# MODELING NONLINEAR SUSPENSION SYSTEM FOR IMPROVED BRAKING DISTANCE ESTIMATION

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## ABSTRACT

Braking distance of a vehicle is an important safety consideration in highway geometric design. Braking distances have been commonly estimated based on vehicle wheel loads and assumed tire-pavement friction. The use of classic vehicle dynamics simulation model, which simplifies tire stiffness as linear elastic function, is a main approach to estimate tire-pavement friction and predict braking distances. The interaction of nonlinear vehicle dynamics with pavement surface roughness has not been considered in analyzing its impact on vehicle braking distance. The impact of pavement roughness induced vehicle vibration on braking distance is the topic of interest in this paper. A nonlinear vehicle dynamics simulation model is proposed where tire stiffness is considered as a nonlinear elastic function in the analysis of vehicle dynamics. The model is implemented in MATLAB 11.0. A hypothetical example is given to illustrate the possible difference between models with linear and nonlinear tire stiffness in calculating braking distance on a wet pavement. Although the model presented is rather simplified considering only nonlinear tire stiffness, and a more elaborate simulation model is required to examine in detail the actual impact of considering nonlinear vehicle dynamics, the example does show that further study is necessary to examine the need for vehicle dynamics simulation in order to reliably predict vehicle braking distances on highways, taking into account the effect of pavement roughness.

**Keywords**: Braking distance, pavement roughness, suspension system, nonlinear tire stiffness

## 1. INTRODUCTION

Braking distance calculation is a topic of importance in safety design of highway geometry. An issue of interest related to this topic is the impact on vehicle dynamics and pavement surface roughness on road vehicle braking distance. Pavement roughness is generally defined as an expression of the surface irregularity. It causes vehicle vibration to be transferred to passengers from vehicle seats and could reduce the ride quality. In addition, the induced vehicle vibration on road could deteriorate the interaction between tire and pavement, resulting in a longer braking distance for vehicles (Purdue Research Foundation, 1974). It is the objective of this paper to investigate the effect of pavement roughness and vehicle dynamics on vehicle braking distance.

In the field of interaction between vehicle dynamics and pavement roughness, research can be classified into two types: experiment and simulation (Pacejka and Besselink, 2012). Experimental studies are useful in providing statistical relationships between vehicle dynamics and pavement roughness. However, being statistical and empirical in nature, the applicability of such relationships is limited to the test conditions of the experimental studies. They do not provide detailed insight into the effect of pavement roughness to vehicle dynamics. Probably the most successful approach has been to produce a limited number of field tests to provide actual vehicle performance data to validate simulation models (Wang et al., 1993; Sun and Deng, 1998). These simulation models are then used to extend experimental results over the range of test conditions.

Until now, many different vehicle dynamics simulation models have been developed for different purposes by researchers. For example, Sun (2002) suggested a walking-beam suspension model to design "road-friendly" vehicle with the recognition of pavement loads. Shi and Cai (2009) proposed a three-dimensional vehicle-pavement coupled model to simulate pavement dynamic loads. Barbosa (2009) used a quarter-car model to identify safe traffic speed limits. These models mainly consider two main aspects: vehicle suspension and tire simulation. Pavement roughness has not been included in these models.

The vehicle suspension system includes wishbones, the spring, and the shock absorber to transport and filter all force and energy between vehicle and pavement. The spring bears the vehicle body to be isolated from road disturbances and thus contributes to driving quality. The damper contributes to both driving safety and comfort. Its task is the damping of body and wheel oscillations, where the avoidance of wheel oscillations directly refers to drive safety. Tire simulation consists of the tire vertical stiffness and damper to simulate tire vertical vibration. In the existing literatures, tire vertical stiffness usually considered as a continuous and proportional way to the required displacement like a linear manner. The behavior of this configuration is demonstrated by the isolation or suppression of the harmonic inputs in the stationary regime. However, when the tire is exposed to severe vehicle dynamics, this configuration is not the one that shows the moderate relationship between tire load and displacement. It is because vertical stiffness has a linear behavior in a local region under steady state, but has strong nonlinear behavior in the whole vibration region from small contact (low load) to full contact (over load) in the experimental measurement (Cenek et al., 2012).

Motivated by the necessity to find a more reliable approach to predict ride comfort and safety, in this study the nonlinear tire stiffness is investigated and its effect on the dynamic response of vehicles under the excitation of pavement roughness is analyzed. The modeling and the behavior of two tire stiffness configurations, one with linear tire stiffness and the other one with nonlinear tire stiffness, are presented. Both types of tire stiffness were analyzed on the wet pavement under different profiles of pavement roughness and simulated in a quarter-car model.

The remainder of this paper is organized as follows: In Section 2, the models with linear and nonlinear tire stiffness respectively are employed for the simulations in the quarter-car model. The simulation results and performance evaluation are shown in Section 3 for the comparison between the two models. Finally, Section 4 provides some concluding remarks and suggestions for future works.

## 2. MODELING OF VEHICLE SUSPENSION SYSTEM

The model used in this paper is a quarter-car model for describing the vertical motions of vehicle. The body and suspensions' vertical movements are described by quarter-car model shown in Figure 1 with two degrees of freedom. $m_2$ represents the mass of sprung vehicle body with suspension stiffness $k_2$ and damping $c_2$. $m_1$ represents the mass of tire with tire stiffness $k_1$ and damping $c_1$. $u$ represents the pavement elevation. $v$ represents the vehicle speed. The vertical movements of the sprung and unsprung masses are represented by $x_1$ and $x_2$ respectively.

Based on the classical mechanical vibration theory, the vibration equation for the system described in Figure 1 can be written as follows:

$$m_1\ddot{x}_1 + c_1(\dot{x}_1 - \dot{u}) + k_1(x_1 - u) - c_2(\dot{x}_2 - \dot{x}_1) - k_2(x_2 - x_1) = f_1 \qquad (1)$$

$$m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_1(x_1 - u) + k_2(x_2 - x_1) = f_2 \qquad (2)$$

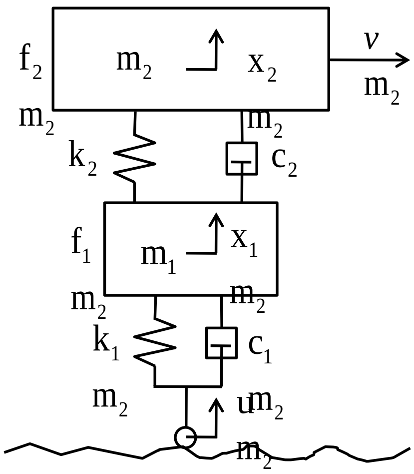



Figure 1. Quarter-car suspension model over rough road surface

Set the state variables of the system to be

$$z_1 = x_1 - u \qquad (3)$$

$$z_2 = x_2 - x_1 \qquad (4)$$

The following equation of the vehicle suspension model can further be obtained:

$$m_1(\ddot{z}_1 + \ddot{u}) + c_1\dot{z}_1 + k_1 z_1 - c_2\dot{z}_2 - k_2 z_2 = 0 \qquad (5)$$

$$m_2(\ddot{z}_1 + \ddot{z}_2 + \ddot{u}) + c_2\dot{z}_2 + k_2 z_2 = 0 \qquad (6)$$

$$c_1\dot{z}_1 + k_1 z_1 = Fc \qquad (7)$$

The contact force between tire and pavement produced by stiffness spring. The tire stiffness spring is considered as two states namely linear and nonlinear throughout this paper. In the linear function, the tire stiffness coefficient is assumed as a constant $k_1$. For simplification purposes, the nonlinearity of tire stiffness is described by a triple piecewise function where the tire stiffness coefficient depends on the sign of the displacement. In order to perform the switching of the tire stiffness coefficients among three different loading states namely under-load, normal-load and over-load representing the nonlinear properties of the tire vertical stiffness, a triple piecewise type function is used as follows:

$$k_1(x_1 - u) = \begin{cases} k_1^+ & \text{if } (x_1 - u)k_1^* > f_1 \\ k_1^* & \text{if } -f_2 < (x_1 - u)k_1^* < f_1 \\ k_1^- & \text{if } (x_1 - u)k_1^* < -f_2 \end{cases} \qquad (8)$$

where $k_1^+$ is the over-loading tire stiffness; $k_1^*$ is the normal-loading tire stiffness; $k_1^-$ is the under-loading tire stiffness; $f_1$ and $f_2$ are threshold forces.

## 3. BREAKING DISTANCE ESTIMATION BASED ON VEHICLE DYAMICS MODELING

By applying to the Newtown's second motion law to vehicle dynamics on the horizontal motion prediction, for a vehicle traveling at a speed of $V_0$, the locked-wheel braking distance (D) required on a road with a constant friction coefficient and a highway grade is given by Eq. (9)

$$D = \int_{v=V_0}^{v=0} \frac{v}{a} \mathrm{d}v \qquad (9)$$

where $V_0$ = initial vehicle speed when the brake is applied; $a$ = rate of deceleration;

It is known that when a vehicle drives on pavement, the contact force between tire and pavement will vary with vehicle vertical vibration under the excitation of pavement roughness. The relationship between the vehicle dynamics and pavement roughness can be obtained from the simulation analysis described in the preceding section as shown in the Equation (10).

$$a = \frac{Fc * u}{m_1 + m_2} \pm Gg \tag{10}$$

with μ = friction coefficient; G = highway grade; and g = acceleration attributed to gravity.

In this study, it is assumed that friction coefficient is a constant value and highway grade is equal to zero. The tire damper effect is not considered. The braking distance as defined in Eq. (9) is computed in terms of numerical integration with a time step of 0.001 s.

## 4. HYPOTHETICAL EXAMPLE

To illustrate the application of the proposed model with nonlinear tire vertical stiffness to improve the vehicle dynamics estimation under excitation of pavement roughness, a numerical analysis is presented in this section. The simulation results of the proposed model with nonlinear tire vertical stiffness are compared with those obtained from the classic model with linear tire vertical stiffness to demonstrate the applicability of the proposed method.

Pavement roughness is applied to model excitation. It considers the profile irregularities of road surface along the traveling direction. Barbosa (2011) measured the pavement roughness in the field as shown in Figure 2. Based on the classification standard by ISO (1995), the spectral density of the measured pavement roughness is equivalent to the ISO quality Level C (ISO 1995) for wavelengths longer than 5 m. It is used in this paper as vehicle excitation base $u(t)$ (Level 1). For illustration purposes, we generate another five roughness excitations of different amplitudes derived from the measured one, given as $1.2u(t)$ (Level 2), $1.4u(t)$ (Level 3), $1.6u(t)$ (Level 4), $1.8u(t)$ (Level 5) and $2.0u(t)$ (Level 6).

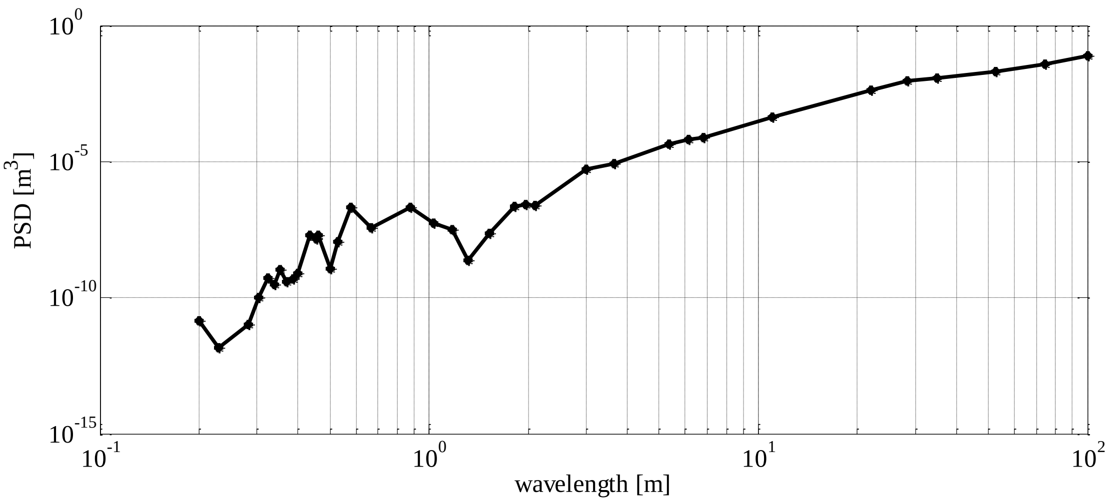



Figure 2. Power Spectral Density Roughness of Measured Pavement (Barbosa, 2011)

The parameters of the model selected for this study are listed in Table 1 based on the data from Barbosa (2011).

**Table 1** Parameters of Vehicle Suspension System

| Characteristics | Symbol | Value | Unit |
|---|---|---|---|
| Vehicle Mass | $m_2$ | 370 | $kg$ |
| Suspension Stiffness | $k_2$ | 18250 | $N/m$ |
| Suspension Damping | $c_2$ | 1025 | $Ns/m$ |
| Tire Mass | $m_1$ | 80 | $kg$ |
| Tire Stiffness | $k_1$ | 80000 | $N/m$ |
| Over-Loading Tire Stiffness | $k_1^+$ | 120000 | c |
| Normal-Loading Tire Stiffness | $k_1^*$ | 80000 | $N/m$ |
| Under-Loading Tire Stiffness | $k_1^-$ | 60000 | $N/m$ |
| Up threshold force | $f_1$ | 400 | $N$ |
| Low threshold force | $f_2$ | 400 | $N$ |

The relationships between the relative displacement and vertical load based on the proposed tire stiffness are clearly illustrated in Figure 3, where nonlinear tire stiffness increase with vertical load.

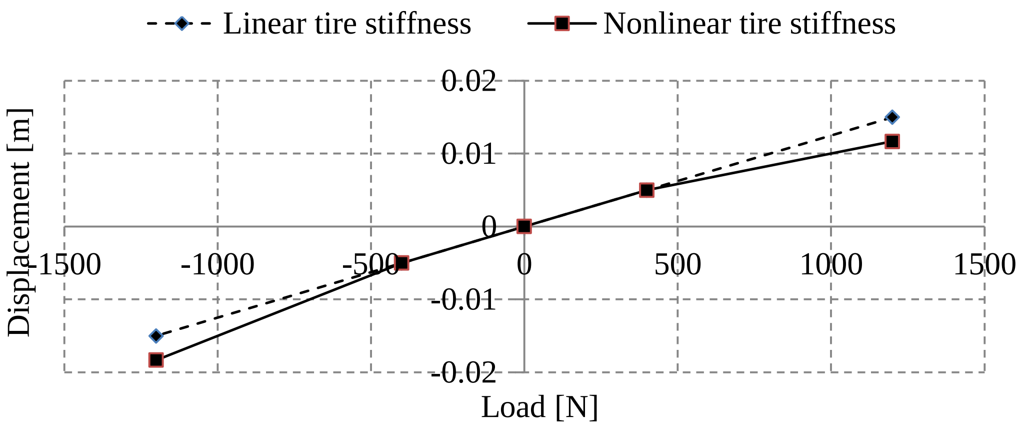



Figure 3. Relationship between load and displacement

On the wet pavement, skid resistance varies with vehicle speed for a given thickness of water-film. The relationship between skid resistance and vehicle speed with water-film thickness of 0.5 mm, as shown in Figure 4, has been verified both experimentally and theoretically by researchers (Anderson et al., 1998; Ong and Fwa, 2007; Rose and Gallaway, 1977). It will be used in the present example for braking distance estimation. It should be highlighted that this relationship between skid resistance and vehicle was obtained for case of sliding locked-wheel on a plane pavement surface without macrotexture-induced tire vibration, and may not be strictly applicable to the present case simulating dynamic tire motions under the influence of pavement roughness. The relationship is used in the present example for illustration purpose to examine the possible differences in braking distance calculation using linear and non-linear tire stiffness models.

To evaluate the dynamic performance of the vehicle suspension system, the root mean square values of acceleration (RMS acc) of tire are defined as follows:

$$J_{RMSacc} = \sqrt{\frac{1}{T}\int_0^T \ddot{x}_1^2(t)\mathrm{d}t} \qquad (11)$$

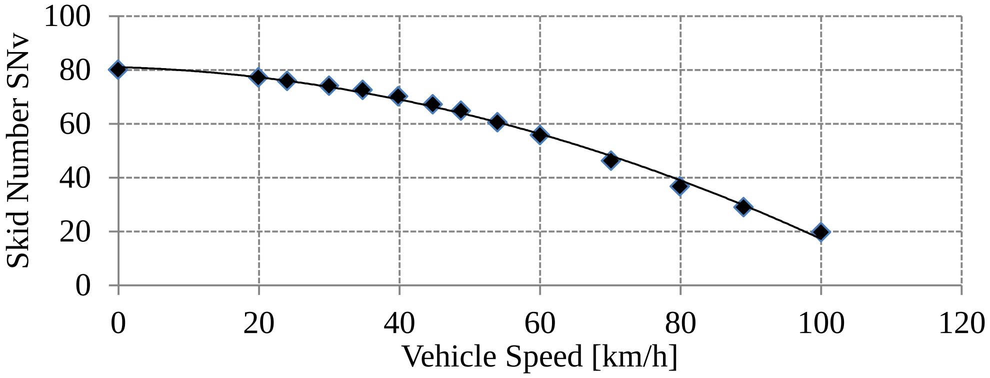
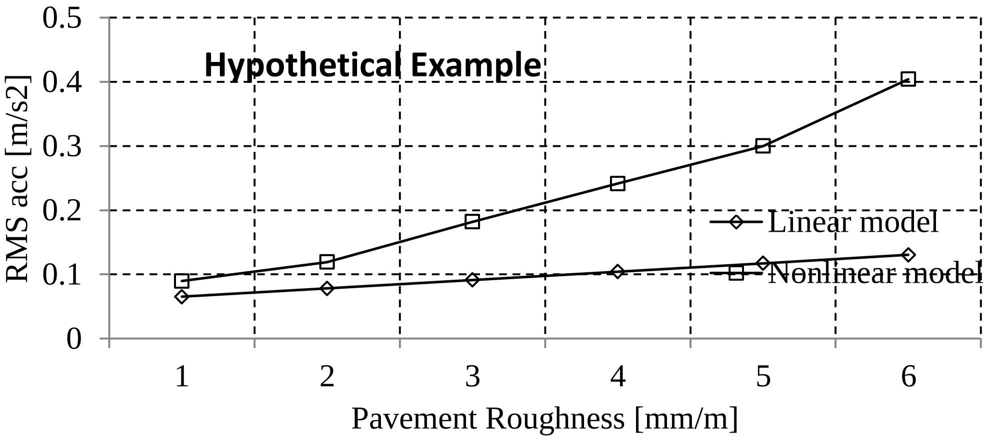
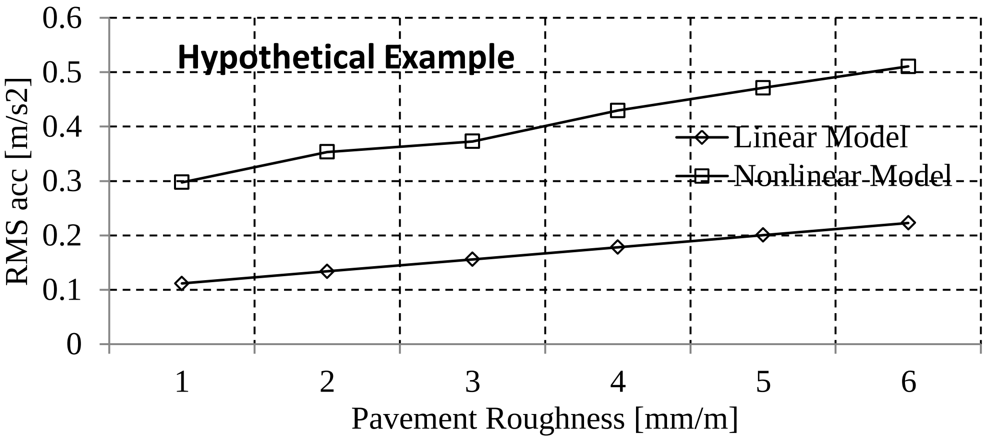



Figure 4. Effect of vehicle speed on wet pavement skid resistance (Ong and Fwa, 2007)

The simulation results of the proposed nonlinear model are compared with those obtained by the linear model. The variations of RMS acceleration under different roughness excitations for the linear and nonlinear models at the vehicle speed of 30 and 50 km/h are plotted in Figure 5. It could be found that the RMS acceleration increases with pavement roughness level for both linear and nonlinear model. The nonlinear model predicts higher vehicle vibration under the same operation conditions. It is also noted that the RMS acceleration increases with vehicle speed and pavement roughness for both models. Vehicle speed has a more significant effect on vehicle vibration than pavement roughness.

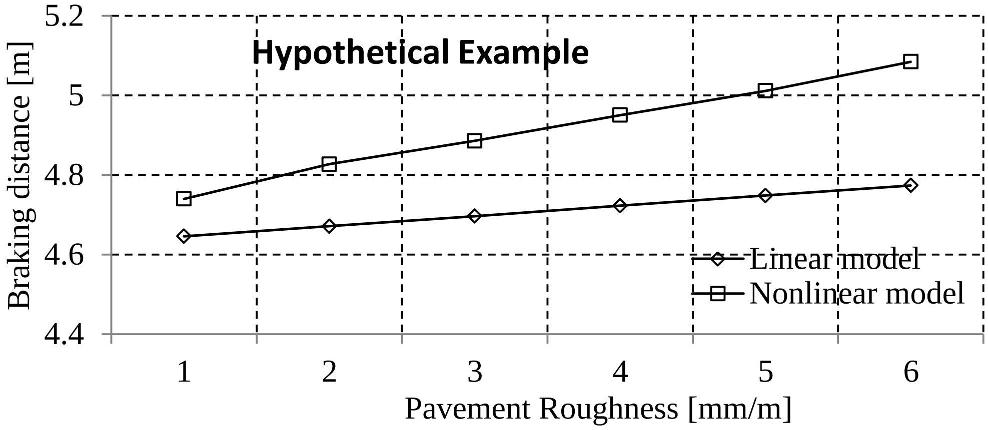



(a) 30 km/h

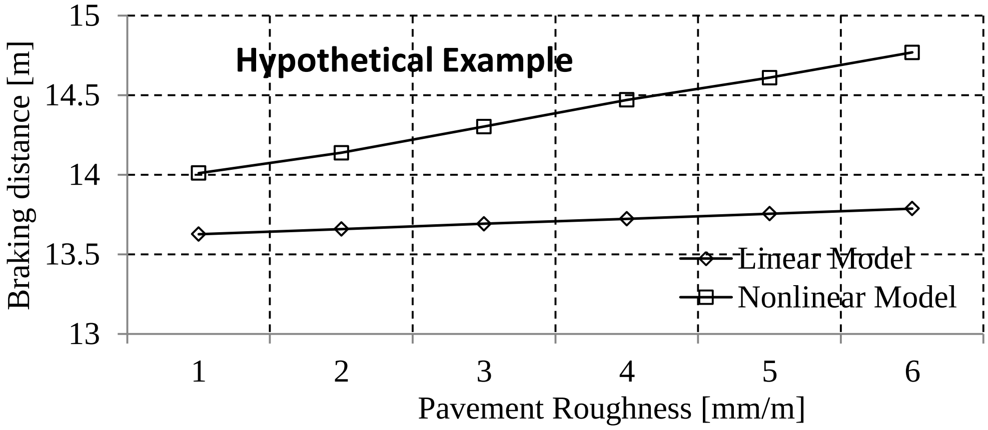



(b) 50 km/h

Figure 5. RMS acceleration variation under different roughness excitations




(a) 30 km/h




(a) 50 km/h

Figure 6. Braking distances under different roughness excitations and initial vehicle speeds on wet pavement

Figure 6 plots the braking distance estimated using the linear and nonlinear models based on Equations 5-9 for various pavement roughness and different initial vehicle speeds. For both the linear and nonlinear models, the braking distance increases with pavement roughness. The rate of increase with roughness appears to increase in the case of nonlinear model. It is also observed that nonlinear model predicts longer braking distance than that by the linear model. The difference of braking distance estimation by between both models increases with pavement roughness level and vehicle speed. It should be emphasized that these findings are based on the hypothetical data assumed in this study, and are yet to be validated by more rigorous study and field experiments.

## 4. CONCLUSIONS

This paper presented the development of a nonlinear vehicle dynamics simulation model for vehicle vibration prediction under pavement roughness excitation. The proposed method is applied to simulate vehicle vertical vibration using a quarter-car simulation model. The model is used to predict the vehicle braking distance on wet pavements using hypothetical parameters and data. Compared with the linear model, the proposed nonlinear model estimated more severe vehicle vibrations. The simulation results suggested that the classic linear model under-estimates vehicle braking distance. In this paper, only the nonlinearity of tire stiffness is considered. The proposed parameter values may not be strictly suitable for vehicle dynamics estimation. Further calibration and validation of the proposed model are required.

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