# EFFECT OF END ANCHORAGE ON CFRP STRENGTHENED CONCRETE BEAMS

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Degree of Masters of Science

Department of Civil Engineering

University of Moratuwa Sri Lanka

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Dissertation submitted in partial fulfilment of the requirements for the degree Master of Science

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### **DECLARATION**

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#### **ABSTRACT**

Reinforced concrete structures are often being subjected to modifications and improvements during their service life. The main causes for improvements are design errors, changes in use, degradation due to corrosion of reinforcing steel, damage due to seismic loads, vehicular impact and excessive wear and excessive loading. Precautions for these issues are mainly in two types; repair and strengthening. Restoring the structures which became structural malfunction is known as repair. Improvements done in structures in order to achieve higher service loads or longer service lives are known as strengthening. As far as strengthening techniques are concerned, concrete jacketing, steel jacketing, precast concrete jacketing, prestressed concrete jacketing and external application of Fibre reinforced polymer (FRP) composite materials are the available upgrading methods.

Structural strengthening with fibre reinforced polymers is a popular strengthening technique worldwide, due to its extensive advantages. The important properties of FRP's are high strength, light weight, good rigidity, corrosion resistance, high elastic modulus etc. FRP's are used to improve structures by means of increasing flexural capacity, enhancing shear capacity and confining concrete columns to improve axial compression load carrying capacity. When flexural strengthening with FRP is concerned, research studies show that a significant strength increment can be achieved with use of CFRP sheets as an external reinforcement. It also improves serviceability of structures.

Failure of a CFRP strengthened beam for flexure can be due to flexure, shear, concrete crushing or debonding. The failure modes can be categorized in to two main types; classical failure and premature failure. Failure of an element due to yielding of steel bars, tensile failure of FRP sheets and crushing of concrete in compression zone are known as classical failure. Failure of element in any other method such as debonding of FRP, peeling off of FRP and concrete cover separation are premature failure modes.

End debonding is the most common failure mode which has been experienced in practice. This mode of failure, limits the capacity by 60% to 80% of ultimate capacity (Mostofinejad 2014, Xiong 2007) of the system and induce sudden failure without prior warning. Different methods have been proposed in literature to delay end debonding. They are Mechanical fasteners, FRP pin and pan shape anchors, Near Surface Mount reinforcement, End wraps and use of wire mesh—epoxy composite. Among these techniques, end wraps are more beneficial since it contribute to shear capacity of the beam and help to improve ductility apart from preventing debonding failure. Although these techniques are advantages, they are not popular in the industry due to lack of technical data to quantify the effects.

Previous research studies emphasise the need of proper design method to predict the strength enhancement gained due to end wraps. There are few studies (Sawada et al,2003 Hawileh et al. 2013) carried out to investigate the interaction between resistance to debonding and the strength gain. Moreover, studies conducted in tropical countries are even less. This has lead to less confidence of using this technique by practicing engineers. Although there are several design guides available on design of externally bonded FRP systems, none of these guides address the effect of end anchorage on flexural strength gain. This study investigates the effect of end anchors on enhancing flexural capacity of reinforced concrete members, flexural strengthened with CFRP sheets.

An extensive experimental program was carried out using reinforced concrete beams to understand the failure behaviour, stress distribution, deflection behaviour and flexural strength enhancement. It was observed that 98.53% strength increment could achieved by the

specimens flexural strengthened with CFRP external reinforcement over control specimens. When the flexural strengthened beams were anchored at the ends, the strength enhancement was 145% compared with that of unstrengthened beams. It was also observed that strain levels at the ends of longitudinal CFRP strips reduces significantly, when end wrap anchorage is provided. End debonding can be fully prevented by providing sufficient amount of end anchorage. The failure mode of beams changed from cover debonding to CFRP rupture, in existence of end wraps.

A new theoretical model was developed based on experimental observations, design guidelines and data collected literature. It is capable of predicting both failure load and failure mode of flexural strengthened and end anchored beams. The model was compatible with experimental results of current study as well as experimental results collected from literature.

Two papers were published from the work of this study and are attached in appendix E.

**Keywords:** CFRP/concrete, flexural performance, de-bonding failure, end anchorage

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### LIST OF ABBREVIATIONS

### **Abbreviation Description**

ACI American Concrete Institute

AFRP Aramid Fibre Reinforced Polymer

BRE Building Research Establishment

CEB Comité euro-international du béton

CFRP Carbon Fibre Reinforced Polymer

EBR Externally Bonded Reinforcement

EMI Electromagnetic Interference

FIB Fédération internationale du béton

FIP Fédération internationale de la précontrainte

FRP Fiber Reinforced Polymer

GFRP Glass Fiber Reinforced Polymer

GI Galvanized Iron

HIT High Heat Treatment

HM High Modulus

HT High Tensile

IHT Intermediate Heat Treatment

IM Intermediate Modulus

JCI Japanese concrete institute

LHT Low Heat Treatment

NSMR Near Surface Mount Reinforcement

RC Reinforced concrete

SHT Super Heat Tensile

UHM Ultra High Modulus

## LIST OF SYMBOLS

Symbol	Description	Units
$A_f$	$nt_f w_f$ , area of FRP external reinforcement	$[mm^2]$
$A_{fv}$	area of FRP shear reinforcement	$[mm^2]$
$A_{s}$	area of non pre-stressed steel reinforcement	$[mm^2]$
$a_s$	cover to steel tension reinforcement	
$a_u$	effective bond length	
$A_{sv}$	provided area of shear reinforcement	$[mm^2]$
$A_{s1}$	area of tensile steel reinforcement	$[mm^2]$
$A_{s2}$	area of compressive steel reinforcement	$[mm^2]$
b	width of beam	[mm]
$b_f$	width of the FRP strip	[mm]
$b_w$	minimum width of cross section over the effective depth	[mm]
С	distance from extreme compression fiber to the neutral .	[mm]
	axis	
$C_1$	Factors obtained through calibration of test results (for CFRP strips, the value is 0.64)	
	Factors obtained through calibration of test results (for	
$C_2$	CFRP strips, the value is 2)	
d	effective depth of the member	[mm]
$d_f$	depth of FRP shear reinforcement	[mm]
•••	distance from centroid of compressive steel to extreme	[mm]
$d_2$	compressive fibre	[111111]
$E_f$	modulus of elasticity of FRP	$[N/mm^2]$
$E_{fu}$	elastic modulus of FRP in the principal fibre orientation	$[N/mm^2]$
$E_{s2}$	modulus of elasticity of steel	$[N/mm^2]$
$f_c$	prism compressive strength of concrete	$[N/mm^2]$
$f_{ctm}$	mean value of the concrete tensile strength	$[N/mm^2]$
$f_{cu}$	characteristic strength of concrete	$[N/mm^2]$
Jcu	onaracionismo suongui oi conciete	[14/111111]

$F_d$	tensile force before the debonding of CFRP sheets	
$f_f$	stress level in the FRP reinforcement	$[N/mm^2]$
$f_{fe}$	effective stress in the FRP	$[N/mm^2]$
$f_s$	stress in non pre-stressed steel reinforcement	$[N/mm^2]$
$F_t$	force in main tension reinforcement	[N]
$F_u$	effective shear force	[N]
$f_y$	characteristic strength of main reinforcement	$[N/mm^2]$
$f_{yd}$	design value of the steel yield strength	$[N/mm^2]$
$f_{yv}$	characteristic strength of shear reinforcement	$[N/mm^2]$
$G_f$	interfacial fracture energy	[Nmm]
h	overall thickness of a member	[mm]
$k_b$	geometry factor	
$k_c$	factor accounting for the state of compaction of concrete	
l	length of specimen	[mm]
$l_{b,max}$	maximum anchorable length	[mm]
$L_e$	active bond length of FRP laminate	[mm]
$L_{ue}$	effective bond length of FRP shear wraps	[mm]
Μ	expected maximum moment	[N mm]
$M_n$	nominal moment strength	[N mm]
$M_{Rd}$	resisting design moment	
$N_{fa,max}$	maximum anchorable FRP force in tensile steel reinforcement	
P	externally applied force	[N]
$S_f$	spacing FRP shear reinforcing	
$t_f$	thickness of FRP	
$V_f$	nominal shear strength provided by FRP stirrups	
W	applied external point load	[N]
w	uniformly distributed self-weight of concrete element	[N/mm]
x	depth of the compression zone	
α	reduction factor, (approximately equal to 0.9)	

α	angle between principal fibre orientation and longitudinal	
	axis of member	
$eta_1$	ratio of the depth of the equivalent rectangular stress	
	block to the depth of the neutral axis	
$\delta_f$	interfacial slip	
$\delta_G$	stress block centroid coefficient	
$\mathcal{E}_f$	FRP strain	
$\varepsilon_{fd,e}$	design value of effective FRP strain	
$\varepsilon_{s2}$	compressive steel strain	
$\theta$	angle of diagonal crack with respect to the member axis	
$ au_f$	interfacial shear stress at failure	
$ au_{max}$	maximum interfacial shear stress	
$arphi_f$	additional FRP strength-reduction factor	
	a factor depends on $\delta_f,\tau_f,$ elastic modulus of FRP and	
λ	thickness of FRP	