

**FORECASTING REFERENCE
EVAPOTRANSPIRATION DURING YALA AND MAHA
SEASONS IN DRY ZONE SRI LANKA: A STATISTICAL
APPROACH**

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Master of Science in Business Statistics

Department of Mathematics

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Declaration of the candidate

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“I have supervised and accepted this dissertation for the submission of the Master Degree”

Signature of the supervisor:

Prof. T. S. G. Peiris

Date

Dedication

“I dedicate this thesis to my husband, parents, supervisors and lecturers and to all of those who are willing to gather knowledge about application of statistical knowledge to forecast reference evapotranspiration during Yala and Maha seasons in dry zone of Sri Lanka.”

Acknowledgment

I am extremely grateful to my supervisor, Prof. T.S.G. Peries, Professor in Applied Statistics and Head, Department of Mathematics, for his valuable guidance, instructions provided and dedication, motivated direct supervision, understanding, and very strong commitment towards this study that played a pivotal role in the success of this endeavour and the staff of the Department of Mathematics of University of Moratuwa for facilitating this work.

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Abstract

Sri Lanka is heavily dependent on both rain-fed and irrigated agriculture and thus irrigation has had a unique contribution towards country's agro economy from history to this date. The established patterns of rainfall in different parts of the country have changed and the demand for agricultural water has to be balanced with the municipal and industrial water demand. The improved procedures for estimating agricultural water requirements both for irrigation and rain-fed agriculture have become an important research particularly due to erratic rainfall patterns and inadequate water resources in dry season. The aim of this study is therefore to develop time series models to predict weekly reference evapotranspiration (ET_0) for Yala and Maha seasons in Polonnaruwa district using climate data from 2010 to 2015. As actual evapotranspiration is not available, those values on weekly basis were computed using Pan Evaporation method based on relative humidity, wind speed and pan evaporation. 85% of the data computed were used for training and balance of 15% was kept for validation. The weekly evapotranspiration during Yala varied from 2.23mm (6 – 12 September 2013) to 5.37mm (1 – 7 May 2015) with mean of 3.62mm and SD of 0.53 and that during Maha varied from 0.76mm (21 – 27 December 2012) to 5.56mm (17 – 23 October 2014) with mean of 2.29mm and SD of 0.85. Both series were able to make stationary by taking one short-term difference and one long-term difference with the length of 26. The identified best fitted ARIMA models for Yala and Maha weekly evapotranspiration were SARIMA (1,1,1) (1,1,1)₂₆. The errors produced by two models were found to be white noise. The percentage errors in both models for validation data set were within the range of $\pm 3\%$ and it was found that the correlations between observed and predicted values for Yala ($r=0.90$) and for Maha ($r = 0.88$) were highly significant ($p<0.05$). The best fitted model identified for the pooled weekly series was SARIMA (0,1,2) (0,1,1)₅₂. Though the errors found to be satisfied all the diagnostic tests, the percentage error was higher in the combined model than the corresponding values for two separate models. Therefore, it is recommended to use the developed separate models to forecast ET_0 on short-term or long-term basis which will be useful for the appropriate water management for real time irrigation scheduling in Dry Zone of Sri Lanka. These models can also be used for estimating irrigation water requirements for different crops. It is suggested to use Artificial Neural Network (ANN) techniques to improve the accuracy of the developed models.

Keywords: Dry Zone, Maha, Reference Evapotranspiration, Yala, SARIMA.

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Abbreviations

ACF	-	Auto Correlation Function
ANN	-	Artificial Neural Network
AR	-	Auto Regressive
ARIMA	-	Auto Regressive Integrated Moving Average
CEA	-	Central Environmental Authority
cm	-	centimeter
CWR	-	Crop Water Requirement
ECA	-	Evaporation Class A
ET	-	Evapotranspiration
ET _o	-	Reference crop evapotranspiration
ET _c	-	Crop Evapotranspiration
FAO	-	Food and Agriculture Organization
FIM	-	First Inter Monsoon
IR	-	Irrigation Requirement
IWRM	-	Integrated Water Resources Management
K _c	-	Crop Coefficient
K _s	-	Water Stress Coefficient
m	-	Meters
MA	-	Moving Average
mm	-	Millimeters
MSE	-	Mean Squared Error
NEM	-	North East Monsoon
PACF	-	Partial Auto Correlation Function
PM	-	Penman-Monteith
RVM	-	Relevance Vector Machines
SAR	-	Seasonal Auto Regressive
SARIMA	-	Seasonal Auto Regressive Integrated Moving Average
SIM	-	Second Inter Monsoon
SMA	-	Seasonal Moving Average
SSE	-	Sums Squared Error

- SVM - Support Vector Machines
- SWM - South West Monsoon