4.3.2 Constant variance	31

	3.4.3.3 Autocorrelation	31
	3.4.3.4 Normality of error terms	32
	3.6 Life tables	33
	3.7 Summary	34
Chapter 4:	Descriptive Analysis of Mortality Data	35
	4.1 Introduction	35
	4.2 Graphical interpretation	35
	4.3 calculation of age specific death rate	36
	4.4 Summary	45
Chapter 5:	Direct Method	46
	5.1 Introduction	46
	5.2 Estimation of parameters using derived formula	46
	5.2.1 For Total population	47
	5.2.2 For males	49
	5.2.3 For Females	51
	5.3 Summary	53
Chapter 6:	Estimation of parameters using least squares method	54
	6.1 Introduction	54
	6.2 Estimation of parameters of Makeham's law for total population	54
	6.2.1 Residual Analysis	58
	6.3 Estimation of parameters of Makeham's law for Males	59
	6.3.2 Residual Analysis	62
	6.4 Estimation of parameters of Makeham's law for Females	63
	6.4.2 Residual Analysis	66
	6.5 Summary	67
Chapter 7:	Life Tables for Sri Lanka	68
	7.1 Introduction	68
	7.2 Construction of life tables	68
	7.2.1 General life table	69
	7.2.2 Life table for males`	75
	7.2.3 Life table for females`	81
	7.3 Summary	86
Chapter 8	Conclusions and Recommendations	87
	8.1 Introduction	87

8.2	Analysis of the results of the direct method	87
8.3	Analysis of the results of least squares method	88
8.4	Conclusions	88
8.5	Recommendations	89
8.6	Further analysis	89
	Reference List	90
	Appendix A1	92
	Appendix A2	100



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LIST OF Figures		Page
Figure 4.1	Graph of deaths in 2001	35
Figure 4.2	Graph of deaths for Male and female	36
Figure 4.3	Graph of age specific death rate with exponential curve	38
Figure 4.4	Graph of age specific death rate with exponential curve for age Group 23-80	39
Figure 4.5	Graph of specific death rate of Males with exponential curve	44
Figure 4.6	Graph of specific death rate of Females with exponential curve	44
Figure 4.7	Graph of age specific death rate for Males and Females	45
Figure 6.1	Fitted line plot of dependant variable	55
Figure 6.2	Unstandardized residual plot	58
Figure 6.3	Fitted line plot of dependant variable for Males	59
Figure 6.4	Unstandardized residual plot	62
Figure 6.5	Fitted line plot of dependant variable for Females	63
Figure 6.6	Unstandardized residual plot	66



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LIST OF TABLES

Page

Table 2.1	Dr. Halley's life table	11
Table 2.2	Northampton life table	12
Table 2.3	Carlisle life table	14
Table 4.1	Death rates for specific ages	37
Table 4.2	Death rates of specific ages for males and females	39
Table 7.1	General life table	69
Table 7.2	Life table for males	75
Table 7.3	Life table for females	81



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LIST OF ABBREVIATIONS

Abbreviation	Description
μ_x	Instantaneous death rate
x	Exact age
$(x, x + n)$	Age group with initial age x with the length of interval n
${}_n m_x$	Death rate for the age group $(x, x+n)$
p_x	Probability that a person aged (x) survived in $(x, x+1)$
q_x	Probability that a person aged (x) died in $(x, x+1)$
l_x	Number of survivors at age x in a life table with radix (starting population) of 100,000 persons
l_{x+1}	Expected number of survivors of age (x)
d_x	Expected number of deaths in age group $(x, x+1)$
L_x	Total expected number of years lives between ages x and $x+1$
T_x	Total expected number of years lived beyond age x , by survivorship group with l_0 initial numbers
e_x	Expected number of years of future lifetime of an individual of the l_x survivors of the group at age x
AD	Anderson Darling statistics
R^2	Coefficient of determination
R^2_{adj}	Adjusted Coefficient of determination
SS_{Reg}	Regression sum of squares
SS_{total}	Total sum of squares
SSR	Residual sum of squares
n	Sample size
p	Number of parameters