2 LITERATURE REVIEW

2.1 Introduction

The heights of the manmade structures have rapidly increased in the past decades due to urbanization and limited land availability. On one side, these tall structures are slender. On the other hand, the wind velocities are increasing with the height and the effect to the tall structures by the wind is significant. Hence the tall structures are subjected to large amounts of cyclic vibrations in their lifespan. Steel is a preferred material for tall structures as its strength to weight ratio is much less than other materials and other benefits. But one of the main problem with the choice of steel for tall structures is the possibility of fatigue damage. Hence tall steel structures should be designed giving much attention to fatigue.

2.2 Phases of fatigue life

Fatigue failure of materials occur in 3 stages. First the cracks nucleated on microscopically small scale (Crack initiation). Then crack propagates in the direction perpendicular to the direct stresses (Crack propagation). Finally rapid fracture happens nearby the end of the crack (fracture).

Earlier fatigue theories considered the fatigue as a single failure and related engineering stresses in the component to fatigue life. Modern fatigue analysis identifies the 3 stages separately (Figure 2.1). Those phases are analysed as below (STL, 2002)

1. Crack initiation – Assuming Local strains and stresses is the cause
2. Crack propagation- Assuming Stresses in the component is the cause
3. Rapid Fracture- Using Fracture mechanics

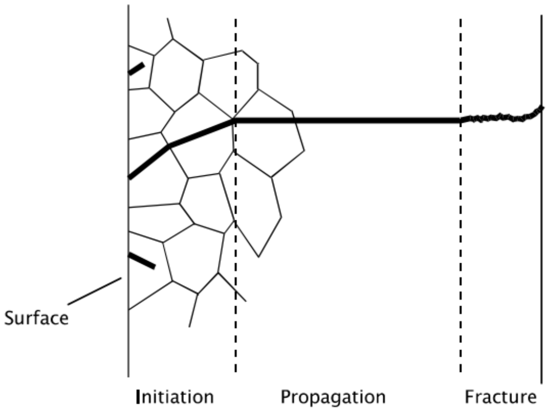


Figure 2.1: Phases of fatigue life

The fatigue life of a component (N_f) is considered as the number of cycles to 1st (N_{in}) and 2nd (N_p) stages stated above since the 3rd stage is a rapid one. The relationship is shown below.

$$Nf = Nin + Np \qquad \text{..............................(2.1)}$$

2.3 Fatigue induce loading on tall structures

Tall buildings are relatively slender hence often flexible. The first few natural frequencies of these structures are relatively low therefore the dynamic stresses generated by dynamic loading will be significant. Dynamic loading such as wind and earthquake induced loading are the main stresses induced in typical tall steel structures and these have to be considered when designing for fatigue.

2.3.1 **Wind induce loading**

Wind is a dynamic and random phenomenon in both time and space. The along wind dynamic effect is determined by gust factor method. This method is used in Canadian and Australian codes. Gust factor relates the peak to mean response in terms of an equivalent static design load. Equivalent static wind load distributions are those loadings that produce the correct expected values of peak load effects, such as bending moments, axial forces in all members, or deflections, generated by the fluctuating wind loading. The effective peak loading distributions associated with the mean wind loading, the fluctuating quasi-static or background response and the resonant response are identified, and combined to give a total effective peak wind loading distribution. The approach can be applied to any type of structure. (Holmes, 2004)

2.3.2 **Earthquake induce loading**

In recent years, much attention is being paid to earthquake loading because of the high seismicity level of many regions where the tall structures were built. Earthquake induced load conditions on steel structures are quite different from other fluctuating loading conditions. Because cyclic loading on structures due to earthquakes involves many fewer cycles (typically less than ten cycles) than conventional low-cycle fatigue and strains that are well in excess of yield. Such conditions can be termed as ultra-low-cycle fatigue (ULCF). (Kanvinde & Deierlein, 2004) Seismic analysis of a structure can be a dynamic analysis or static analysis. In static analysis equivalent static load analysis can be carried out. In dynamic analysis response spectrum analysis or time history analysis can be carried out. The response spectrum analysis method yields much more accurate results than the equivalent static approach and it is essential for flexible structures where dynamic effects are significant. This increase in accuracy is largely due to combining specific vibratory modes from the structure with the spectral accelerations determined for the site.

2.4 Endurance curves (S-N curve)

Endurance curve shows the relationship between the stress level in a component and the number of cycles to failure under that stress. Series of tests of specimen under constant amplitude cyclic loading of different intensities are done to produce these S-N curves. A typical S-N curve is shown in the figure 2.2 below.

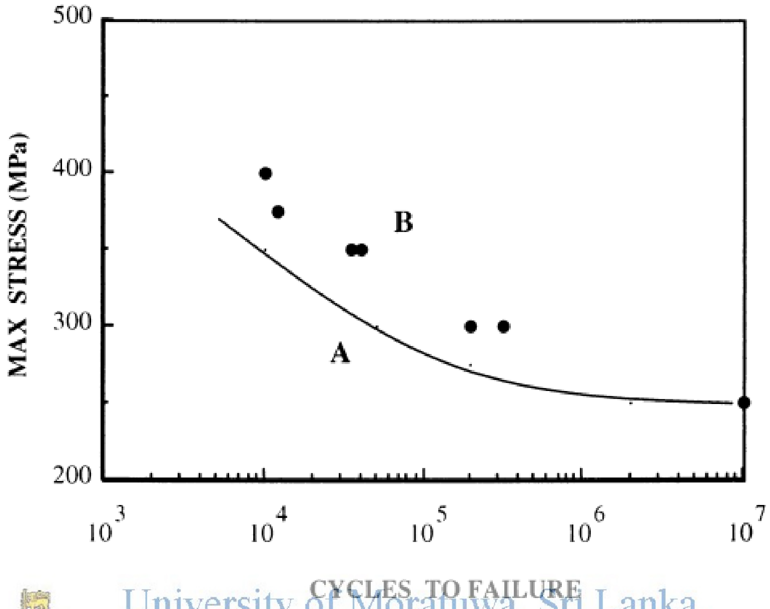


Figure 2.2: An example S-N curve

The number of cycles to failure rapidly reduces with the increasing stress amplitude as shown in the figure below. The slope of this curve reduces with increasing number of cycles to failure and finally it becomes almost parallel to x axis. This stress value is referred to as the endurance limit .If the stress levels are less than the endurance limit it is assume that no failures will occur under any number of cycles. But recent research has showed that this endurance limit is a false phenomenon and every stress level has a certain number of cycles to failure. (Bathias, 1999) Still endurance limit is used for steel as the slope of the curve is negligible.

2.4.1 **Generalised Fatigue data**

Fatigue endurance curves are produced to a certain component should be able to generalise to use for other shapes, different loading histories and other materials.

2.4.1.1 Generalisation for loading

Common structures are subjected to varying loads in their lifespan. The method adopted to use constant amplitude S-N curves to analyse complex loading histories is called the Palmgren-Miner cumulative damage hypothesis or Miner’s rule. It assumes that the each successive cycle generates additional damage which accumulates in proportion to the number of cycles until failure occurs. (Holmes, 2004).

Miners rule is given by the following equation,

$$\sum\left(\frac{n_i}{N_i}\right) = 1 \ldots\ldots\ldots\ldots\ldots \text{(2.2)}$$

Where *ni* is the number of stress cycles of the considered amplitude and *Ni* is the number of cycles required to cause failure under the same amplitude.

If we can estimate the number of cycles and amplitude levels of the designed structure in its lifetime, endurance curves can be used to calculate the miner sum to check whether fatigue failure happens or not. If the loading history is a complex, a method like Rain flow cycle counting can be used considering stress.

2.4.1.2 Generalisation for shape

Fatigue failure of a component often initiate from a place where change of the section is present (holes, grooves and fillet radii). This is because those features produce local stress and strain concentrations. Some local plasticity must occur in the loading history to initiate a fatigue crack. Modern fatigue theorise such as critical location, local stress-strain relate fatigue endurance to local stress concentrations. (STL, 2002). Conventional S-N curve analysis uses a relationship between engineering stresses and fatigue life (Engineering–stress method).

The local stress distribution is different to nominal stress distribution in components with varying section such as shown in figure 2.4. But typical fatigue tests have been performed for smooth cylindrical specimens. Those test results can be used for components with complex local stress distributions by using stress concentration factors. Elastic stress concentration factor K_t is defined as,

$$\mathbf{Kt} = \frac{\textit{local stress at notch}}{\textit{nominal stress}} \ldots\ldots\text{(2.3)}$$

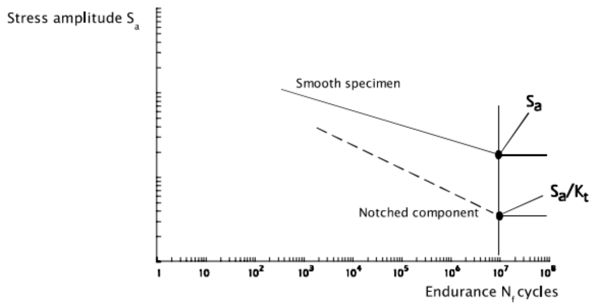


Figure 2.3: SN curves for notched and smooth specimen

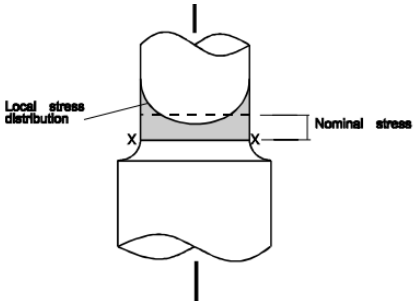


Figure 2.4: Local stress distribution and nominal stress of a component

Stress concentration factors for large number of engineering details have been published. Elastic stress concentration factors can be used to modify the endurance limit as shown in figure 2.3 since there will be little plasticity at the endurance limit stress amplitude. Calculation of notch fatigue strength at a lower amplitude is complex as the plastic stresses will be there. Both local strain and engineering stress fatigue methods contain equations for calculating the effect of notches

2.4.1.3 Generalisation for mean stress

Tests of high mean stresses show shorter fatigue lives. Most standard fatigue tests are done for zero mean stress constant amplitude cyclic vibrations. Mean stress correlation factors are available for load ratios (R) given by,

$$Load\ ratio\ (R) = \frac{Pmin}{Pmax} \ldots\ldots\ldots\ldots\ldots\ldots (2.4)$$

Change of SN curve for different load ratios is shown in the figure 2.5 below,

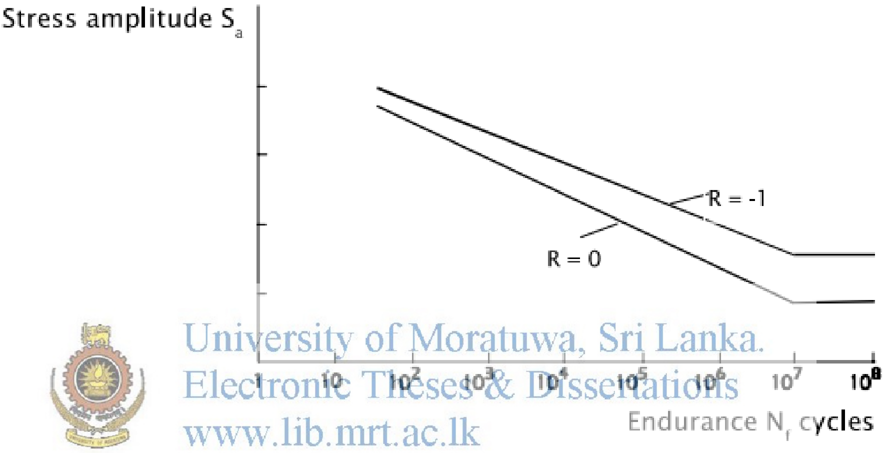


Figure 2.5: SN curves for different load ratios

2.4.1.4 Generalisation for other factors

Crack initiation is a surface phenomenon. Any process which affects the surface will have a significant impact on fatigue strength. They are,

- Quality of the surface finish-ground: machined/rolled/as-forged/as-cast/grinded
- Surface treatment: planting/cladding
- Residual stresses
- Operating environment: temperature/corrosive environment

The cyclic frequency of the applied loading and the waveform can be ignored in many calculations. Smooth specimen test data is almost always obtained from the tests on uniaxial loaded specimens. Special methods must be adopted when these data are used for biaxial stresses.

Laboratory endurance strength ($S_{e'}$) of the materials obtain from S-N diagram (or the likes) are therefore corrected for actual conditions by using correction factors as shown below,

$S_e = K_a x K_b x K_c x K_d x K_e x K_f x S_e$..................(2.5)

Where,

K_a = Surface Correction factor

K_b = Size Correction factor

K_c =Reliability Correction factor

K_d = Temperature Correction factor

K_e = Stress concentration Correction factor

K_f = Miscellaneous Correction factor

$S_{e'}$ = Endurance Strength of material specimen under laboratory condition

S_e = Endurance Strength of material specimen under actual running condition

(Azeez, 2013)

2.5 Uniaxial strain life fatigue analysis

Local strain life fatigue analysis presumes that the local stress concentration stain life behaviour is similar to a larger uniform with uniform stresses and strains similar to that. Local strain life methods are suitable for Finite element models because the stress strain relationship at all locations are known.

2.5.1 True stress and strain

When a cylindrical test specimen is loaded in tension or compression its length and cross sectional area changes. There are 2 concepts as true stress/strain and engineering stress/strain. They are defined by the equations 2.6 to 2.9 below,

$$\textbf{Engineering stress} = \frac{\textit{Applied load}}{\textit{Original cross sectional area}} \;\ldots\ldots\ldots\text{(2.6)}$$

$$\textbf{True stress} = \frac{\textit{Applied load}}{\textit{Actual cross sectional area}}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\text{(2.7)}$$

$$\textbf{Engineering strain} = \frac{\textit{Total change in gauge length}}{\textit{Original gauge length}}\ldots\ldots,\ldots\;\text{(2.8)}$$

$$\textbf{True strain} = \int \frac{\textit{Instantaneous change of gauge length}}{\textit{Instantaneous gauge langth}}\;\ldots\ldots\ldots\text{(2.9)}$$

True stress strain curve obtained from a single load application is called a monotonic curve.

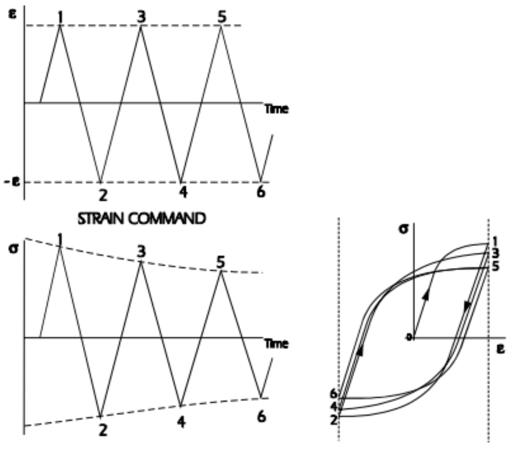
Ramberg and Osgood proposed true stress strain relationship defined as,

$$\boldsymbol{\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K}\right)^{\frac{1}{n}}}\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\textbf{(2.10}$$

Where,

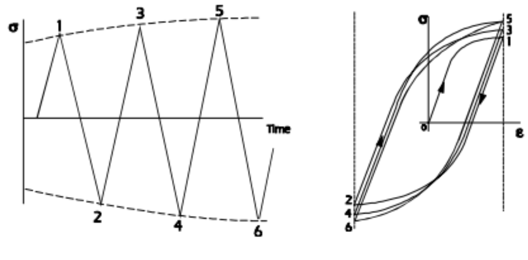
E- The elastic modulus

K- The strain hardening coefficient
n- The strain hardening exponent



stress response cyclic stress-strain

Figure 2.7: Cyclic softening



stress response cyclic stress-strain

Figure 2.6: Cyclic hardening

When a material is cyclically loaded it hardens (figure 2.7) or softens (figure 2.6) at first but often comes to a stable hysteresis loop in which the same response is shown in all cycles (figure 2.8).

The curve constructed through the tips of the stable hysteresis loops at different strain amplitudes is called the *stable cyclic stress-strain curve*. It is represented by,

$$\boldsymbol{\varepsilon = \frac{\sigma}{E} + \left(\frac{\sigma}{K'}\right)^{\frac{1}{n'}}} \ldots\ldots\ldots \ \mathbf{(2.11)}$$

Where K’ is the cyclic strain hardening coefficient and n’ is the cyclic strain hardening exponent. Other symbols have the usual meaning.

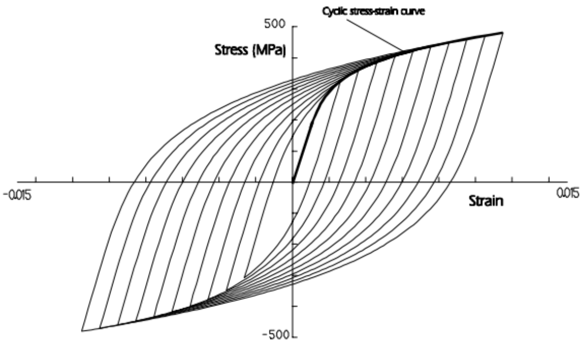


Figure 2.8: Stabilized cyclic response

2.5.2 Low cycle fatigue and high cycle fatigue

When significant plastic straining occurs in a component it is considered as subjected to low cycle fatigue since it fails in lower number of cycle. The fatigue life of the material is less dependent on the stress level but the strain. The analytical procedure used to address low cycle fatigue behaviour is referred to as strain life method. (Hossain & Ziehl, 2012) Manson and Coffin showed that the relationship between plastic strain amplitude and endurance can be expressed as given below,

In high cycle fatigue the stresses and strains are largely confined to the elastic range.

$$\frac{\Delta\varepsilon_p}{2} = \varepsilon'_f(2N_f)^c \quad \text{………........…..(2.12)}$$

Therefore the number of cycles to failure is high (typically more than 100,000). However the fatigue endurance in high cycle fatigue is less dependent on the strain but highly dependent on the stress level. The analytical procedure used for these type of fatigue is called as stress life method (Hossain & Ziehl, 2012) Basquin proposed the relationship between elastic stress amplitude and endurance as given below,

$$\frac{\Delta\sigma}{2} = \sigma'_f(2N_f)^b \quad \text{………………….(2.13)}$$

Combing the 2 equations the total (elastic & plastic) strain-life relationship (figure 2.9) is defined as,

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f}{E}(2N_f)^b + \varepsilon'_f(2N_f)^c \quad \text{….…(2.14)}$$

Local strain-life analysis is a fatigue crack initiation criterion. There are several correlations such as Morrow's mean stress correlation to take account of the mean stress effect.

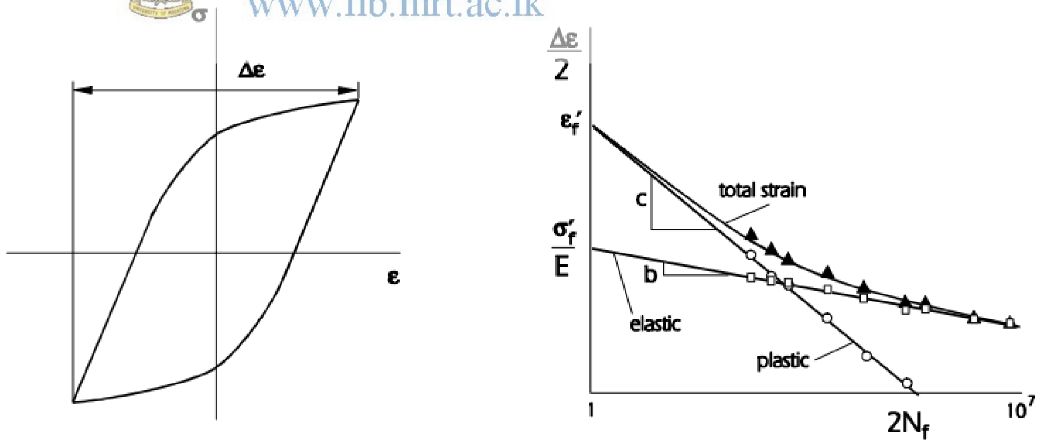


Figure 2.9: Strain life Relationship

The strain excursions of a cyclic strain applied are modelled using the cyclic stress-strain curve and hysteresis loop (figure 2.10) equation. Then actual fatigue cycles in that loading cycle can be identified. Rain flow cycle method too can be used for this. The strain ranges of these fatigue cycles are used in the strain life equations to find the fatigue life for each of the cycle. Then miners rule

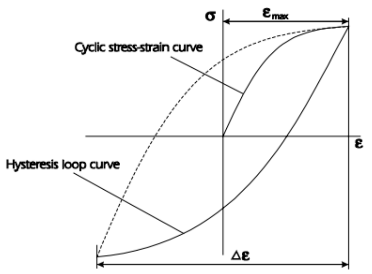


Figure 2.10: Hysteresis loop

can be applied to add the damage of all the fatigue cycles and find the number of cycles of loading the component can survive. Similar method is used in FE-safe software. (STL, 2002)

Morrow's mean stress correction is

$$\frac{\Delta\varepsilon}{2} = \frac{\sigma'_f - \sigma_m}{E}(2N_f)^b + \varepsilon'_f(2N_f)^c \quad \text{………….. (2.15)}$$

The hysteresis loop shape is defined by

$$\Delta\varepsilon = \frac{\Delta\sigma}{E} + 2\left(\frac{\Delta\sigma}{2K'}\right)^{\frac{1}{n'}} \quad \text{…………...………….. (2.16)}$$

2.6 Multi-axial fatigue

2.6.1 Introduction

Many actual structures experience multi-axial fatigue, bending, torsion as well. Use of uniaxial methods for multiracial fatigue may give overestimates of the life. The fatigue cracks are usually initiated from the surface. Combination of in-plane stresses and out-of-plane stresses on the surface creates tri-axial stress distribution. Multi-axial fatigue theories concentrate on this condition. Different criterions have been proposed by researchers.

2.6.2 Brown miller combined strain criterion

The brown-miller equation assumes that the maximum fatigue damage occurs on the plane which experience the maximum shear strain amplitude, and that the damage is a function of both this shear strain and the strain normal to this plane.

Complete Brown-Miller strain-life equation is,

$$\frac{\Delta\gamma_{max}}{2} + \frac{\Delta\varepsilon_n}{2} = 1.65\frac{\sigma'_f}{E}(2N_f)^b + 1.75\,\varepsilon'_f(2N_f)^c \quad \text{………….(2.17)}$$

The brown-miller equation gives the most realistic life estimates for ductile metals.

Considering the effects of the mean stress, if morrows mean stress correlation is used with the brown miller equation the equation transforms to,

$$\frac{\Delta\gamma_{max}}{2} + \frac{\Delta\varepsilon_n}{2} = 1.65\frac{(\sigma'_f - \sigma_{n,m})}{E}(2N_f)^b + 1.75\,\varepsilon'_f(2N_f)^c \quad \text{……..(2.18)}$$

Where $\Delta\gamma_{max}$ is the maximum shear strain range and $\Delta\varepsilon_n$ is the range of strain normal to the maximum shear strain. Other parameters have same meanings as the uniaxial strain life equation.

Methods like critical plane method can be used with brown miller equation to consider the critical planes when complex loading is applied. Critical plane method resolve the strain in to number of planes, and calculate the damage in each plane.

2.7 Codes of practices

Calculation of wind induced fatigue according to Australian codes requires calculation of number of stress cycles, stress ranges for each cycle. Finally fatigue analysis can be done according to the AS4100 code using the derived data.

Calculation of the number of cycles: The probability of the wind speed being in a required band may be calculated from hourly mean wind speeds. Most wind data can be fitted to a Weibull distribution. These probabilities can be transferred to number of stress cycles in the design life.

Calculation of stress ranges for each wind speed band: Suitable methods should be used for along wind and cross wind response. AS1170.2 (1989) describes gust factor method which is suitable for along wind response estimation. Cross wind response can be estimated using sinusoidal lock-in and random excitation models. If the structure is too complicated, wind tunnel testing may be required.

Fatigue analysis: Australian code AS4100 defines a concept called the detailed category (f_{rn}) for different components which means the fatigue strength at 2x10^6 cycles on the S-N curve. Detail category considers local stress concentration details, size and the shape of the maximum acceptable discontinuity, residual effects, and the welding effects.

The miners rule is used to calculate the combined effect of the all stress ranges found from wind analysis. The following flow chart clearly shows the method used in AS4100. (Mendis & Dean, 2000). See Appendix 3 for the methodology adopted in AS 4100 to estimate the fatigue life.