

STUDY OF DEEP BEAMS USING FINITE ELEMENT APPROACH

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DECLARATION

I declare that this is my own work and this thesis does not incorporate without acknowledgement any material previously submitted for a Degree or Diploma in any other University or institute of higher learning and to the best of my knowledge and belief it does not contain any material previously published or written by another person except where the acknowledgement is made in the text.

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Date: 18th December 2017

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The above candidate has carried out research for the Masters under my supervision.

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Prof I.R.A Weerasekera

ABSTRACT

Beams are common structural elements in most structures and generally they are analysed using classical beam theories to evaluate the stress and strain characteristics of the beam. But in the case of deep beams, higher order shear deformation beam theories predicts more accurate results than classical beam theories due its more realistic assumption regarding the shear characteristics of the beam.

In this study a hyperbolic shear deformation theory for thick isotropic beams is developed where the displacements are defined using a meaningful function which is more physical and directly comparable with other higher order theories. Governing variationally consistent equilibrium equations and boundary conditions are derived in terms of the stress resultants and displacements using the principle of virtual work. This theory satisfies shear stress free boundary condition at top and bottom of the beam and doesn't need shear correction factor.

.A displacement based finite element model of this theory is formulated using the variational principle. Displacements are approximated using the homogeneous solutions of the governing differential equations that describe the deformations of the cross-section according to the high order theory, which includes cubic variation of the axial displacements over the cross-section of the beam. Also, this model gives the exact stiffness coefficients for the high order isotropic beam element. The model has six degrees of freedom at the two ends, one transverse displacement and two rotations, and the end forces are a shear force and two components of end moments.

Several numerical examples are discussed to validate the proposed shear deformation beam theory and finite element model of the beam theory. Results obtained for displacements using the present beam theory and the finite element model are compared with results obtained using other beam theories, 2D elastic theory and 2D and 3D finite element models. Solutions obtained using the proposed beam theory and finite element model are in close agreement with the solutions obtained using 2D elastic theory and 2D and 3D finite element models of 'ABAQUS'.

Keywords: Beam theory; Finite Element; Higher order; Shear Deformation.

DEDICATION

To My Parents

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TABLE OF CONTENT

Declaration	i
Abstract	ii
Dedication	iii
Acknowledgement	iv
Table of Content.....	v
List of figures	vii
List of Tables	ix
Notations	x
1. INTRODUCTION	1
1.1. Background	1
1.2. Overview of the finite element method (FEM).....	2
1.3. Objectives.....	4
1.4. Methodology	4
1.5. Outline of this Thesis	5
2. LITERATURE REVIEW	7
2.1. Review of beam theories.....	7
2.2. Review of finite element models	11
2.3. Theoretical aspects related to formulation of finite element.....	15
2.3.1. Strong forms Vs. Weak forms	15
2.3.2. Variational methods and variational principle	16
2.3.3. Weighted residual method.....	16
2.3.4. Euler - Lagrange equations and boundary conditions	18
2.3.5. Finite element formulation using minimum total potential energy principle.....	21

3.	THEORETICAL FORMULATION	23
3.1.	Assumptions made in the theoretical formulation	24
3.2.	Displacement field	24
3.3.	Governing equations and boundary conditions.....	25
3.3.1.	The stress resultant-displacement relations.....	25
3.4.	General solutions for static flexure of beams.....	28
4.	FINITE ELEMENT FORMULATION	30
4.1.	Derivation of shape functions	30
4.2.	Construction of weak form.....	34
4.3.	Finite element model.....	35
4.3.1.	Stiffness matrix terms.....	37
4.3.2.	Load vector.....	39
4.4.	Using the finite element model for 2D frame analysis.	39
4.4.1.	Transformation from local system to global system	41
5.	RESULTS AND DISCUSSION	43
5.1.	Validation of the proposed theory.....	43
5.1.1.	Numerical examples I.....	43
5.1.2.	Comparisons and findings of the theoretical solutions.....	54
5.2.	Validation of the finite element model.....	55
5.2.1.	Numerical examples II	57
6.	CONCLUSIONS AND FUTURE WORKS	71
6.1.	Conclusions	71
6.2.	Future work.....	72
	REFERENCES.....	73
	APPENDIX A	xii
A.1.	MATLAB program for stiffness matrix	xii

A.2. MATLAB program to analyse the general beam problem	xiv
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LIST OF FIGURES

Figure 2.1-Kinematics of various beam theories	10
Figure 2.2 - Typical function $y(x)$ and the approximate solution $y(x)$	19
Figure 3.1-The beam under consideration	23
Figure 4.1-Genralized displacements and generalised forces on a typical element	37
Figure 4.2-Generalised displacements in the local and global coordinates ...	41
Figure 5.1-simply supported beam problem	43
Figure 5.2 -The coordinate system for 2D elasticity solution for example 1(a)	44
Figure 5.3-Transverse displacement along centre line for example 1(a).....	45
Figure 5.4-Shear strain along centre line for example 1(a).....	45
Figure 5.5-Shear stress variation across the depth at $x = 0$ for example 1(a)	46
Figure 5.6-Axial stress variation across the depth at $x = 0.5L$ for example 1(a)	46
Figure 5.7-Transverse displacement along centre line for example 1(b).....	47
Figure 5.8-Shear strain variation along centre line for example 1(b)	47
Figure 5.9-Transverse displacement along centre line for example 1(c).....	48
Figure 5.10-Shear strain variation along centre line for example 1(c)	48
Figure 5.11-Shear stress variation across the depth at $x = 0$ for example 1(c)	49
Figure 5.12-Axial stress variation across the depth at $x = 0.5L$ for example 1(c)	49
Figure 5.13-Cantilever beam problem	51
Figure 5.14-Transverse displacement along centre line for example 2(a).....	52
Figure 5.15-Shear strain along centre line for example 2(a).....	53
Figure 5.16-Transverse displacement along centre line for example 2(b).....	53
Figure 5.17-Shear strain along centre line for example 2(b)	54

Figure 5.18 -Transverse displacement along centre line using present FE for example 1(a).....	57
Figure 5.19 -Transverse displacement along centre line using	58
Figure 5.20 -Transverse displacement along centre line using present FE for example 1(a).....	58
Figure 5.21-Shear strain along centre line using.....	59
Figure 5.22-Transverse displacement along centre line using present FE for example 1(c).....	60
Figure 5.23- Transverse displacement along centre line using ABAQUS for example 1(a).....	61
Figure 5.24- Transverse displacement along centre line using ABAQUS for example 1(b)	62
Figure 5.25- Shear strain along centre line using ABAQUS for example 1(a).....	62
.....	
Figure 5.26- Shear strain along centre line using ABAQUS for example 1(b).....	63
.....	
Figure 5.27-Stepped beam problem	64
Figure 5.28 - Shear strain along centre line for example 4	65
Figure 5.29 - Transverse displacement along centre line for example 4	65
Figure 5.30 - Results for tranverse displacement and shear strain uisng ‘ABAQUS’ 2D model.....	66
Figure 5.31- Continuos beam probelem.....	67
Figure 5.32 -Transverse displacement along centre line for example 5	68
Figure 5.33 - Shear strain along centre line for example 5	68
Figure 5.34 - Results for tranverse displacement and shear strain uisng.....	69
Figure 5.35 - Axial stress variation for thin beam	70
Figure 5.36 - Axial stress variation for deep beam	70

LIST OF TABLES

Table 5.1. Maximum value for displacements and stresses for example 1(a)	
.....	.50
Table5.2. Maximum value for displacements and stresses for example 1(b)	
.....	..50
Table5.3. Maximum value for displacements and stresses for example 1(c)	
.....	..50

NOTATIONS

A - area of cross section of beam.

b - width of beam.

D - nodal degree of freedom.

D^e - nodal degree of freedom in local coordinate system.

E - elastic modulus.

f - force vector due to distributed load.

F - force vector due to concentrated load.

f^e - force vector due to concentrated load in local coordinate system.

F^e - force vector due to concentrated load in local coordinate system.

G - shear modulus.

h - depth of beam.

I - second moment of area about centroidal axis.

K_s - shear correction factor.

L - length of beam element.

N_i - shape function.

$p(x)$ - axially distributed load

$q(x)$ - transverse distributed load.

S - element stiffness matrix.

S_e - stiffness matrix in local coordinate system.

T - transformation matrix.

U - strain energy.

$u(x)$ - axial displacement at centre line of beam.

$u(x, z)$ - axial displacement at coordinate (x, z) .

V - work done by the external forces.

$w(x)$ - transverse displacement of beam at centre line.

$w(x, z)$ - transverse displacement at coordinate (x, z) .

$\frac{dw}{dx}$ - bending rotation of cross section.

$\phi(x)$ - total rotation of cross section.

$\phi(z)$ - function which describes the distribution of transverse shear stress along the thickness of beam.

Ω - problem domain.

Γ - boundary of domain.

Π - total potential energy.

ε_{xx} - normal strain.

γ_{xz} - shear strain.

σ_{xx} - normal stress.

τ_{xz} - shear stress