STUDY OF DEEP BEAMS USING FINITE ELEMENT APPROACH

Sriharan Jasotharan

168015U

Degree of Master of Science

Department of Civil Engineering

University of Moratuwa

Sri Lanka

December 2017

STUDY OF DEEP BEAMS USING FINITE ELEMENT APPROACH

Sriharan Jasotharan

168015U

Thesis submitted in partial fulfilment of the requirements for the degree
Master of Science in Civil Engineering

Department of Civil Engineering

University of Moratuwa

Sri Lanka

December 2017

DECLARATION

I declare that this is my own work and this thesis does not incorporate without acknowledgement any material previously submitted for a Degree or Diploma in any other University or institute of higher learning and to the best of my knowledge and belief it does not contain any material previously published or written by another person except where the acknowledgement is made in the text.

Also, I hereby grant to University of Moratuwa the non-exclusive right to reproduce and distribute my thesis, in whole or in part in print, electronic or other medium. I retain the right to use this content in whole or part in future works (such as articles or books)

……………………………………… Date: 18th December 2017

S.Jasotharan

The above candidate has carried out research for the Masters under my supervision.

……………………………………… Date: 18th December 2017

Prof I.R.A Weerasekera

ABSTRACT

Beams are common structural elements in most structures and generally they are analysed using classical beam theories to evaluate the stress and strain characteristics of the beam. But in the case of deep beams, higher order shear deformation beam theories predicts more accurate results than classical beam theories due its more realistic assumption regarding the shear characteristics of the beam.

In this study a hyperbolic shear deformation theory for thick isotropic beams is developed where the displacements are defined using a meaningful function which is more physical and directly comparable with other higher order theories. Governing variationally consistent equilibrium equations and boundary conditions are derived in terms of the stress resultants and displacements using the principle of virtual work. This theory satisfies shear stress free boundary condition at top and bottom of the beam and doesn't need shear correction factor.

.A displacement based finite element model of this theory is formulated using the variational principle. Displacements are approximated using the homogeneous solutions of the governing differential equations that describe the deformations of the cross-section according to the high order theory, which includes cubic variation of the axial displacements over the cross-section of the beam. Also, this model gives the exact stiffness coefficients for the high order isotropic beam element. The model has six degrees of freedom at the two ends, one transverse displacement and two rotations, and the end forces are a shear force and two components of end moments.

Several numerical examples are discussed to validate the proposed shear deformation beam theory and finite element model of the beam theory. Results obtained for displacements using the present beam theory and the finite element model are compared with results obtained using other beam theories, 2D elastic theory and 2D and 3D finite element models. Solutions obtained using the proposed beam theory and finite element model are in close agreement with the solutions obtained using 2D elastic theory and 2D and 3D finite element models of 'ABAQUS'.

Keywords: Beam theory; Finite Element; Higher order; Shear Deformation.

DEDICATION

To My Parents

ACKNOWLEDGEMENT

I would like firstly to express my deep gratitude to my supervisor Prof. Ruwan Weerasekera, for his guidance, support and encouragement during the past year. He opened a door for me to a fascinating academic world. With the wide vision on the science and engineering, he always can find an exciting research direction. He motivated to think independently and guided me wherever I needed to keep me on the track in achieving my research goals. Talking with him was always enjoyable and also inspiring, not only in the academics but also the life.

I would like to thank Dr. Chinthaka Mallikarachchi and Dr. Rangika Halwatura for their valuable feedbacks on my research which have been very helpful.

I would like to give some special thanks to my colleagues and friends. Mr. Ubamanyu and Mr. Dharmadasa who helped me whenever I face difficulties with ‘ABAQUS’ software in getting solutions. Also their feedbacks on my presentations and research papers related this work helped me to improve my work further.

I would like to thank Department of Civil Engineering, for providing the facilities and resources required during this period which helped me to carry out my research in a good environment. I also acknowledge Senate Research Committee (SRC), University of Moratuwa for their financial support.

Last but not least, I would like to express my gratitude to my parents for their understanding. I owe everything to my parents for what I have become.

S.Jasotharan
18.12.2017

TABLE OF CONTENT

LIST OF FIGURES

LIST OF TABLES

NOTATIONS

A - area of cross section of beam.

b - width of beam.

D - nodal degree of freedom.

D^e - nodal degree of freedom in local coordinate system.

E - elastic modulus.

f - force vector due to distributed load.

F - force vector due to concentrated load.

f^e - force vector due to concentrated load in local coordinate system.

F^e - force vector due to concentrated load in local coordinate system.

G - shear modulus.

h - depth of beam.

I - second moment of area about centroidal axis.

K_s - shear correction factor.

L - length of beam element.

N_i - shape function.

p(x) - axially distributed load

q(x) - transverse distributed load.

S - element stiffness matrix.

S_e - stiffness matrix in local coordinate system.

T - transformation matrix.

U - strain energy.

u(x) - axial displacement at centre line of beam.

u(x, z) - axial displacement at coordinate (x, z).

V - work done by the external forces.

w(x) - transverse displacement of beam at centre line.

w(x, z) - transverse displacement at coordinate (x, z).

$\frac{dw}{dx}$ - bending rotation of cross section.

$\phi(x)$ - total rotation of cross section.

$\Phi(z)$ - function which describes the distribution of transverse shear stress along the thickness of beam.

Ω - problem domain.

Γ - boundary of domain.

Π - total potential energy.

ε_{xx} - normal strain.

γ_{xz} - shear strain.

σ_{xx} - normal stress.

τ_{xz} - shear stress