

**DEVELOPMENT OF A THEORETICAL PACKING MODEL  
INCORPORATING THE EFFECT OF VIBRATION, SHAPE AND  
SURFACE TEXTURE**

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Thesis submitted in partial fulfilment of the requirements for the  
degree of Doctor of Philosophy in Civil Engineering

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## **DECLARATION**

“I declare that this is my own work and this thesis does not incorporate without acknowledgement any material previously submitted for a Degree or Diploma in any other University or institute of higher learning and to the best of my knowledge and belief it does not contain any material previously published or written by another person except where the acknowledgement is made in the text.

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The above candidate has carried out research for the Doctoral thesis under my supervision.

Name of the supervisor: Prof. W.K. Mampearachchi

Signature of the supervisor:

Date:

## **Abstract**

Determination of packing density of a particulate mixture is still an open problem for researchers and scientists. The complex and random nature of particle behavior in a mixture and effect of various external factors have made it more and more complicated to develop theoretical and analytical models to predict the packing density. This study focused on the effect of vibration frequency, particle shape and surface texture on packing density. Initially, laboratory experiments were carried out to determine the use of packing concepts in concrete mixture design for interlocking concrete block pavers (ICBP). The approach found to be successful. However, determination of packing density of aggregate mixtures in laboratory was time consuming and difficult. Hence, the use of packing models to determine the packing density was studied. Validity of existing packing models for the aggregate mixtures was studied and as a result the 3-parameter model was found to be the only model that incorporates loosening effect, wall effect and wedging effect and the percentage error of 3-parameter model found to be lesser than that of Toufar model and compressible packing model. Hence, the 3-parameter model was selected for the modification. The results obtained from experiments were then analyzed and relationships were developed isolating the effect of vibration, surface texture and particle shape. Three effects were combined, and the packing density variations were obtained to incorporate the effects and modify the 3-parameter model. The packing density and vibration shows a 3<sup>rd</sup> order polynomial behavior while shape and surface texture shows a linear relationship with packing density. The developed model was validated for more than 300 independent data. The behavior of loosening effect, wall effect and wedging effect with vibration, surface texture and shape were also analyzed. The wall effect is affected by both surface texture and vibration frequency. The loosening effect is affected only by particle shape and the wedging effect does not affect by any of these factors.

Key words: Packing density, Vibration, Shape, Surface texture, Packing model

## **DEDICATION**

*To my mother who could not get a better sleep for the last 28 years.*

*To my father who went through all the trouble to made me who I am today.*

*To my wife who encouraged and supported me through thick and thin.*

*To my supervisor who believed in me and empowered me to achieve this dream.*

*To people of Sri Lanka whom I owe a debt of gratitude for paying for my education.*

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## LIST OF ABBREVIATIONS

<b>Abbreviation</b>	<b>Description</b>
ICBP	Interlocking concrete block pavers
DEM	Discrete element models
3D	Three dimensional
UHPC	Ultra high-performance concrete
W/C	Water/Cement
CSF	Cement spacing factor
LPDM	Linear packing density model
SSM	Solid suspension model
CPM	Compressible packing model
3PM	Three parameter model
SA	South Africa
IS	Indian Standards
SLS	Sri Lanka Standards
FHWA	Federal highway administration
BPN	British pendulum number
SN	Skid number
MTD	Mean texture depth