

PORTFOLIO OPTIMIZATION USING QUADRATIC PROGRAMMING



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Master of Science.



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DECLARATION

I hereby certify that this dissertation does not incorporate any material previously submitted for a Degree or Diploma in any University, without acknowledgement, and to the best of my knowledge and belief it does not contain any material previously published or written by another person or myself except where due reference is made in the text.

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ABSTRACT

Investment analysis is concerned, portfolio optimization is very important in order to get maximum profit. In the proposed research the optimization will be done in two main steps. The first part is the modelling mean variance so called reward and risk. The second part is finding optimum solution. The data set published by the Colombo Stock Exchange was used for this research paper as the raw data. The following five companies are selected for the analysis without biases those are Commercial bank, John Keells, Lanka Hospital, The Sri Lanka Telecom and The United motors. These companies represent several fields in the Sri Lankan market such as banking, group of companies, health service, semi government companies, automobile sector.

The objective of the research is to find the optimum allocation of the portfolio. The risk should be minimized and the reward should be maximized at the same time. As a strategy to do both of these simultaneously, the linear combination with controlling arbitrary constant is used. That particular linear combination is a convex quadratic function. In order to find the solution of this, the numerical method is used via MATLAB inbuilt 'm file'.

The developed model of the Markowitz portfolio optimization model¹ could be formulated in order to find the optimum allocation of investment amounts for any number of investment channels. The model can be used by investment researchers and could be applied to gain an analytical idea about the efficient frontier. The model has a parameter that can change emphasis on risk minimization or reward maximization.

The portfolio optimization finds the optimum allocation of money to be invested. The optimum allocation depends on several factors, according to Markowitz, the return as well as risk, should be considered simultaneously. The main model for this research is 'Markowitz Portfolio Selection Model'. The objective function of the above model consists a linear combination of risk and return. Since the risk is a quadratic expression, the objective function can also be considered as a quadratic function. Then the normal optimization can not be applied and the non linear optimization (quadratic optimization) must be applied. The main constraint that can be identified is the budgetary constraint along with other limitations, such as boundary restraints. The model has the advantage of changing the budget at any time and the user can use the total budget as a unit, then the optimum allocation fractions, for each investment can be found. The optimization calculation is carried out through 'Matlab', computer aided calculation software.

The output of the optimization model is the ratio of the total investment amount to be allocated, the allocated in the percentages of the total portfolio for Commercial Bank, John Keells, Lanka Hospital, Sri Lanka Telecom and United Motors respectively as 0%, 0%, 62%, 38%, and 0%. The minimum function value is -0.0907, and the function stands for the linear combination of the risk and the reward.

¹ Harry Markowitz (1952, 1959) developed his portfolio-selection technique, which came to be called modern portfolio theory (MPT). Prior to Markowitz's work, security-selection models focused primarily on the returns generated by investment opportunities. Standard investment advice was to identify those securities that offered the best opportunities for gain with the least risk and then construct a portfolio from these.

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L. P. Ranasinghe



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