DESIGN A MODEL AND MATHEMATICAL APPROACH FOR VECTORED TRUST CONTROLLED TRI ROTOR AERIAL PLATFORM

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ABSTRACT

It is a challenging factor to address the more or less of vectored trust-controlled platforms for Stability. There are unnecessary parts in any UAV plat form and must be identified and apply removal of such parts or substitute with aerodynamically more suitable parts will lead to design of good aero stable platform. Testing platform for the different attitude for disturbances and effective analyzing can lead to develop best tri-rotor vectored trust-controlled platform. Since project is to develop superior tri-rotor system to perform better stability and effective control of motion various mathematical and control approaches to be introduced. A Discussion and an analytical approach towards developing tri-rotor system is a need of future multi-rotor platforms. In coming years there will be a great advancement on tri-rotor systems. Still there are many areas to study when developing a stable tri-rotor platform with precise six directional control. Design will demonstrate stability of the vectored trust-controlled tri-rotor over other multi rotor platforms and enhance the present capabilities of tri-rotor platform to prove it as the future of the multi rotor UAV platforms. To demonstrate the mathematical approach discussed here, it is used a platform enable to carry out further studies on to tri copters.

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Ex. Squadron Leader. Hashitha Maduranga Madanayaka

B.Tech(EEE), Pg.Dip(Aero), C.Eng, MIE(SL), MIEEE

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CHAPTER 1
Introduction

Chapter one reveals the today and future of the autonomous flying systems. the Autonomous flying systems today play a vital role in military systems, surveillance, agriculture and so on. Fixed wing autonomous flying systems have dominated the fields in the military and surveillance since they are able of having long endurance and able to carry heavy payloads. But they have got disadvantages as well. They are mainly recognized as systems needing runway or suitable area to takeoff or landing and not able to hover at a given place. Rotor type autonomous system or Vertical Take Off and Landing (VTOL) autonomous systems has dominated the field of Agriculture since they need to take off and land at agriculture fields and need of hovering due to weather. Due to above rotary wings capabilities, thinking process and engineering approach have changed towards developing rotary wing autonomous platforms. With the development of rotary wing platforms, can overcome problems faced with fixed wing areal platforms. Today autonomous rotor flying has more developed as multi rotor platforms rather developing single rotor flying machines. These autonomous flying systems are trying to engage in day to day life since they have been able to overcome major drawbacks discussed earlier. There are still wide engineering areas to be addressed and carry out further studies such as, reduction of disturbances with precise attitude and motion control, increasing of endurance, make able to carry heavy payloads and pinpoint navigation. Controlling of platform attitude and motion has major area to be studied. Disturbances developed due to head and cross wind components, identifying aerodynamically effective surfaces in the platform and synchronizing rotors for the disturbances are necessary areas to be studied to find engineering solutions.

1.1 Objectives of Research Project

There are many versions of multi rotor designed platforms such as tri-rotor, quad rotor, octal rotor and even more rotors installed platforms. Above all tri-rotor and quad rotor platforms are more popular and considered to be probable future of autonomous rotor machines. This research is to identify and demonstrate the suitability for the development of tilted tri-rotor platforms over quad rotor. There are many advantages on single tri-rotor platform over single quad rotor platforms. They are lesser cost involved, since tilting mechanism only one rotor with servo motor is needed for pitch and yaw movement where as in quad rotor two rotors must synchronize to achieve the same effect. This improves the efficiency of six directional movements, result in fast response of the platform. These factors affect more on if the systems become operational and need of carrying more payloads. Further, compared to quad rotors, tri-rotor UAVs are smaller in size, less complex, less costly and have longer flight time due to the reduction in number of rotors and power, which makes tri-rotor vehicles ideal for future deployments.

Thrust vectoring has been used in various designs of aerial vehicles to maximize the capability of UAVs. Thrust vectoring mechanism is used to give UAVs the capability of taking-off and landing in very narrow areas. A simple technique of tilt-rotor mechanism can be used to obtain thrust vectoring where the propulsion units are inclined in certain angles using an additional control motor to get the desired thrust in different directions. Tilt-rotor mechanism is mainly used in tri-rotor systems to control the horizontal forces and yaw torque of the vehicle. Hence it is required to carry out studies with the focus of improving tilt-rotor mechanism in order to obtain advance trust vector control mechanism.

Tri-rotor platforms need to be studied for how it stands for disturbances, especially wind components with different attitudes as pitch variations, roll variations and yawing. Need to minimize disturbances occur due to asymmetry of the platform since pitching movements may differ around the vertical axis if platform is not aerodynamically symmetrical. Yaw control of tri-rotor systems is a challenge due to the asymmetric configuration of the system. The reactive yaw moments in quadrotor systems are decoupled from pitch and roll moments. Whereas pitch, roll and yaw moments are highly coupled in tri-rotor systems. Most of the cases it is suggested to have four loops to attitude control and guidance for tri-rotor platforms. Hence intended to study existing control loops and improve control loops in order to find best control mechanism.

Studying of existing mathematical models is required for multi rotor platforms and finding, generating best control method to overcome primary issues like gyro stability and control loops generation with different wind situations.

Selection of other sensors such as differential pressure sensor and static pressure sensors, also play a huge role in stability of the platform when operating in higher altitudes. Coupling of these devices together is to be done with precise involvement of active filters.

It is required to carry out different methods (change of pitch, roll and yaw movements) in different backgrounds (windy weather, gasping weather) and need to log data for the analysis and need to feed corrections until desired output is to be achieved through hardware simulation. Through this it will able to find out disturbances and will be able to smooth the disturbances by analyzing the logged data.

1.2 Scope of the Research

Following tasks have undertaken for the research

First task of the research is to carry out feasibility study of the existing multi rotor platforms and mathematical models already developed.

Study six directional movement of the tri rotor with tilted rotors and further improve movements. Focus trust vector control with the tilt-rotor mechanism and design and improve a methodology for yaw and pitch movements.

Mathematically develop unique control mechanism to overcome existing problems identified through feasibility studies.

Simulation of control theory developed and analyzes the system's capabilities as a theoretical approach. Upgrade control systems and mathematical models further and smooth simulation results.

Selection of hardware for hardware simulation of the project and carryout hardware simulation.

Analyze test flights and feed the data while changing rates such as pitch to roll, pitch to altitude etc.

1.3 Methodology and Approach

Attitude (altitude, pitch, roll and yaw) Control of the vectored trust-controlled tri-rotor is usually done through control of the tilt mechanism and control of the speeds of the rotors. As it is well understood that altitude of the platform can be achieved through sum of trust generated through the platform. This can be done through increase of speed of rotors and maintain all three rotors in same speed and level. Roll control can control with single rotor given speed, varying the rotor speeds of the two front rotors. Pitch control is generated through front two rotors maintaining the same speed and third rotor is varying the speed but maintaining the level. Tilted rotor produces the yaw moment of the platform. Trust line of the tilted rotor is changed and being kept all rotors in same speed to generate yaw moment. Based on these basic control principles initial mathematical models are to be derived.

It is also needed to design mathematical design for hover the platform. In order to do this equilibrium-point of hovering state must be figured out. It is intended to use mathematical operations to find out trim point of the platform from the derived equilibrium point of hovering state. Researchers on this field is experienced that they have obtained asymptotically stable system for hovering the platform and linear-quadratic regulator (LQR) controller gain is used to guarantee the stability of the system. Research is intended to use LQR controller to stabilize platform and locate the trim point of the platform for hovering.

Developing initial mathematical model for the multi rotor systems is to carry out using the suitable mathematical method and is used to model the flight dynamics of the rotorcraft. Through this initial mathematical model is to be developed. There are different control methods used by many researchers and it is intended to understand and further develop the use of these control methods such as feedback states, sliding modes and backstepping for all dynamic positions of UAV.

Linear and nonlinear mathematical models will have to obtain and develop further and following control and mathematical methods are to be studied for obtaining linearization of the nonlinear mathematical models.

a. Backstepping Control Low
b. Lyapunov function

In related to above theories, normally there are various different methods those use for developing control method for actuator dynamics. Linear and Nonlinear mathematical models are studied with knowledge of following theories. When developing mathematical model, linearization of feedback of the nonlinear system is done through a mathematical model developed with initially fed reference signal and resulting error signals. Linearization of the nonlinear function is to be done using Backstepping control low. Through this system convergence and stability can be achieved for error function. Here most of the functions may Ordinary Differential Equations (ODE). To solve these equations such as Lyapunov function is intended to use.

Since it is a challenge to develop platform withstand against windy environments, it is required to develop mathematical models for the in-house or zero wind condition. Based on the results obtained from the zero wind condition it is intended to obtain and develop mathematical functions for different windy situations. Control dynamics are to be obtained in order to suit for the selected wind conditions and limit the operation as seen from the virtual testing. It is well identified that in zero wind condition there are limitations obtained for the attitude of platform. These angle limitations for pitch, roll and yaw movements are to be studied and based on them suitable limitation is to select for the initial zero windy environments.

Build control theory and approach towards initial mathematical model to the vectored trust control platform and simulate using Matlabsimulink. Further update control system and simulate with Matlab Simulink. Design of circuits and simulate with the use of tools like Proteus/Altium, MP LAB IDE 3.

Selection of hardware tri-rotor vectored trust-controlled platform and purchasing of hardware after comprehensive study. Here selecting hardware like Inertia Measuring Units (IMU) have to be done with extra care since they differ from unit to unit. It is intended to study possibilities of using emerging technologies like Fiber Optic Gyros (FOG) since their performance is recommended to be superior to IMUs. It is intended to practically demonstrate hardware selection using test beds for rotor selection (trust against blade angle and speed) and IMU selection for rate of change.

Designing of multi-rotor platform should be done with the use of aircraft grade materials or fiber developed surfaces. This will result in achieving light weight platform.

Hardware simulation with designed circuits is to be done in two systematic ways. They are to be Dynamically uncontrolled and Dynamically controlled. Dynamically uncontrolled is to be done to check basic operations of the platform and as to check initial mathematical model developed. Dynamically controlled is to be done as to check and demonstrate improved mathematical models.

Intended simplified approach to follow the entire research is as in the following figure 01.

There are various mathematical and control approaches those used for development of tri-rotor UAV platforms. Hence, it is understood and intended to study relevant approaches to build trust vector-controlled tri-rotor UAV platform and to develop own control methodology for the UAV platform to withstand against disturbances.

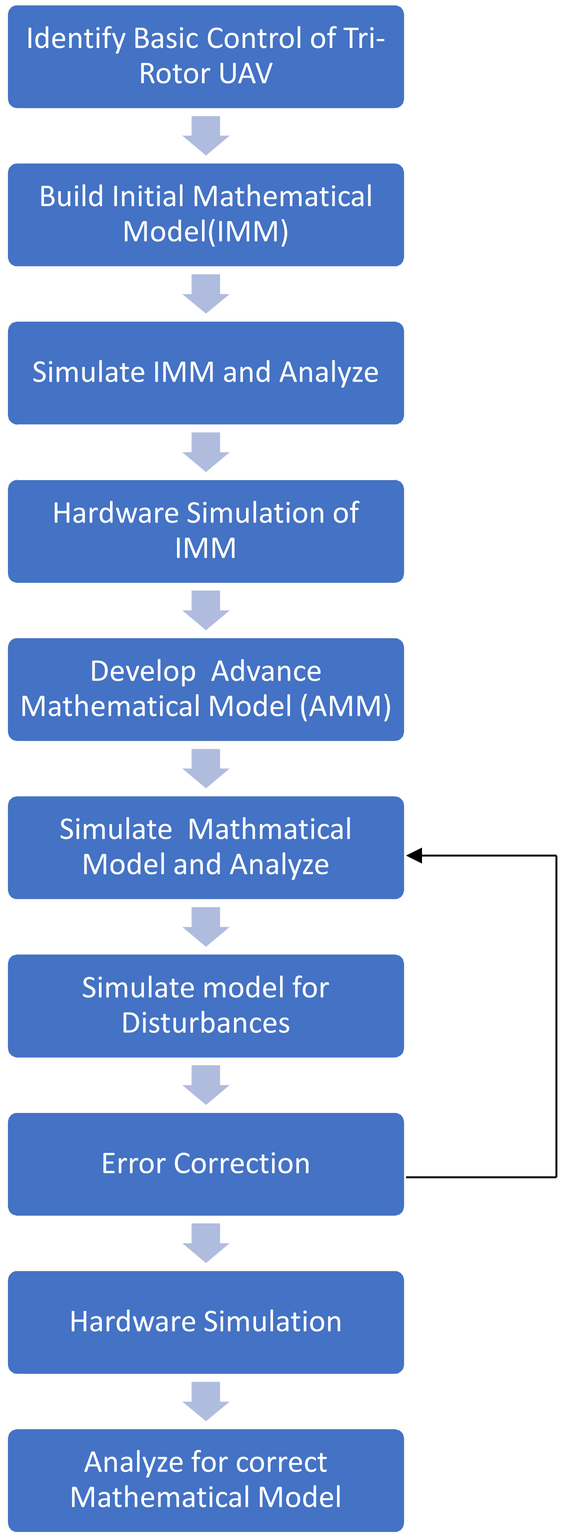


Figure1: Methodology

1.4 Time Frame and Work Plan

The time frame and work plan as following table 1 and derived by the assumptions the workloads associated with relevant tasks.

Table 1: Time frame and Work Plan

Commencing the Project	
Milestone 1	Building initial mathematical model and control theory and simulation from existing data and theories
Milestone 2	Expand mathematical model for problems identified. Improve control theory and simulate.
Milestone 3	Select Hardware for Physical Testing.
Milestone 4	Hardware simulation and check controls (Dynamically Uncontrolled and Dynamically Controlled).
Milestone 5	Testing in non-windy environments analyze logged data.
Milestone 6	Corelate and analyze calculated data vs Logged Data to prove the mathematical model
Milestone 7	Final analysis and conclusion
Preparation of Thesis and Submission	

1.5 Problem Identification

Autonomous flying systems today play a vital role in military systems, surveillance, agriculture and so on. Fixed wing autonomous flying systems have dominated the fields in the military and surveillance since they are able to have long endurance and able to carry heavy payloads. Also they have got disadvantages as well. They are mainly recognized as systems needing runway or suitable area to takeoff or landing and not able to hover at a given place. Rotor type autonomous system or Vertical Take Off and Landing (VTOL) autonomous systems has dominated the field of Agriculture since they actually need to take off and land at agriculture fields and need of hovering due to weather. Due to above rotary wings capabilities, thinking process and engineering approach have changed towards developing rotary wing autonomous platforms. With the development of rotary wing platforms, can overcome problems faced with fixed wing areal platforms. Today autonomous rotor flying has more developed as multi rotor platforms rather developing single rotor flying machines. These autonomous flying systems are trying to engage in day to day life since they have been able to overcome major drawbacks discussed earlier.

In Sri Lankan military perspective Sri Lanka Navy (SLN) and Army (SLA) request such autonomous flying objects since they need to deploy them in the operation areas for surveillance. Their need is to have systems capable of long endurance as well as Infrared payload caring capability. Navy requirement is to deploy them in the deep water sea, since they have to cover more area of surveillance. They have identified the importance of these kind of rotary flying objects since they minimizes their patrolling which in deed lead to save lot of money in case of fuel and reduce the risk under taken.

1.6 Identified suitable solution

There are many UAV versions and they differ from one another and they have been classified depend on size of drones, range, applications and technology used. Based on the size of the UAVs classified semi small drones (small as a bug size like 50mm wide), small drones (Lesser than 2m wide), Medium Drones (UAVs can carry 200kg payloads) and Big drones (Size almost like normal aircraft size). Range wise classification is as substantially close range (operates 5km range), Close range (range between 5km – 50km), Short Range (50km – 150km), Mid-range (150km – 650km) and Long range (greater than 650km). Application wise classification mainly divide in to followings such as,

1. Military applications,
2. Commercial applications

Military applications are real time video communications to ground troops as surveillance aid, armed with essential weapons or gears, reconnaissance of unknown areas and assisting lost battalions. With above military capabilities UAVs have become frontline tool for mission commanders to take decisions and launch coordinated military missions. Below figure 02 shows military use and distribution of UAVs depend on development and usage by 2017.

Commercial applications are such as infrastructure, agriculture, Transport, Security, Media Entertainments, Insurance, Telecommunications and Mining. (Ref: Unmanned Aerial Vehicles: A Survey on Civil Applications and Key Research Challenges - HazimShakhatreh, Ahmad Sawalmeh, Ala Al-Fuqaha, Zuochao Dou, EyadAlmaita, Issa Khalil, Noor Shamsiah Othman, Abdallah Khreishah, Mohsen Guizani). There are Law Altitude Platforms (LAP) and also High-Altitude Platforms use in these commercial areas. LAPs are classified on operational altitude below 10km and HAPs are classified as operational altitude above 10km. Below table 01 represents the commercial use of applications in UAV industry. Here VTOL means Vertical Take Off and Landing and mainly they are rotor platforms.

Multirotor or VTOL platforms and fixed wing platforms are classified mainly as different technology used platforms. Multirotor platforms are able to hover at certain location with long endurance and mainly operates at Low altitudes.

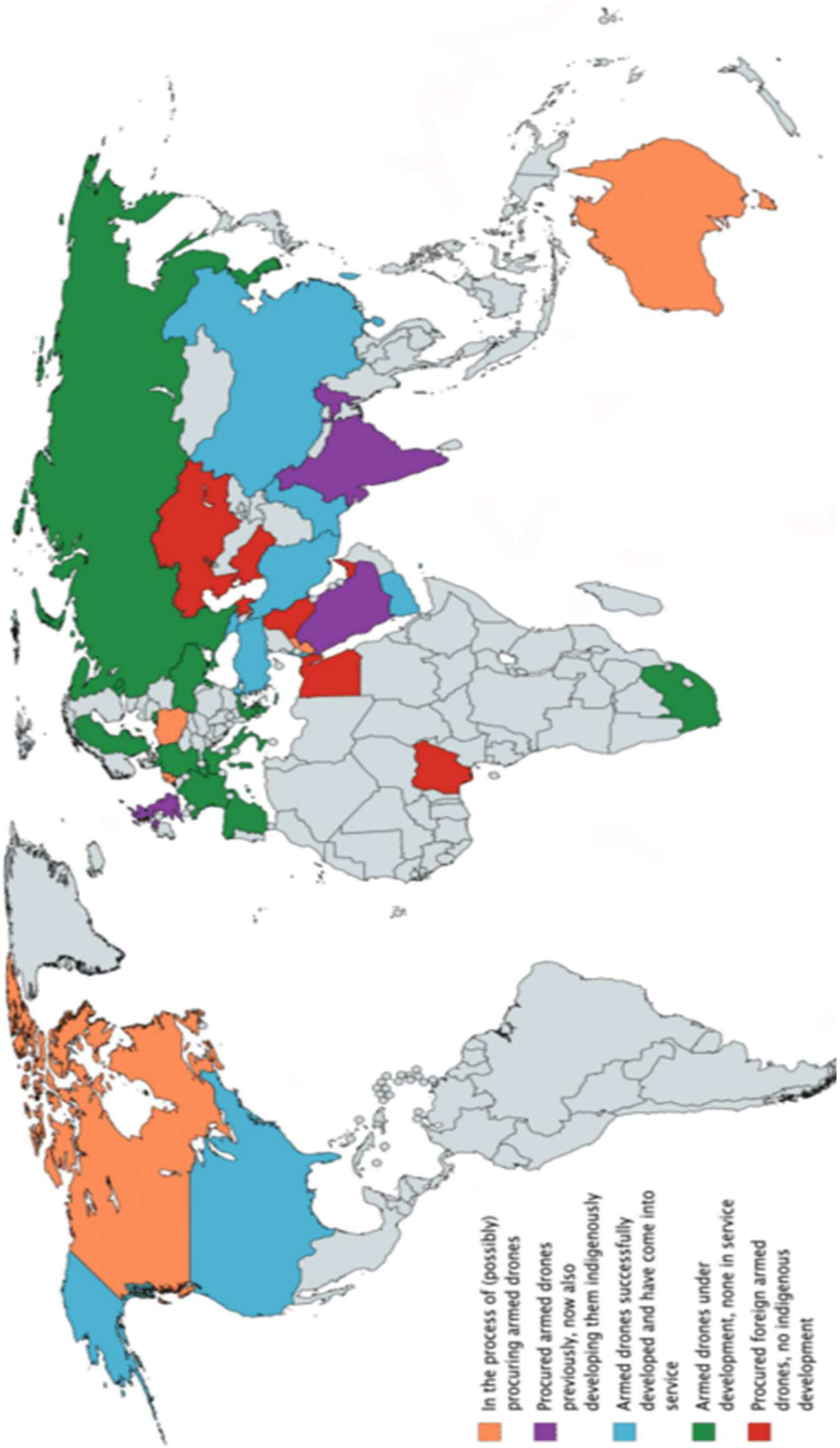


Figure 2: Armed drones proliferation as of May 2017

Source: [1]

Issues	HAP			LAP		
Type	**Air Ship**	**Air Craft**	**Balloon**	**VTOL**	**Aircraft**	**Balloon**
Endurence	Long Endurence	15-30 Hours (JP Fuel) > 7 days (Solar)	Long Endurence Up to 100 days	Few Hours	Few Hours	From 01 day to few days
Max. Altitude	Up to 25km	15 - 20 km	17 - 23 km	up to 04 km	Up to 05 km	Up to 1.5 km
Payload (kg)	Hundreds of kg's	Aup to 1700kg	Tens of kg's	Few kg's	Few kg's	Tens of kg's
Flight Range	Hundreds of km's	From 1500km to 25000km	Up to 17million km	Tens of km's	Less than 200km	Tethered Balloon
Deployment Type	Need runway	Need runway	Custom Build Auto lunachers	Easy to Deploy	Easy to Deploy by Catapult	Easy to Deploy 10-30 min
Fuel type	Helium Balloon	JP - 8 Jet Fuel Solar Panel	Helium Balloon Solar Panel	Batteries Solar Panel	Fuel Injection Engine	Helium
Operatuional Complexity	Complex	Complex	Complex	Simple	Medium	Simple
Coverage Area	Hundreds of km's	Hundreds of km's	Hundreds of km's	Tens of km's	Hundreds of km's	Several Tens of km's
UAV Weight	Few Hundreds of kg's	Few Hundreds of kg's	Tens of kg's	Few kg's	Tens of kg's	Tens of kg's
Public Safety	Considered Safe	Considered Safe	Need Global Regulations	Need Safety Regulations	Safe	Safe
Applications	Testing Environmental Effects	GIS Imaging	Internet Delivery	Internet Delivery	Agriculture Application	Aerial base station
Examples	HiSentine180	Global Hawk	Project Loon Balloon (Google)	LIDAR	EMT Luna X-2000	Desert Star 34cm Helikite

Table 2: Platform Classification of UAV Types, And Performance Parameters

Source: [1]

Chapter 2

Literature Survey

There are many versions of multi rotor designed platforms such as bi rotor, tri-rotor, quad rotor, octal rotor and even more multi-rotors platforms. The purposes and control of each platform is differing from each other. Many multi rotor platforms are developed for increasing the payload capability. Below picture is shown available multi-copter platforms and some of they are developing for the commercial use.

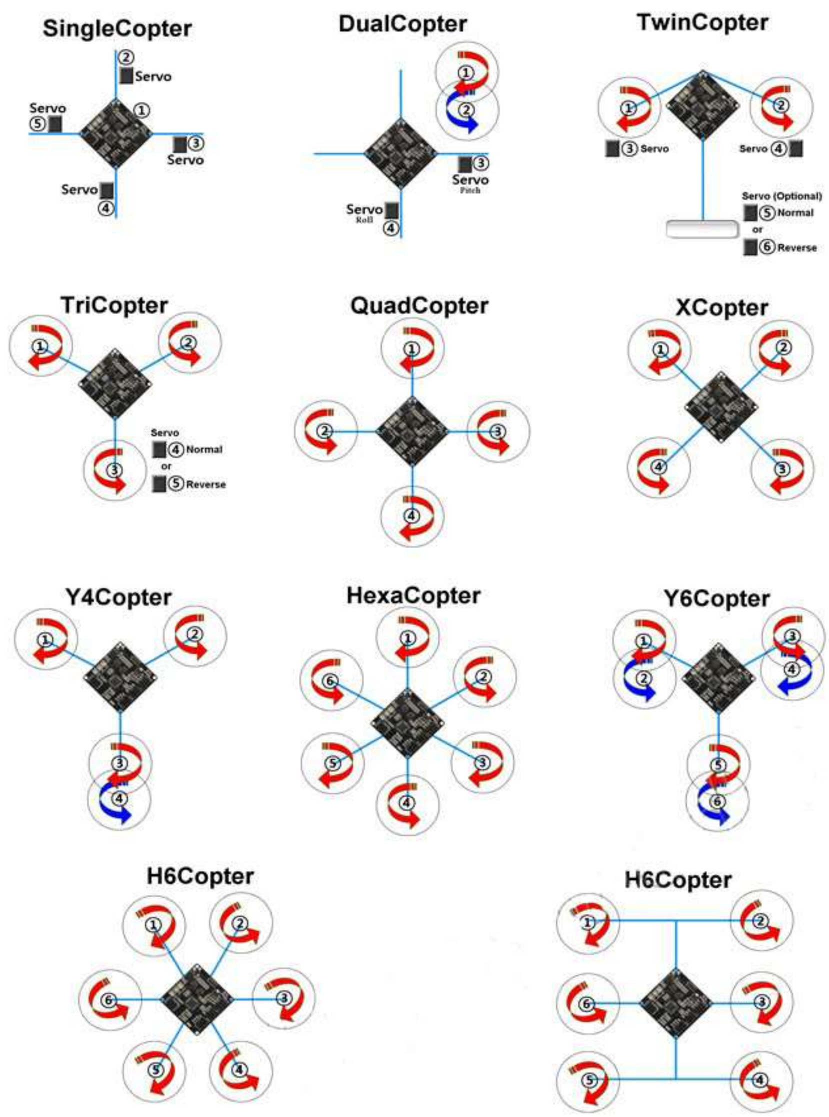


Figure 3: Multi-copter Platforms
Source: [2]

Some of multi rotor platforms are designed to even carry a payload like human. Above all tri-rotor and quad rotor platforms are more popular and considered to be probable future of autonomous rotor machines. There are many advantages on single tri-rotor platform over single quad rotor platforms. They are as follows.

- Less power requirement
- Reduced air frame drag
- Less likelihood of rotors being affected by the airframe itself
- Only three arms leaves space for a wide camera angle
- Less cost involvement
- More endurance
- Small in size

They are lesser cost involved, since tilting mechanism only one rotor with servo motor is needed for pitch and yaw movement where as in quad rotor two rotors must synchronize to achieve the same effect. This improves the efficiency of six directional movements, result in fast response of the platform. These factors effect more on if the systems become operational and need of carrying more payloads. Further, compared to quadrotors, tri-rotor UAVs are smaller in size, less complex, less costly and have longer flight time due to the reduction in number of rotors and power, which makes tri-rotor vehicles ideal for future deployments. Even in tri-rotor design also splits in to two types mainly “T” shape and “Y” shape. They are as in below figures 4 and 5.

Figure 4: T shape Tri Rotor plat form
Source: rcgroups.com

Figure 5: Y shape Tri Rotor platform
Source: joypicta.pw

Most common and popular tri copter design is Y shape due to support towards balancing since the symmetry over all three legs. Tilted rotor or trust vector-controlled tri-rotor platforms are becoming further popular with the integration of controlled yaw movement with single rotor. Here, in this research is focused to develop mechanical approach for tilted tri rotor platform.

2.1 Existing Designs

a. Existing Designs

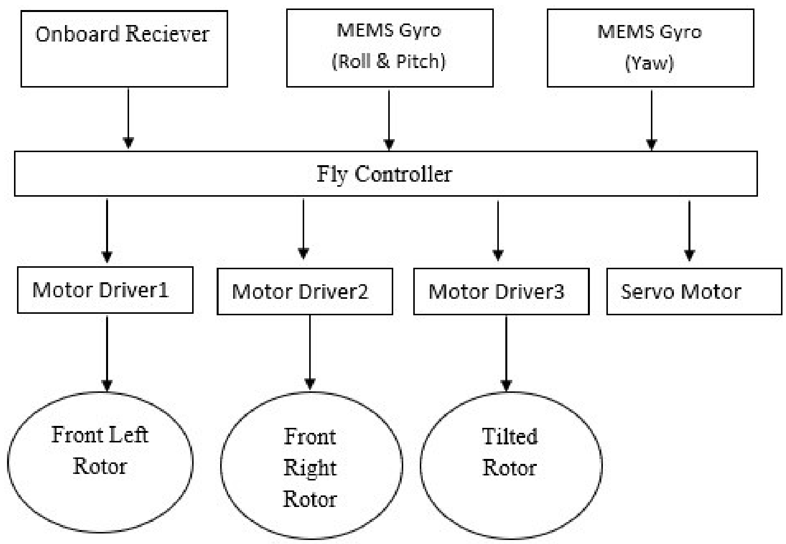


Figure 6: Hardware Platform

In the design it is mainly we have to consider about mathematical approach of the design and controller design. So tri-rotor platform closed loop system can be shown as follow. Here U_1, U_2,U_3 and U_4 are controlled thrust force, actuated roll, pitch and yaw moments respectively.

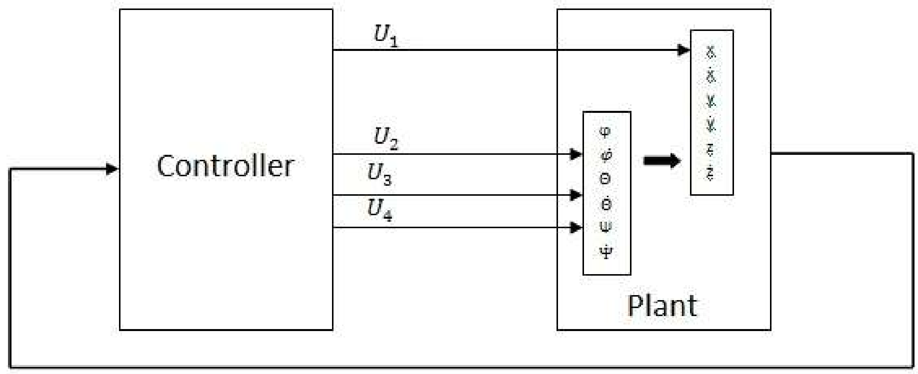


Figure 7: Tri copter closed loop system

b. Proposed controller design

There are few control design implementations which are popular among the multirotor platform designers.

- Proportional Integrative Derivative (PID) Controller based methods
- Model Predictive Controller (MPC) based methods
- Linear Quadratic Regulator (LQR) based methods

In this design project choose the PID controller-based method since the limitations lies with MPC and LQR based method. PID controller can be implemented to overcome many limitations faced by MPC and LQR. Proposed PID controller structure is as follow.

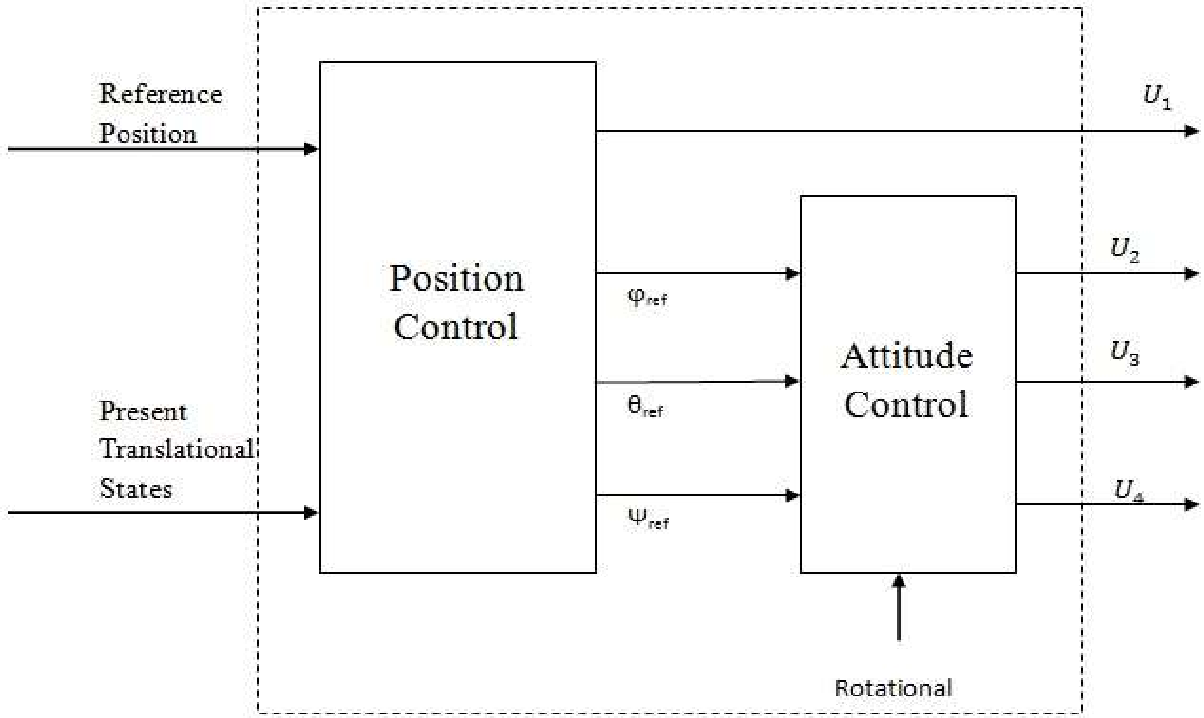


Figure 8: Proposed PID controller structure

2.2 Mathematical model of the physical system

a. Inertia calculation

Lets say that rotor blade has “r” radius, each leg length is “l”, weight of the motor and its accessories are “m” , central control board and related items weight “m_o” and width in to length of the control board is “b x a” then the inertia is calculated among three axis is as follow.

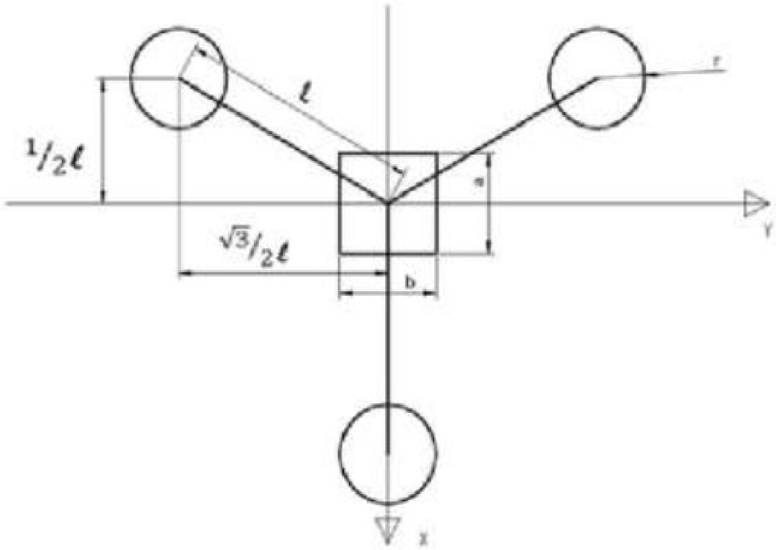


Figure 9: Inertia calculation

Source: [10]

$$I_{xx} = \frac{3}{2}ml^2 + \frac{1}{12}m_0b^2 + \frac{1}{12}m(3r^2 + h^2) \qquad (01)$$

$$I_{yy} = \frac{3}{2}ml^2 + \frac{1}{12}m_0a^2 \qquad (02)$$

$$I_{zz} = \frac{1}{12}m_0(a^2 + b^2) + 3ml^2 \qquad (03)$$

b. Rotational co-ordinate system

If global rotational frame is denoted by "G" and local rotational frame is denoted by "B". The relation between tri-rotor's (Body) co-ordinate system and co-ordinate system of earth can be described in a mathematical way with the rotation matrix $Q_{xyz}^{\varphi\theta\Psi}$.

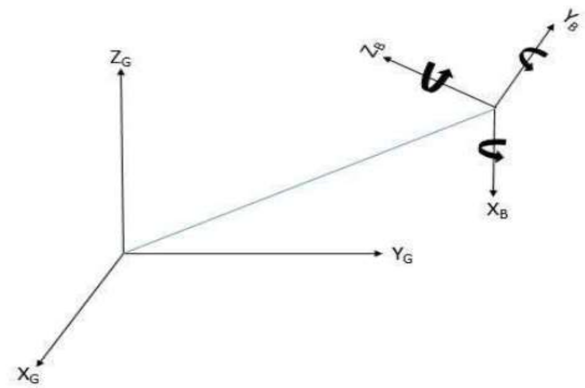


Figure10: Rotational Co-ordinate system

Source: [10]

$$Q_{xyz}^{\varphi\theta\psi} = Q_{\varphi}^{x} Q_{\theta}^{y} Q_{\psi}^{z} \qquad (04)$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & c\varphi & s\varphi \\ 0 & -s\varphi & c\varphi \end{pmatrix} \begin{pmatrix} c\theta & 0 & -s\theta \\ 0 & 1 & 0 \\ s\theta & 0 & c\theta \end{pmatrix} \begin{pmatrix} c\psi & s\psi & 0 \\ -s\psi & c\psi & 0 \\ 0 & 0 & 1 \end{pmatrix} \qquad (05)$$

$$= \begin{pmatrix} c\theta c\psi & c\theta s\psi & -s\theta \\ s\varphi s\theta c\psi - c\varphi s\psi & s\psi s\theta s\varphi - c\varphi c\psi & s\varphi c\theta \\ c\varphi s\theta c\psi + s\theta c\psi & s\psi s\theta c\varphi - c\varphi c\psi & c\varphi c\theta \end{pmatrix} \qquad (06)$$

c. Translational model

Mass multiplied by with the time derivative of the “translational speed vector” result is equal to the directional force vector.

$$m\dot{V}_G = \sum f \qquad (07)$$

The gravitational force is separated from the externals because it is acting on the mass centre and therefore not inducting any torque on the centre of mass.

$$m\dot{V}_G = \sum f = f_G + f_{gravity} = Q_{xyz}^{\varphi\theta\Psi} (f_b)_{ext} + [0\ 0 - mg] \qquad (08)$$

d. Rotational model

It can derive the rotational equation of motion from Euler's equation for rigid body dynamics by considering the tri-rotor as a rigid body in the body frame. Hence if "τ" is the momentum, "I" is inertia and "ω" is the rotational velocity then

$$\tau = I\dot{\omega} \qquad (09)$$

$$\dot{\omega} = \begin{pmatrix} \dot{\varphi}_x \\ \dot{\theta}_y \\ \dot{\Psi}_z \end{pmatrix} \qquad (10)$$

$$I = \begin{pmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{pmatrix} \qquad (11)$$

Therefore, $$\tau = \begin{pmatrix} I_{xx} & 0 & 0 \\ 0 & I_{yy} & 0 \\ 0 & 0 & I_{zz} \end{pmatrix} \begin{pmatrix} \dot{\varphi}_x \\ \dot{\theta}_y \\ \dot{\Psi}_z \end{pmatrix} \qquad (12)$$

e. Velocity model

If total force acting on the tri-rotor body is $\sum F$ then the relation between rotational gravity mode and thrust acting on the tri-rotor can be shown as,

$$\sum F = F_{thrust} + F_{gravity} \qquad (13)$$

Experimental results have shown that F_{thrust} is directly proportional to rotational speeds, hence

$$F_{thrust} = k\,({\dot{\varphi}_x}^2 + {\dot{\theta}_y}^2 + {\dot{\Psi}_z}^2) \qquad (14)$$

Here rotational velocity in the body frame and inertial frames are approximated as equal. It is needed to find out relationship between the rotational velocity as well as transactional velocities in order to identify the position of the platform. This is given from the Newton – Euler formula.

$$\begin{pmatrix} F \\ \tau \\ 0 \end{pmatrix} = \begin{pmatrix} m_t & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} a \\ \propto \\ 0 \end{pmatrix} + \begin{pmatrix} mv\omega \\ I\omega^2 \\ 0 \end{pmatrix} \qquad (15)$$

Matlab code for simulation for above mathematical approach is as shown in Appendix I. From above mathematical model position can be found and simulation is as shown and location of tri rotor platform can be visualized in 3D as below diagram.

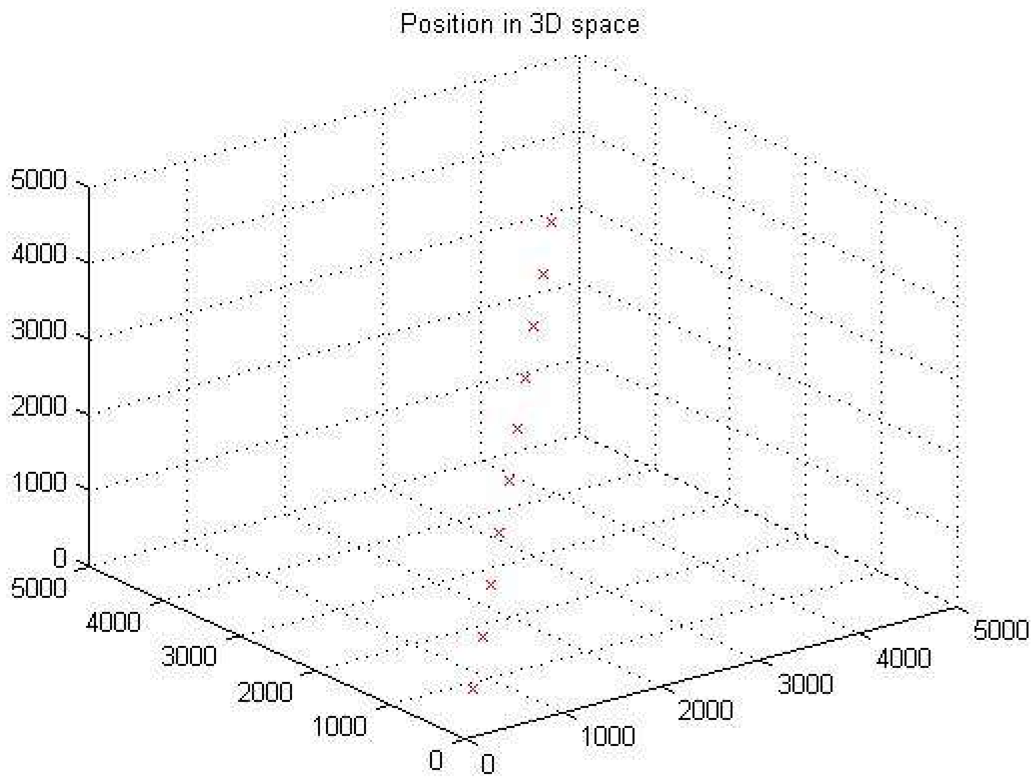


Figure 11: Matlab simulation for 3D space

2.3 Design Project Overview

Prior to hardware implementation it was intended from the design project under the Master of Scienceprogramme to study on other possibilities on predicting the mathematical behavior and predict control loop gains.

In control system it is needed to find the controllers for roll, pitch and yaw in defining of attitude control and horizontal and vertical control in defining of position control. Basically, the control system can be identified as following control system as in the Figure 7. Here R(s), E(s), U(s), D(s) and Y(s) are reference signal, error signal, controlled signal, external disturbance and desired output respectively. G_c and G_p are controller transfer function and tri- rotor plant transfer function. Here we assume that measurement error is negligible.

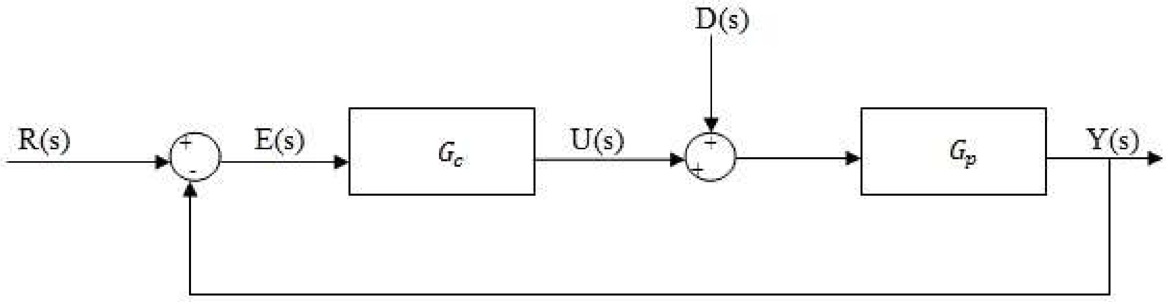


Figure 12: Control Architecture

From above control block diagram can obtain following control relationships. If "G" is the total system transfer function and the "S" is the sensitivity of the system then,

$$G = \frac{G_c G_p}{1+G_c G_p} \quad (16)$$

$$S = \frac{1}{1+G_c G_p} \quad (17)$$

$$Y(s) = G \cdot R(s) + S \cdot D(s) \quad (18)$$

a. Attitude control

Attitude control is the control of roll, pitch and yaw movements. Here for the design of attitude control can consider in to control of roll and pitch movements separately and control of yaw movement separately since we apply PID based control method. Since as mentioned earlier that rotational velocity in the body and inertial frames are approximated as equal. As mentioned previously also U_2,U_3 and U_4 are moment control signals and if "τ" is the moment disturbance due to aerodynamic instability. So the angular acceleration and moment disturbance can relate with following equations.

$$\ddot{\varphi} = \frac{U_2}{I_{xx}} + \frac{\tau_{aero}^{\varphi}}{I_{xx}} \quad (19)$$

$$\ddot{\theta} = \frac{U_3}{I_{yy}} + \frac{\tau_{aero}^{\theta}}{I_{yy}} \qquad (20)$$

$$\ddot{\Psi} = \frac{U_4}{I_{zz}} + \frac{\tau_{aero}^{\Psi}}{I_{zz}} \qquad (21)$$

Roll and pitch control

Following facts have been considered for designing PID controller for the control of roll and pitch.

- Need speed up response and stability
- Need not to consider about accumulated error
- Since there is no concern on accumulated error can consider about proportional and derivative (PD) controller
- Since the use of PD controller sudden and rapid compensation is possible

So the controller in time domain will be like as follow

$$u(t) = \left(K_p + K_d \frac{d}{dt}\right) e(t) \qquad (22)$$

So in the "s" domain controller transfer function can write as,

$$G_c = K_p + K_d s \qquad (23)$$

From motor control theory it is possible to take the transfer function of the uncontrolled plant as follow, Here "***l, p, h*** and ***k***" are known values and can be obtained from the DC motor specifications.

Therefore, $G_p = \frac{l}{ps^2+hs+k}$ (24)

Further this equation can be written as, $G_p = \frac{1}{\frac{p}{l}s^2+\frac{h}{l}s+\frac{k}{l}}$ (25)

From eqn.(1), “G” can be obtained as,

$$G = \frac{G_c G_p}{1+G_c G_p} \qquad (26)$$

$$G = \frac{K_p + K_d s}{\frac{p}{l}s^2 + \left(\frac{h}{l}+K_d\right)s + \frac{k}{l} + K_p} \qquad (27)$$

Optimum tuning method

To tune the PD controller, it is chosen the Integral Time Absolute Error (ITAE) criterion. ITAE criterion provides following advantages over other optimization methods such as Zieglar - Nichols method and Cohen-Coon method. [9]

- Smaller overshoots
- Smaller Oscillations
- Best selectivity (according to performance indices)
- Very sensitive to the changes

Following table indicate the coefficients for Optimum forms of the Closed Loop Transfer Function based on the ITAE criteria (Zero Steady State Step Error Systems). [9]

From above table since we have second order characteristic equation in the transfer function “G” following equation is obtained.

$$s^2 + 1.4\omega_n s + \omega_n^2 \equiv s^2 + \frac{\left(\frac{h}{l}+K_d\right)}{\frac{p}{l}} s + \frac{\left(\frac{k}{l}+K_p\right)}{\frac{p}{l}} \qquad (28)$$

Here ω_n is nothing but natural undamped frequency and it is to be selected as in following equation,

$$\omega_n = 4 / \varepsilon t_0 \qquad (29)$$

Following facts must considered when selecting the ω_n, to minimum settling time t_0 and for low overshoots ε (usually selected between $0.7 < \varepsilon < 0.9$). Hence, ω_n can be found.

From ITAE criteria PD controller constants can be found out,

$K_d = \frac{1.4\omega_n p - h}{l}$ (30) and

$K_p = \frac{\omega_n^2 p - k}{l}$ (31)

Hence PD controller can be implemented. Hence "G" and "S" can find.

From above equations it is obtained roll and pitch angles compensated for disturbances.

Based on above design approach PD controller is implemented and simulated in Matlab. Matlab .m code as in Appendix II.

Following diagrams show Bode plots for system transfer function and sensitivity function when controller is implemented. From this it is obtained that the functions are stable.

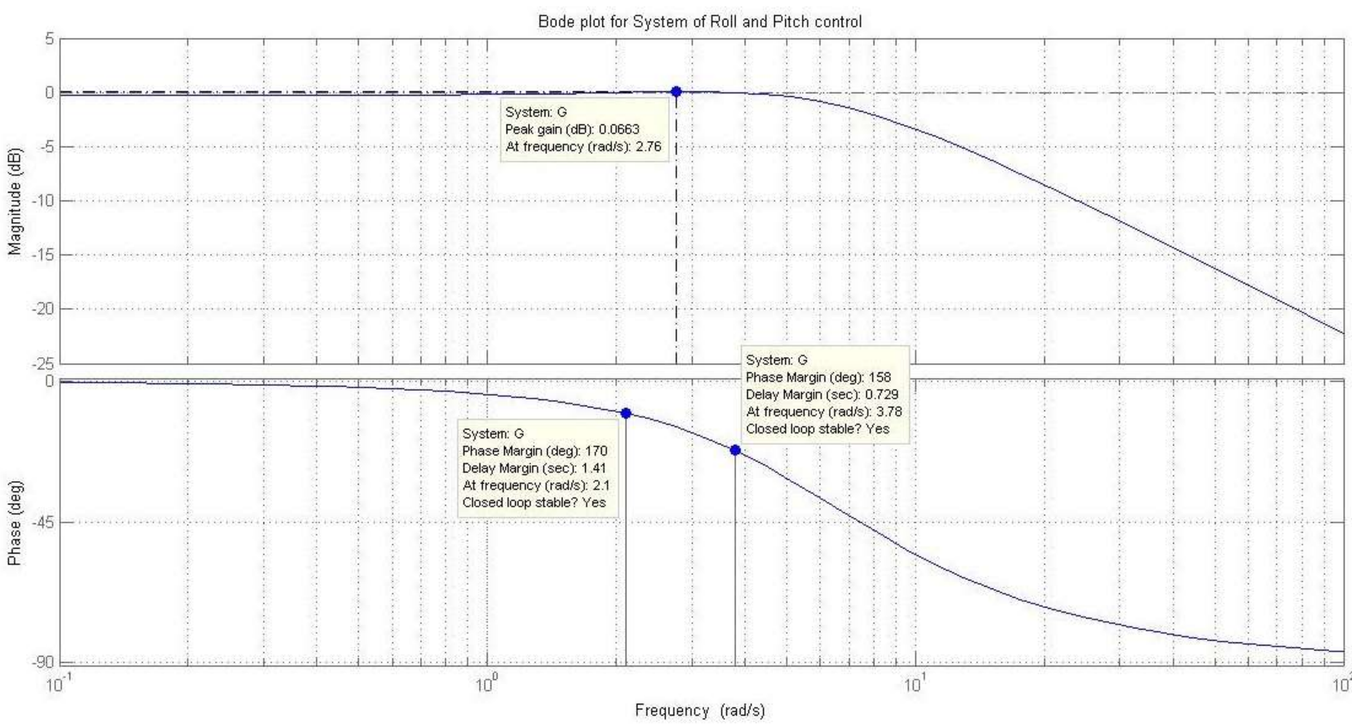


Figure 13: Bode plot for roll and pitch

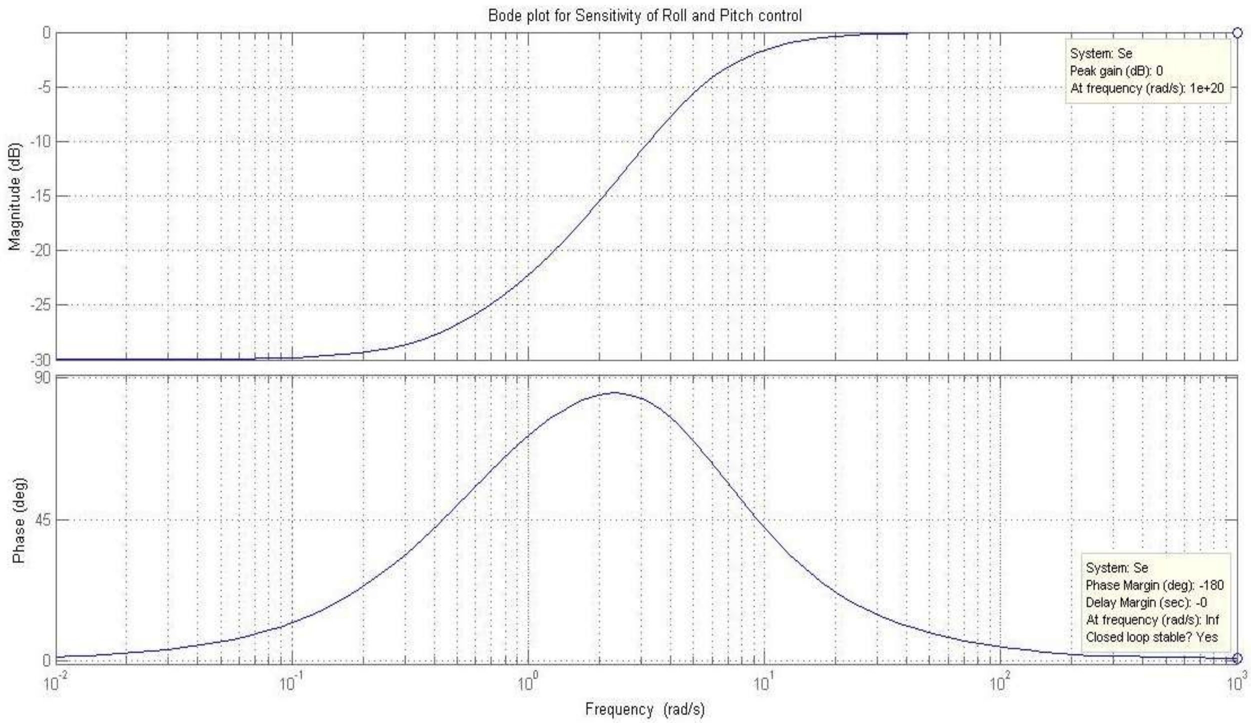


Figure 14: Bode Plot for sensitivity of roll and pitch

Root locus for system transfer function and sensitivity function is as follow.

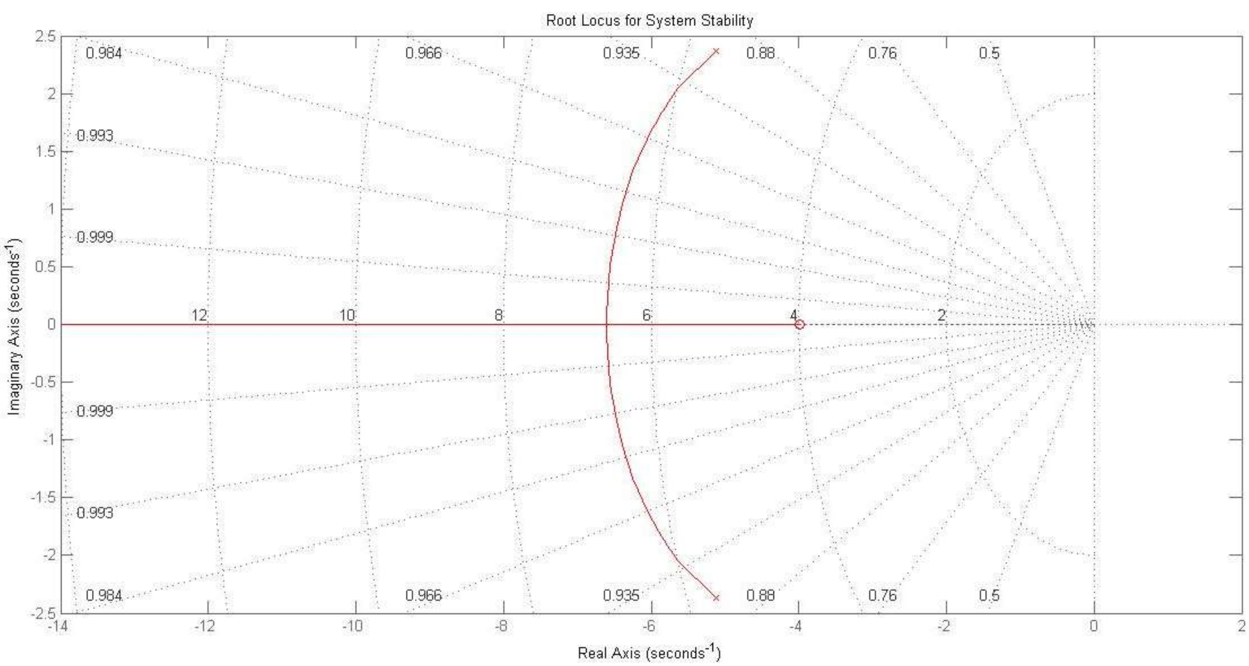


Figure 15: Root locus for system stability

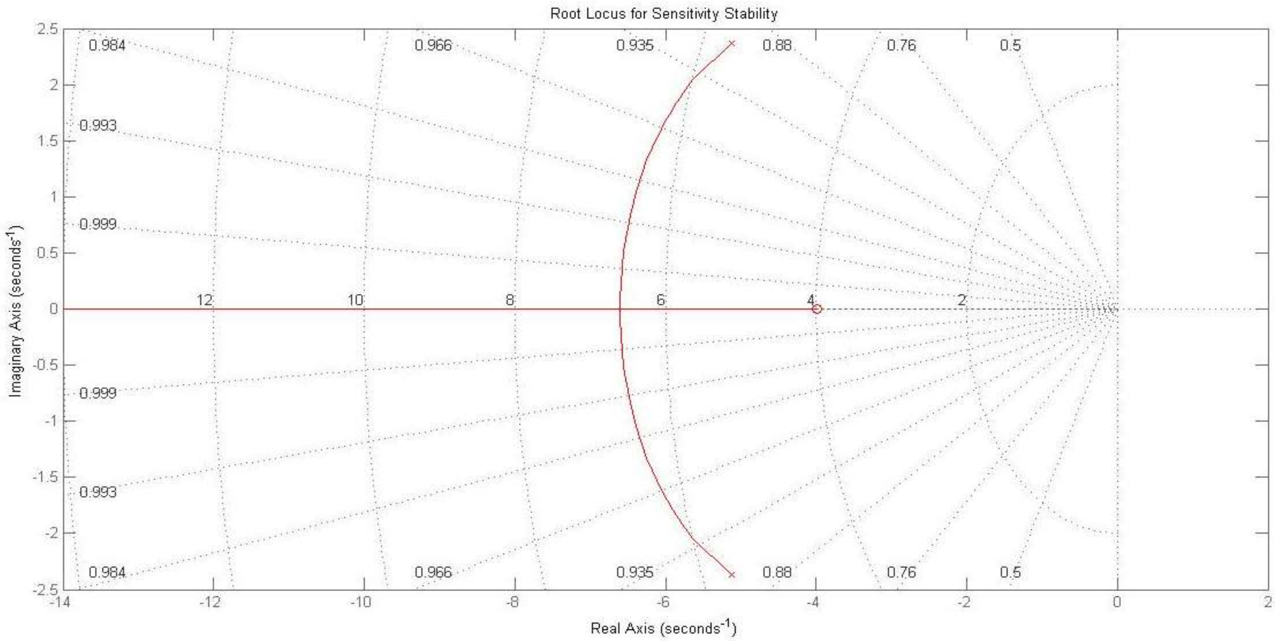


Figure 16: Root locus for sensitivity stability

Step responses for the roll and pitch functions are as follow.

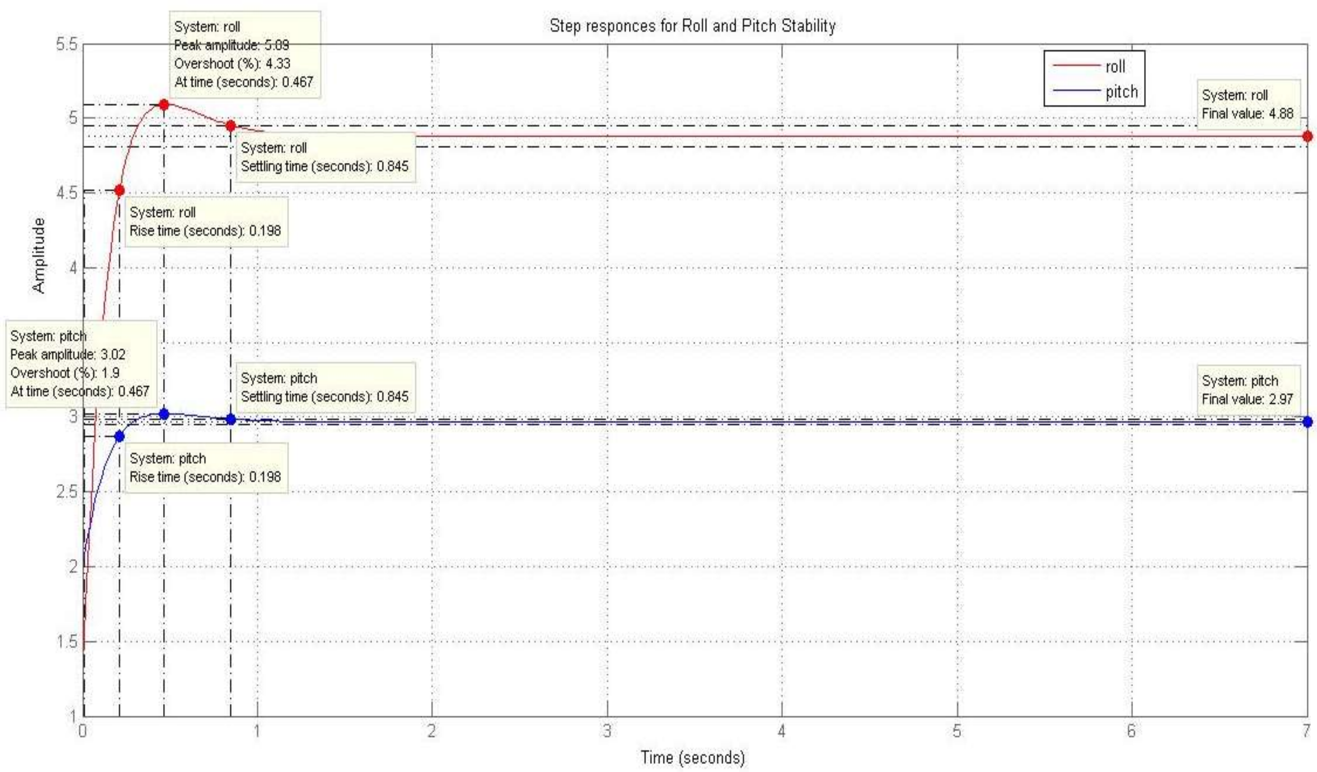


Figure 17: Step response for roll and pitch stability

Yaw control

For yaw control it is needed to consider about following facts.

- Here in proposed control structure, the position is controlled without the use of yaw angle.
- Integral action must be employed since need of considering the accumulated error
- Hence choose the PID controller for controlling the yaw movement.

So the controller in time domain will be like as follow

$$u(t) = \left(K_p + K_d \frac{d}{dt} + K_i \int dt\right) e(t) \qquad (32)$$

So in the "s" domain controller transfer function can write as,

$$G_c = K_p + K_d s + \frac{K_i}{s} = \frac{(K_d s^2 + K_p s + K_i)}{s} \qquad (33)$$

Considering the motor control for yaw control can get the plant transfer function as similar to PD roll and pitch control,

$$G_p = \frac{1}{\frac{p}{l}s^2 + \frac{h}{l}s + \frac{k}{l}} \qquad (34)$$

Hence G is obtained as,

$$G = \frac{G_c G_p}{1 + G_c G_p} \qquad (35)$$

$$G = \frac{K_d s^2 + K_p s + K_i}{\frac{p}{l}s^3 + \left(\frac{h}{l} + K_d\right)s^2 + \left(\frac{k}{l} + K_p\right)s + K_i} \qquad (36)$$

For the coefficients for Optimum forms of the Closed Loop Transfer Function based on the ITAE criteria (Zero Steady State Step Error Systems) gives,

$$s^3 + 1.75\omega_n s^2 + 2.15\omega_n^2 s + \omega_n^3 \equiv s^3 + \frac{\left(\frac{h}{l}+K_d\right)}{\frac{p}{l}} s^2 + \frac{\left(\frac{k}{l}+K_p\right)}{\frac{p}{l}} s + \frac{K_i l}{p} \qquad (37)$$

As in PID control natural undamped frequency,

$$\omega_n = 4 / \varepsilon t_0 \qquad (38)$$

Following facts must considered when selecting the ω_n, to have minimum settling time t_0 and for low overshoots ε (usually selected between $0.7<\varepsilon< 0.9$). Hence, $\omega_n$can be found.

From ITAE criteria PID controller constants can be found out as,

$$K_d = \frac{1.75\omega_n p - h}{l} \qquad (39)$$

$$K_p = \frac{2.15\omega_n^2 p - k}{l} \qquad (40)$$

$$K_i = \frac{\omega_n^3 p}{l} \qquad (41)$$

Hence PID controller can be implemented. Hence "G" and "S" can find. If submit "G" and "S" to Y, it is obtained yaw angle compensated for disturbances. Based on above design approach PID controller is implemented and simulated in Matlab. Matlab .m code as in Appendix III.

Following diagrams show Bode plots for system transfer function and sensitivity function when controller is implemented. From this it is obtained that the functions are stable.

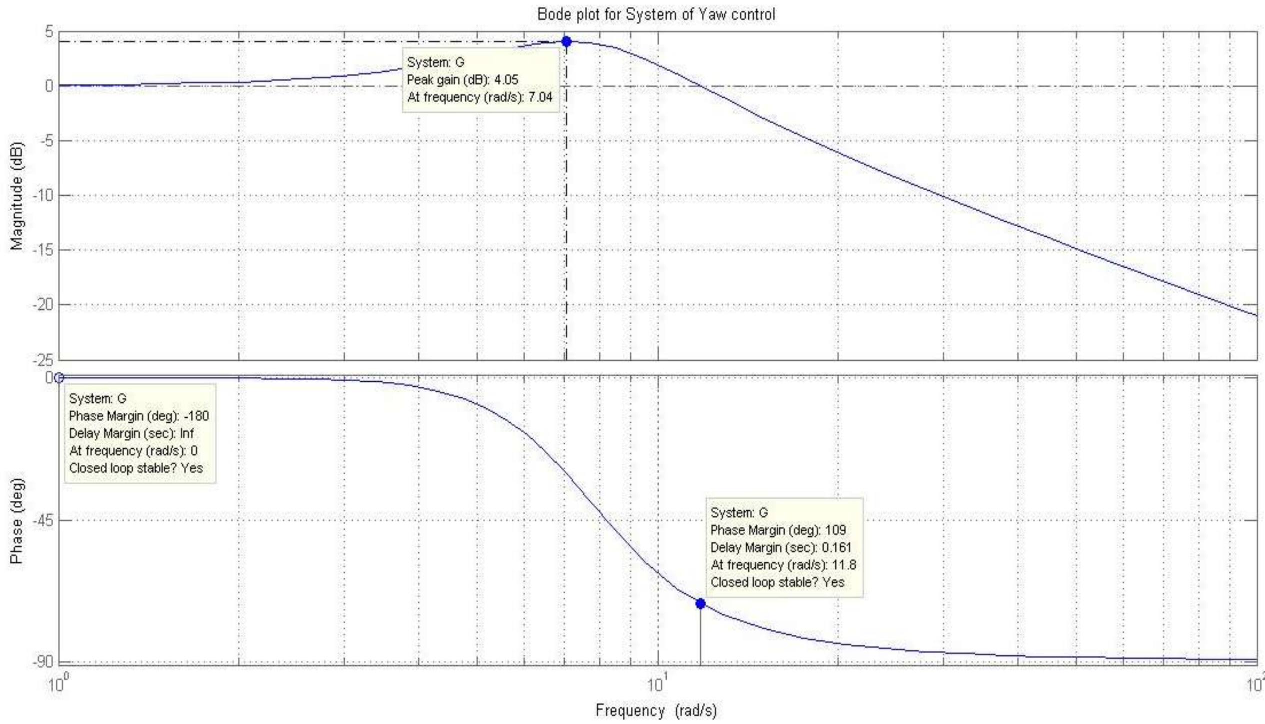


Figure 18: Bode plot for System yaw control

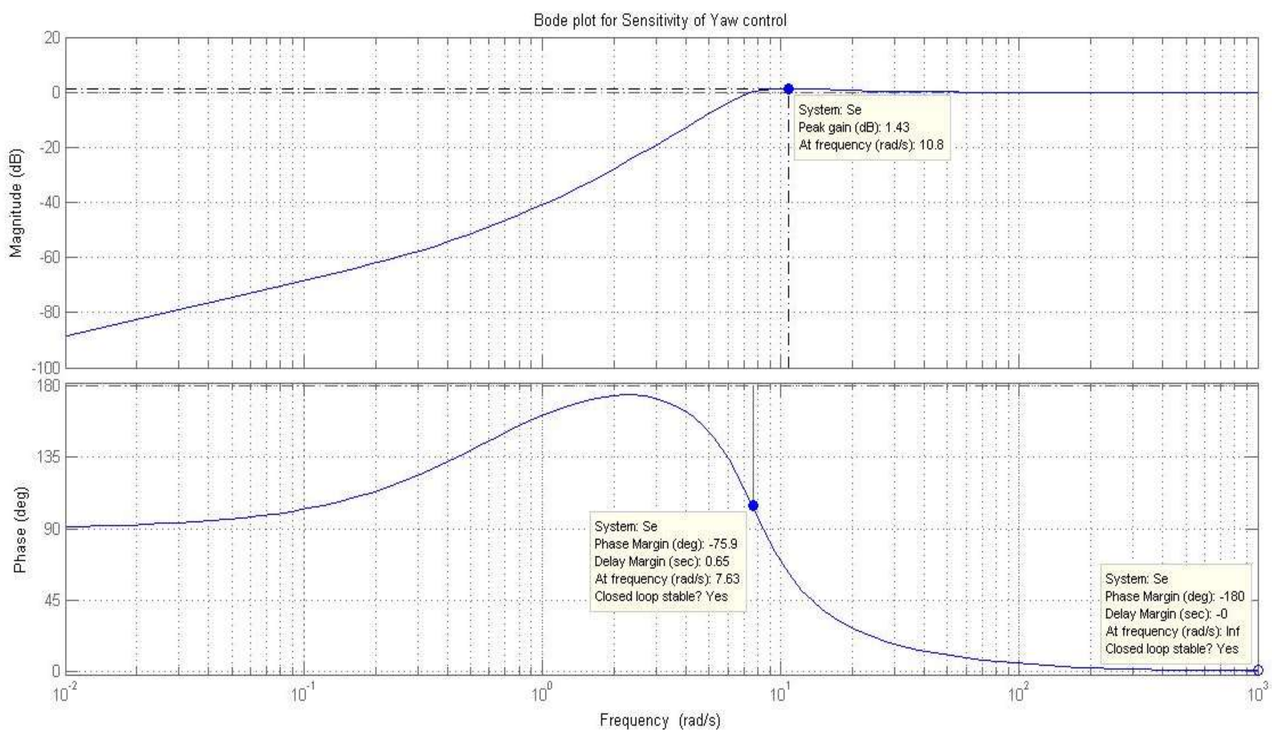


Figure 19: Bode plot for Sensitivity of yaw control

Root locus for system transfer function and sensitivity function is as follow.

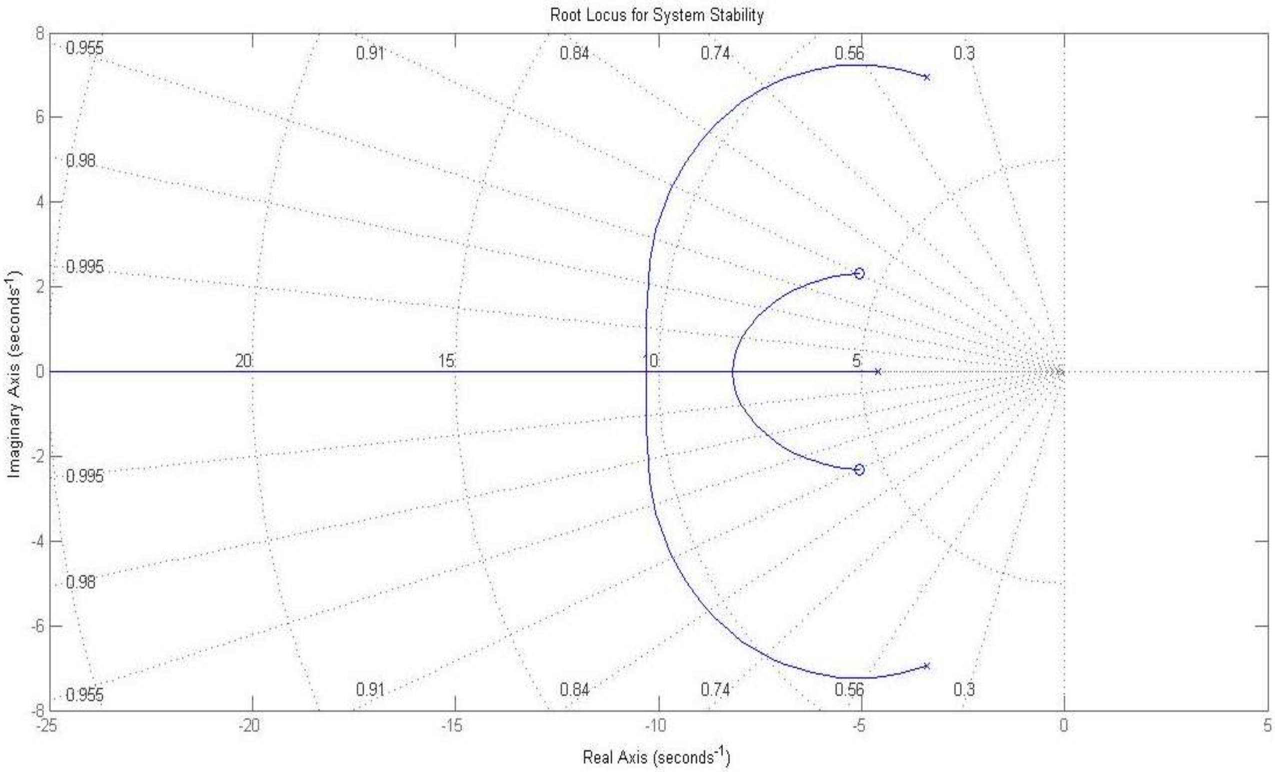


Figure 20: Root locus for system stability

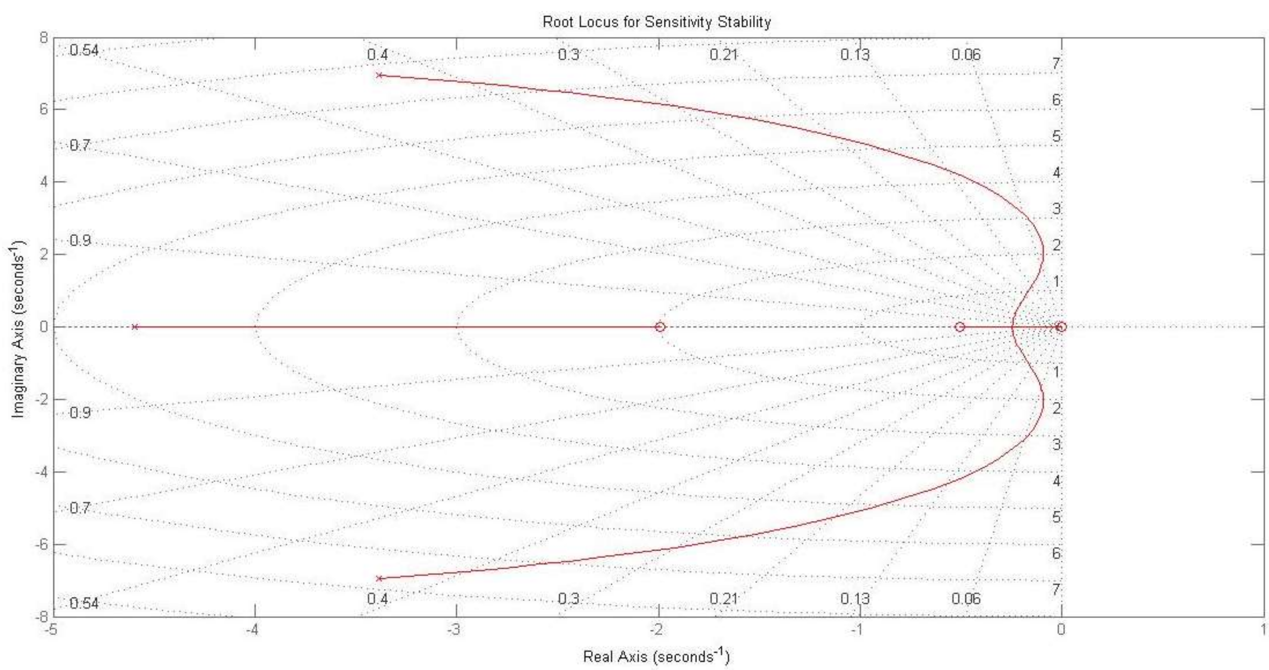


Figure 21: Root locus for sensitivity stability

Step responses for the yaw function are as follow.

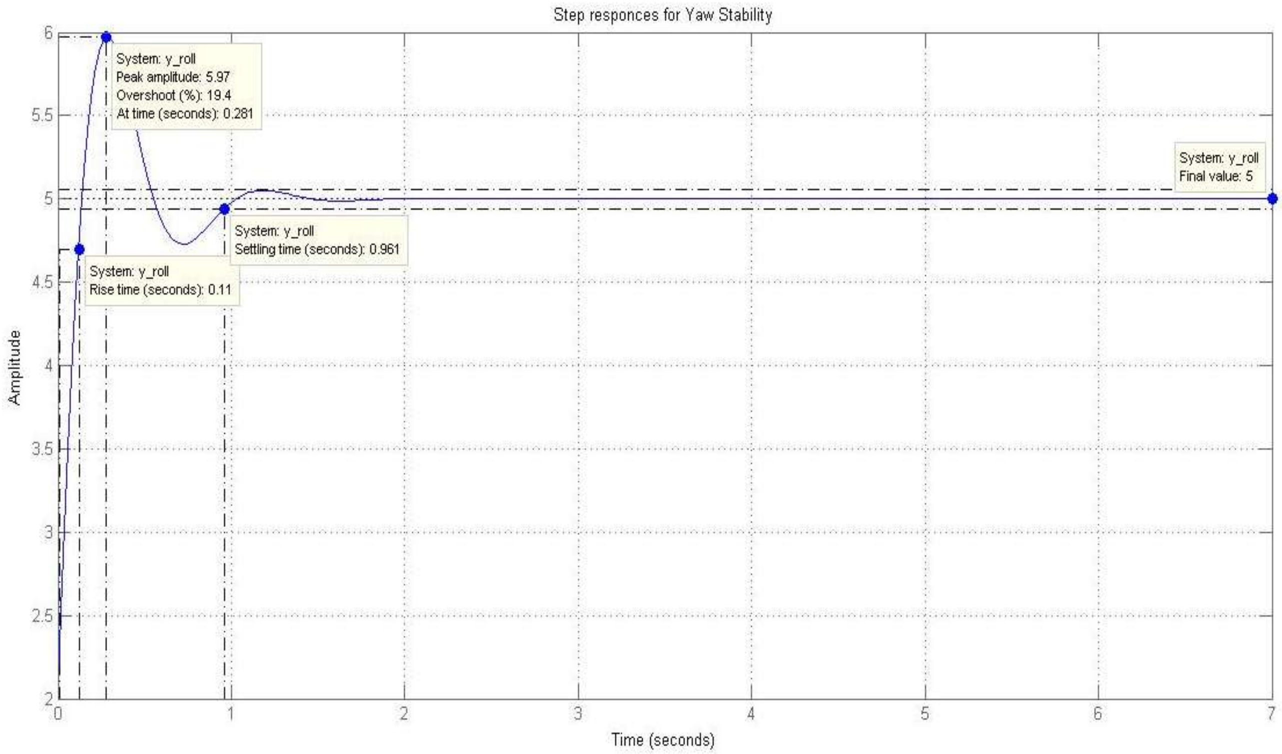


Figure 22: Step response for yaw stability

2.4 Learnings from the Design Project

From Above it is identified and there are too many assumptions. They are as below and have discussed the implementations.

1. Assuming the overshoots ε (selected between $0.7<\varepsilon< 0.9$)

2. Assuming the most fitting method for finding PIDs for control system is Time Absolute Error (ITAE) criterion [8]

3. Assuming control system transfer function constants

To predict and system functions cannot rely on above assumptions. Hence, different approach must be implied to get desired control system and should have hardware to prove the implementations.

Chapter 3

New System Design Approach and Hardware Verification

3.1 New Control System Architecture

As it was learned that research cannot rely on too many assumptions and hence, needed to find new approach for designing the controller for the research and verify the mathematical approach with hardware implementation. The new control architecture is as below.

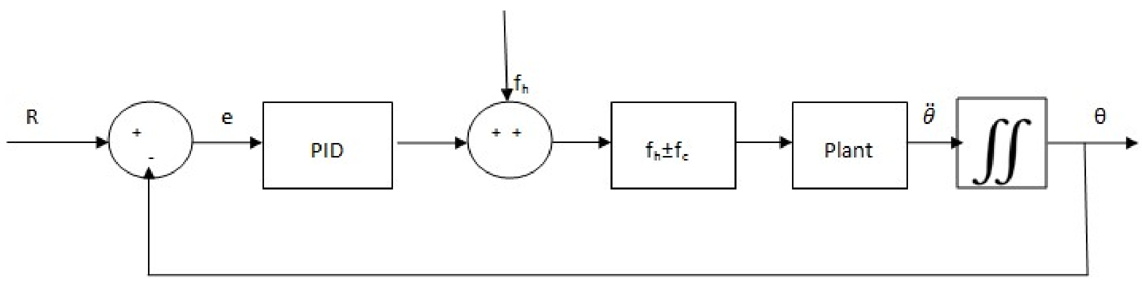


Figure 23: New control architecture

Here, the basic control mechanisms of tri copter were studied further for developing experimental results.

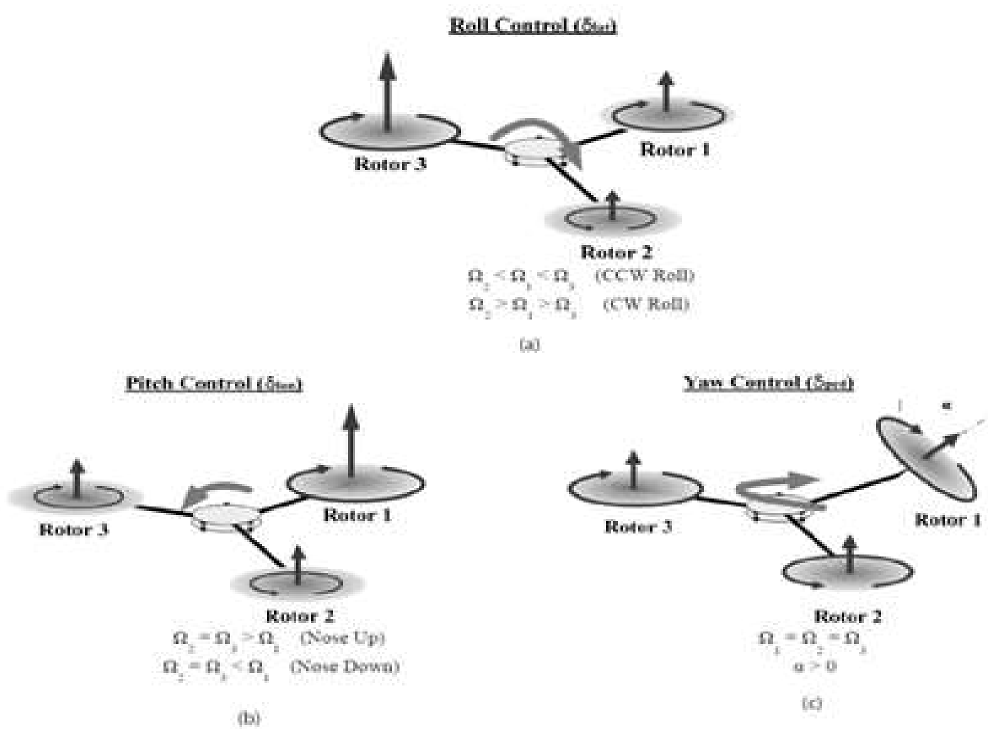


Figure 24: Basic motions of tri rotor

Source: [11]

Lets consider “a” and “b” control scenarios from above Figure 24 and forces acting on rotors can be described as below.

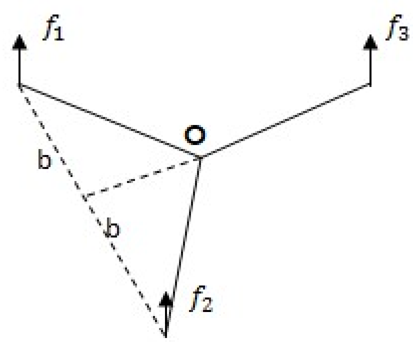


Figure 25: Momentum around center of the platform

From above figure 25, let’s take the momentum acting around “o”,

$M_{xx} = f_1 - f_2$ (42)

But from the control architecture we know that,

$$f_1 = f_c + f_h$$

$$f_2 = f_c - f_h$$

Hence, we arrived on,

$M_{xx} = 2f_c b$ (43)

Dividing both side by Moment of Inertia gives,

$\frac{M_{xx}}{I_{xx}} = \frac{2f_c b}{I_{xx}}$ (44)

But also,

$\frac{M_{xx}}{I_{xx}} = \ddot{\theta}$ (45)

As shown in the control architecture,

$$f_c = K_p\theta + K_d\dot{\theta} + K_i \int \theta \, dt \qquad (46)$$

For roll and pitch, since there is no concern on accumulated error can consider about proportional and derivative (PD) controller.

$$f_c = K_p\theta + K_d\dot{\theta} \qquad (47)$$

Therefore, from equations above we can get,

$$\ddot{\theta} = \frac{2b}{I_{xx}}\{K_p\theta + K_d\dot{\theta}\} \qquad (48)$$

This equation leads to second order differential equation,

$$\ddot{\theta} - \frac{2bK_d}{I_{xx}}{}_d\,\dot{\theta} - \frac{2bK_p}{I_{xx}}\,\theta = 0 \qquad (49)$$

This can be written as below,

$$s^2 - \frac{2bK_d}{I_{xx}}s - \frac{2bK_p}{I_{xx}} = 0 \qquad (50)$$

This is like second order equation of,

$$s^2 + 2\varepsilon\omega_0 s + \omega_0^2 = 0 \qquad (51)$$

Hence can be taken as,

$$\omega_0 = \sqrt{(\frac{-2bK_p}{I_{xx}})} \qquad (52)$$

And also,

$$\omega_0 = \sqrt{(\frac{-2bK_p}{I_{xx}})} \qquad (53)$$

$$2\varepsilon\omega_0 = \frac{-2bK_d}{I_{xx}} \qquad (54)$$

From both above equations substituting to each other we can get,

$$\varepsilon^2 = \frac{-2bK_d{}^2}{I_{xx}K_p} \qquad (55)$$

From this it can be identified that, poles are real-negative and depend on K_p and K_d. Here, K_p and $K_d$are constants and cannot have two values. Hence, it can say that ε = 1, henceforth,

$$I_{xx}K_p = -2bK_d^2 \qquad (56)$$

3.2 Finding the Correction Function

For finding the correction function, it is intended to carry out test using same Brush Less DC (BLDC) motor and the propeller used for the tri copter hardware platform. The experimental set up is as below,

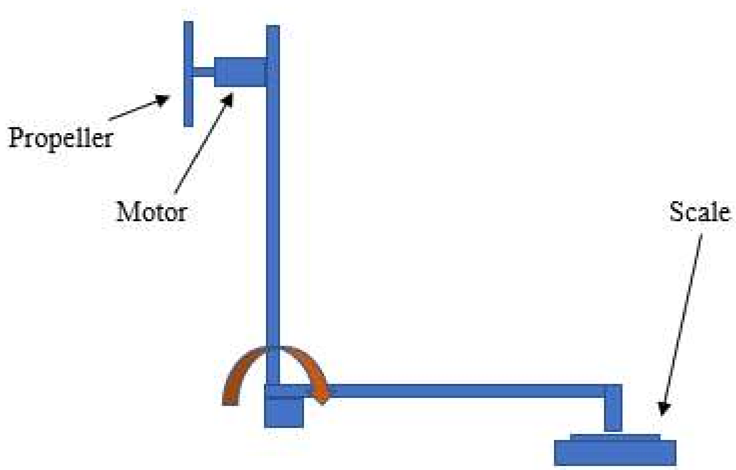


Figure 26: Experimental set up description

Below table mentioned hardware have been used for the actual set up,

Table 3: Components used for Rotor correction function identification

Item	Description
Motor	2212, KV 1400
Propeller	8045
Metal Structure	Made using iron box bars
Arduino programing boards (02)	Arduino Mega

Potentio meter	100kΩ
Jumper wires	Male to Male & Male to female
Bread Board	2 x 5 & 2 x 2 normal bread board
LCD Display	8 x 2 characters
Infrared (IR) sensor	C 03
Weighing Scale	Maximum load sense 5000g

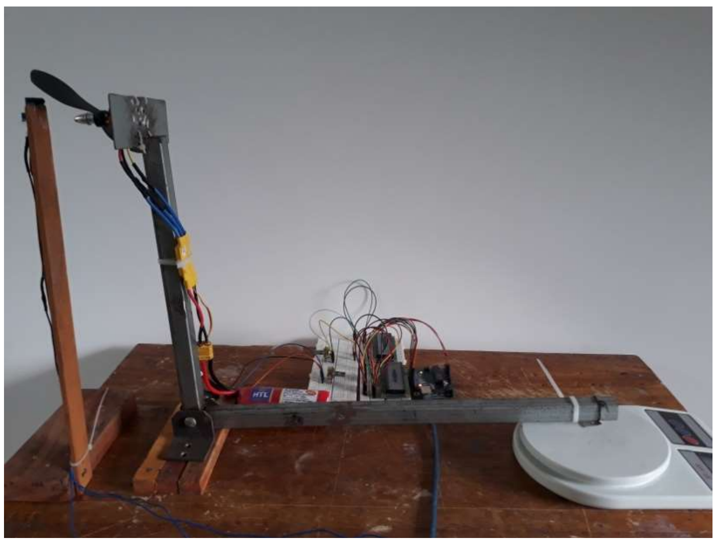


Figure 27: Actual Set Up

For the experiment two Arduino programs were written respectively for IR sensor data read in LCD display and to ARM plus controlling RPM of the motor and they are as Appendix IV and Appendix V. From above experiment have taken RPM vs Thrust data are taken as below table and since we are to consider the momentum around "O" from below set up trust force is actually calculated as below equation.

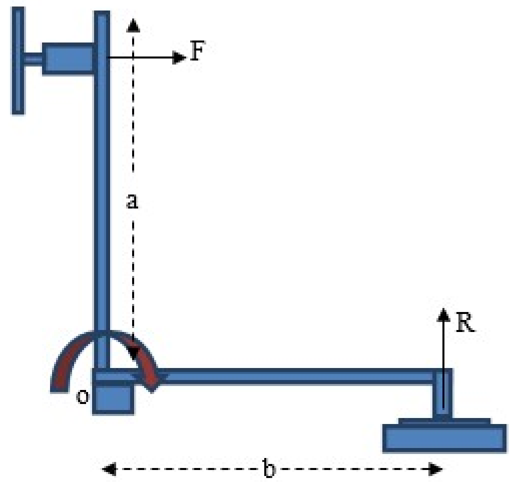


Figure 28: Momentum calculation around the pivoting point

$Fa = Rb$ (57)

Hence,

$F = \frac{Rb}{a}$ (58)

In the physical hardware set up a = 33cm and b = 45cm, from all above below table values are obtained for total thrust is obtained as in below table.

Table 4: Experimental Data of Trust vs Angular Speed from the Physical Set up

RPM	Angular Speed	Scale Reading (g)	F (g)	Single Rotor (KG)	Total Force (N)
1022	106.97	196	237.6	0.238	7.13
1514	158.47	222	269.1	0.269	8.07
1814	189.87	152	184.2	0.184	5.53
2650	277.37	194	235.2	0.235	7.05
2867	300.08	228	276.4	0.276	8.29
3004	314.42	248	300.6	0.301	9.02
3210	335.98	270	327.3	0.327	9.82
5578	583.83	796	964.8	0.965	28.95
6458	675.94	826	1001.2	1.001	30.04
8122	850.10	864	1047.3	1.047	31.42

15180	1588.84	1010	1224.2	1.224	36.73
17540	1835.85	1052	1275.2	1.275	38.25
18703	1957.58	1074	1301.8	1.302	39.05
19643	2055.97	1086	1316.4	1.316	39.49
29305	3067.26	1148	1391.5	1.392	41.75
31773	3325.57	870	1054.5	1.055	31.64

From above table the below graph is drawn using simple Matlab programme by linear approximation [12] as in Appendix VI.

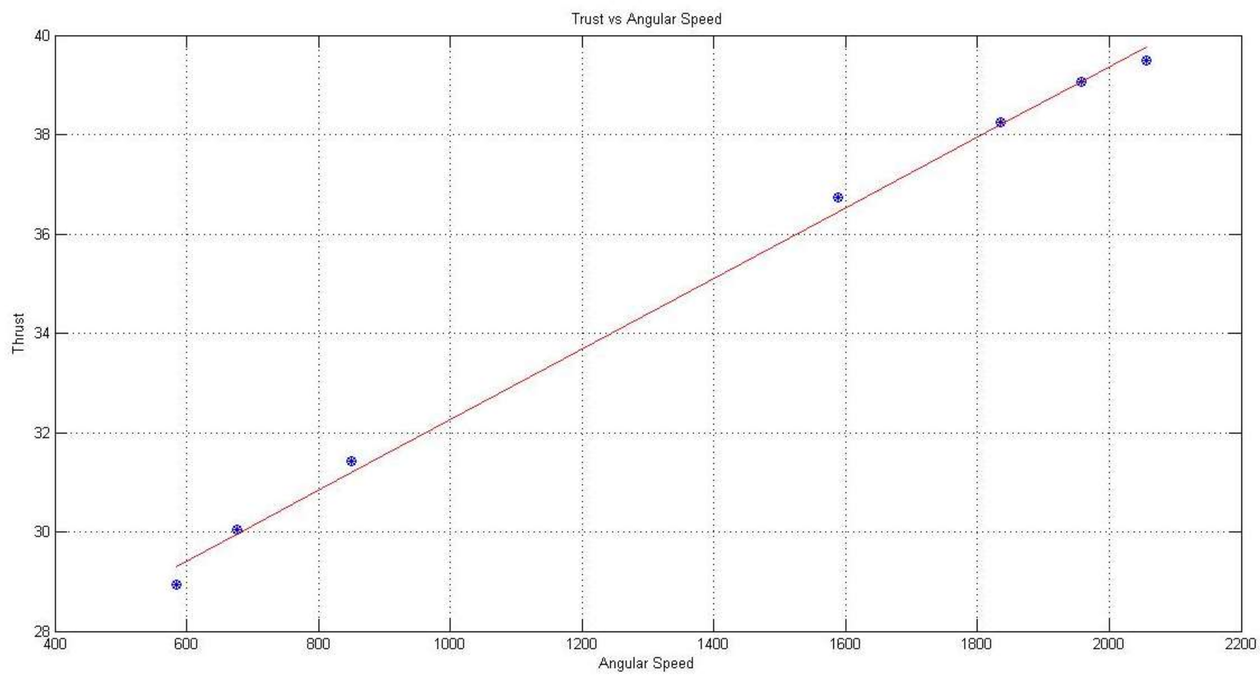


Figure 29: Trust vs angular speed graph

This graph is gives identical linear relation between thrust generated and angular speed. Thus, this can be identified as the correction force function discussed above.

$$f_c = K_d\dot{\theta} + K_p\theta \qquad (59)$$

Hence, the gradient m = K_d,

Which is,

$$m = K_d = \frac{29.96-3\ \ .2}{850-1589} = 0.007 \qquad (60)$$

from equation,

$I_{xx}K_p \ = \ -2bK_d{}^2$ (61)

We get, $K_p \ = 0.14$(Note, Ixx is obtained as 0.0054)

For pitch control similar method is continued around ZZ axis.

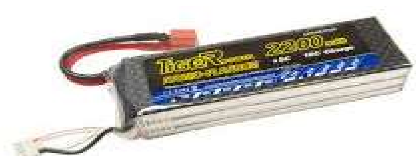
Hence, finally for PD gains for roll & pitch are $K_d = 0.007$and$K_p \ = 0.14$.

This shows that, for system can be used for even yaw calculation by assuming a quadratic graph.

3.3 Hardware Implementation

For hard ware implementation following table no 3 main components have been used,

Table 5: Tri Copter Hardware

Item	Image
Turnigy-Talon Tricopter V.1	
Ardupilot 2.6	
LiPo Batteries (11.1V)	
Fly Sky FS – T6 Transmitter and Receiver	

Before having the Turnigy Talon Tricopter V.1, it had experienced the locally made tri copter frame and could not get desired controls and eventually the tri copter lead to an unrecoverable crash. The locally made tri copter frame is as below.

Figure 30: Locally made frame

The Turnigy Talon Tricopter V.1, used frame with its electronics is below,

Figure 31: Assembled Tri copter with accessories

Chapter 4

Results

4.1 Auto tuned data

The tri copter has been auto tuned to observe the results to check above calculated data with the experiment. The mission planner which is an open source software available gives the auto tuned data as below figure 32.

Figure 32: Autotuned mission planer layout PID values

Mission Planner auto tuned roll variation and pitch variations as in below figures 33 and 34.

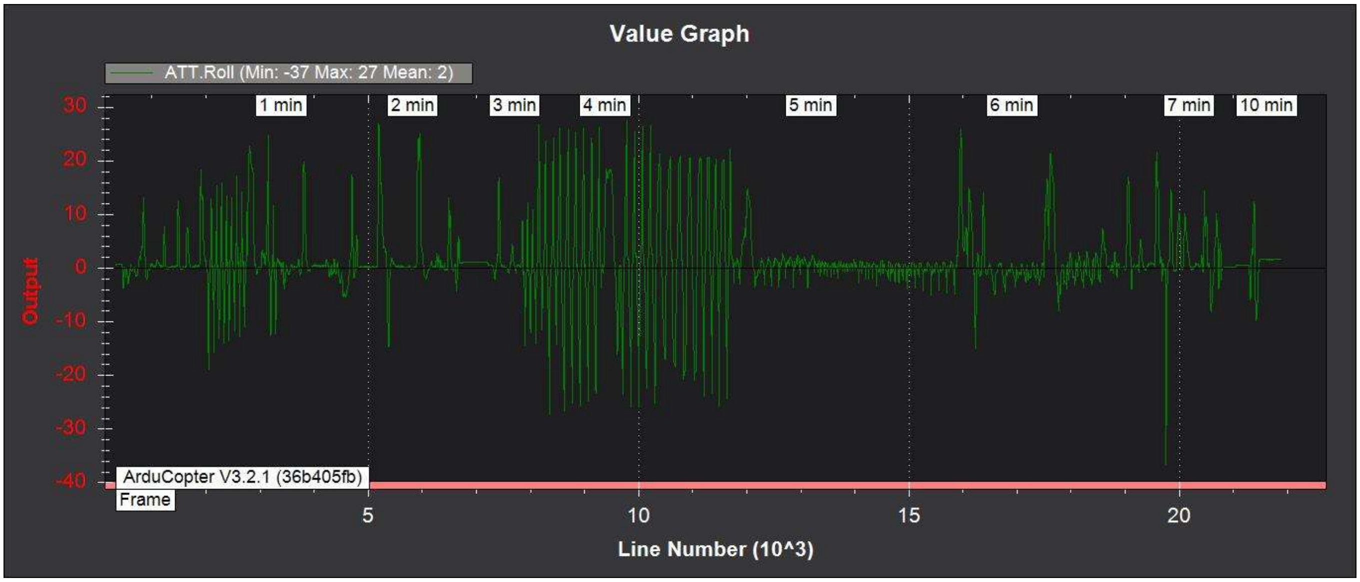


Figure 33: Auto tuned Roll variation

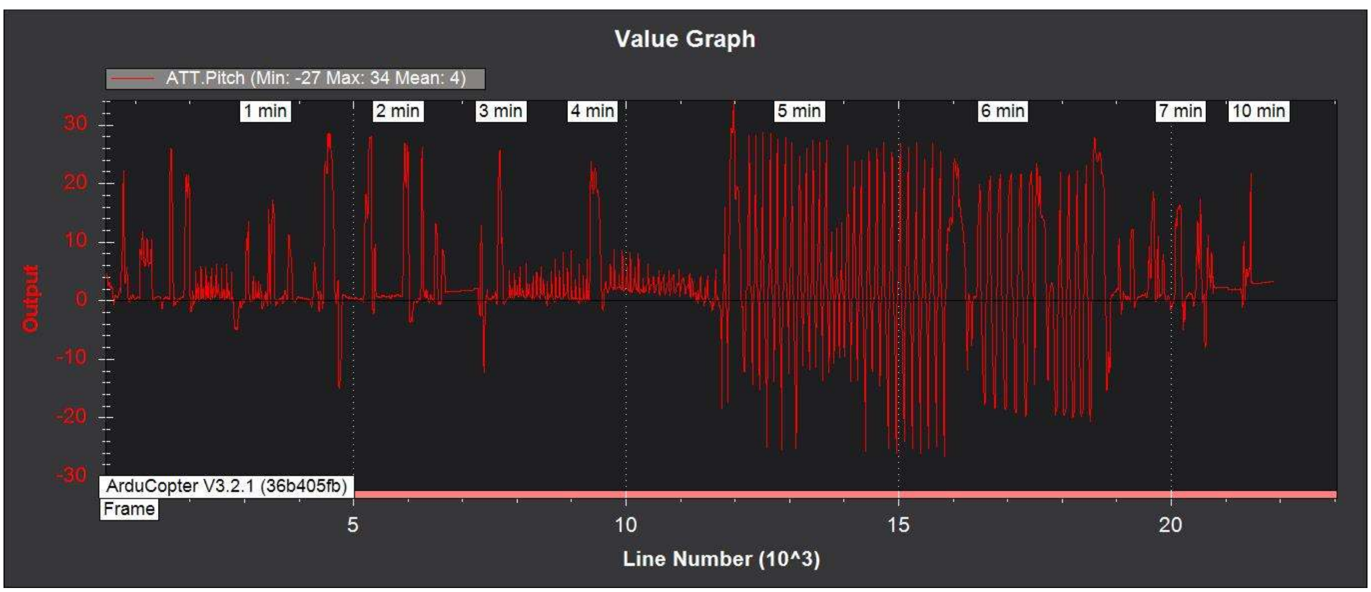


Figure 34: Auto tuned Pitch variation

4.2 Conclusion

It is understood with the above studies if we are to find control mechanism which would fit to ITAE criteria is not the best method since, it is involved with lot of assumptions. Mathematical derivation of a control architecture cannot be based on assumptions with the sensitivity of the problem. As it is understood flying plat forms are having six degree of motion and very sensitive to changes. Hence, best method to find the suitable control architecture should be always based on the mathematical results obtained by experiments and verified tests.

Platform of the tilted tricopter has to have a geared mechanism to control tilted platform, since there need to have a smooth control with minimized friction. It was observed on locally developed tricopter platforms lacks smooth controls. Further, if use only carbon fiber made platform will be able to reduce the weight of the platform and eventually benefited with required thrust generation by choosing less RPM rotors.

In the experiment setup described under section 3.2 has to do with carefully observing variation on RPM is being monitored. Sometimes if there are slight changes with IR sensor position may lead to have different readings with RPM vs Thrust generated. Hence, by changing RPM several times and taking data for thrust is recommended.

From above explained section 3.2 results it is evident that calculated PD values match with tuned roll and pitch values. Hence the graph obtained from the experimental data are coincides with practically obtained tuned data from correction function. Hence this proves the accuracy of the experimental results. This can be extended to find yaw data as well, then the equation becomes quadratic and need additional filtering method for test data. Hence, concluded the research by assuming control system will fit and suit for yaw control as well, since data coincides each other.

Table 6: Hardware verified PD values

Controller	P	D
Roll	0.14	0.007
Pitch	0.14	0.007

4.3 Future Works

It is required to carry out different methods (change of pitch, roll and yaw movements) in different backgrounds (windy weather, gasping weather) and need to log data for the analysis and need to feed corrections until desired output is to be achieved through hardware simulation. Through this it will able to find out disturbances and will be able to smooth the disturbances by analyzing the logged data.

Also, tri copters can be further analyzed depend on the hardware architecture used by the tri coper by changing the angles of rotor arms and by use of each arm twin rotors. These analyses also need to carry out with disturbances to correctly identify the behavior of rotor platforms.

APPENDICES

Appendix I

```
% Mathematical approach for the tri rotor platform
clc
close all
clear all

rpm1 = 800; %Rpm of rotors
rpm2 = 600;
rpm3 = 600;

fi = 30; %Position angles in degrees
theta = 45;
si = 20;

%Forces acting on the plaform at B frame
F_xB = 1;
F_yB = 1;
F_zB = 1;

m = 0.9 %(kg)
l= .45 %(m)
g = 9.812

fi_r = pi*(fi/180);
theta_r = pi*(theta/180);
si_r = pi*(si/180);
```

```
Ix = 0.02396 %Inertia values
Iy = 0.01271
Iz = 0.01273

Rot_x = [1 0 0;0 cos(fi_r) sin(fi_r);0 -sin(fi_r) cos(fi_r)]
Rot_y = [cos(theta) 0 -sin(theta);0 1 0;sin(theta) 0 cos(theta)];
Rot_z = [cos(si_r) sin(si_r) 0;-sin(si_r) cos(si_r) 0;0 0 1];

Rot_t = Rot_x*Rot_y*Rot_z;

I = [Ix*Ix -Ix*Iy -Ix*Iz;-Iy*Ix Iy*Iy -Iy*Ix;-Iz*Ix -Iz*IyIz*Iz]

%Linear accelarations towards x,y,z directions

U_dot_G = (1/m)*(F_xB*cos(theta_r)*cos(si_r)+F_yB*cos(theta_r)*sin(si_r)-
F_zB*sin(theta_r));
V_dot_G = (1/m)*(F_xB*(sin(fi_r)*sin(theta_r)*cos(si_r)-
cos(fi_r)*sin(si_r))+F_yB*(sin(fi_r)*sin(theta_r)*sin(si_r)-cos(fi_r)*sin(si_r))-
F_zB*cos(theta_r)*sin(si_r));
W_dot_G = (1/m)*(F_xB*(cos(fi_r)*sin(theta_r)*cos(si_r)-
sin(fi_r)*sin(si_r))+F_yB*(cos(fi_r)*sin(theta_r)*sin(si_r)-sin(fi_r)*cos(si_r))-
F_zB*cos(theta_r)*cos(si_r));

%linear accelaration
aclrn = ((U_dot_G)^(2)+(V_dot_G)^(2)+(W_dot_G)^(2))^(1/2);

%Rotatioal velocities %%%%%%%%%%

w1 = rpm1/60;
w2 = rpm2/60;
w3 = rpm3/60
```

```
%Calculation of linear forces
%Forces on frame B
F_B = [F_xB;F_yB;F_zB];

F_g = [0;0;m*g]; %Gravitational force

F_t = Rot_t*F_B + F_g; %Total force

%Here we have assumed that fi = 45,theta = 45 and si = 45

vx=(w1*w1+w2*w2+w3*w3);
vy=(w1*w1+w2*w2+w3*w3);
vz=(w1*w1+w2*w2+w3*w3);

for t=1:10
x=vx*t;
y=vy*t;
z=vz*t;
plot3(x,y,z,'rx')
title('Position in 3D space')
grid on
hold on
end
xlim([0,5000])
ylim([0,5000])
zlim([0,5000])
```

Appendix II

```
%%Matlab code for controller design of roll and pitch control
clc
clear all

e =  0.9; % Damping factor
ts = 0.8; % Settling time
l =0.01; %
p = 0.01; %
h = 0.025; %
k = 0.0101; %

fi_ref = 5; % Reference roll angle
theta_ref = 3; % Reference pitch angle

mo_roll = 1.25; % Roll moment disturbance
mo_pitch = 2;   % Pitch moment disturbance

Wn = 4/(e*ts); % Finding natural undamedfrequancy

Kd = 1.4*Wn-h; % Finding D-constant of PD controller
Kp = Wn^(2);  %% Finding P-constant of PD controller

s = tf('s');
Gc = Kp+Kd*s;
[numc,denc] = tfdata(Gc,'v');
TF_Gc = tf([numc],[denc]);
```

```
Gp = (l/(p*s^(2)+h*s+k));
[nump,denp] = tfdata(Gp,'v');
TF_Gp = tf([nump],[denp]);

Gcp1 = TF_Gc*TF_Gp;
fprintf ('Gp =');
Gcp2 = (1+Gcp1);
G = feedback(Gcp1,1); % Obtaining system transfer function
figure(1)
bode(G)
title('Bode plot for System of Roll and Pitch control')
grid on

figure(2)
rlocus(G,'r')
title('Root Locus for System Stability')
grid on

[numG,denG] = tfdata(G,'v');
TF_G = tf([numG],[denG]);
Se = 1/Gcp2; % Obtaining sensitivity transfer function
[numSe,denSe] = tfdata(Se,'v');
figure(3)
bode(Se)
title('Bode plot for Sensitivity of Roll and Pitch control')
grid on
TF_Se = tf([numSe],[denSe])

figure(4)
```

```
rlocus(G,'r')
title('Root Locus for Sensitivity Stability')
grid on

y_roll = minreal(fi_ref*G + (mo_roll*Se));
y_pitch = minreal(theta_ref*G + (mo_pitch*Se));

figure(5)
dur = 7
step(y_roll,dur,'r')
grid on
hold on
step(y_pitch,dur,'b')
title('Step responces for Roll and Pitch Stability')
legend('roll','pitch')
```

Appendix III

```
%%Matlab code for controller design of yaw control
clc
clear all

e =  0.77;
ts = 0.8;

%system is nothing but motor to be controlled, hence motor parameters
l =0.01;
p = 0.01;
h = 0.025;
k = 0.0101;

si_ref = 5;
mo_yaw = 2;

Wn = 4/(e*ts);

Kd = (1.75*(Wn)*p-h)/l;
Kp = (2.15*(Wn^(2))*p-k)/l;
Ki = (Wn^(3)*p)/l;

s = tf('s');
Gc = (Kd*s^(2)+Kp*s+Ki)/(s);
[numc,denc] = tfdata(Gc,'v');
```

```
TF_Gc = tf([numc],[denc])

Gp = (l/(p*s^(2)+h*s+k));
[nump,denp] = tfdata(Gp,'v');
TF_Gp = tf([nump],[denp]);

Gcp1 = TF_Gc*TF_Gp;
Gcp2 = (1+Gcp1);
G = feedback(Gcp1,1);
figure(1)
bode(G)
title('Bode plot for System of Yaw control')
grid on

[numG,denG] = tfdata(G,'v')
TF_G = tf([numG],[denG]);
Se = 1/Gcp2;
[numSe,denSe] = tfdata(Se,'v');

figure(2);
bode(Se)
grid on
title('Bode plot for Sensitivity of Yaw control')
TF_Se = tf([numSe],[denSe])

y_roll = minreal(si_ref*G + (mo_yaw*Se))
dur = 7;
figure(3)
step(y_roll,dur,'b')
title('Step responces for Yaw Stability')
grid on
```

```
figure(4)
rlocus(G,'b')
title('Root Locus for System Stability')
grid on

figure(5)
rlocus(Se,'r')
title('Root Locus for Sensitivity Stability')
grid on
```

Appendix IV

```
#include<LiquidCrystal.h>

LiquidCrystallcd(8,9,4,5,6,7);

constintdataIN = 2;

unsignedlongprevmillis;
unsignedlong duration;
unsignedlonglcdrefresh;

int rpm; // RPM value

booleancurrentstate; // Current state of IR input scan
booleanprevstate; // State of IR sensor in previous scan

voidsetup()
{
pinMode(dataIN,INPUT);
lcd.begin(16,2);
prevmillis = 0;
prevstate = LOW;
}

voidloop()
{
// RPM Measurement
currentstate = digitalRead(dataIN); // Read IR sensor state
```

```
if( prevstate != currentstate) // If there is change in input
  {
if( currentstate == HIGH ) // If input only changes from LOW to HIGH
     {
       duration = ( micros() - prevmillis );
       rpm = (60000000/duration);
prevmillis = micros(); // store time for next revolution calculation
     }
  }
prevstate = currentstate; // store this scan (prev scan) data for next scan

// LCD Display
if( (millis()-lcdrefresh ) >= 100 )
   {
lcd.clear();
lcd.setCursor(0,0);
lcd.print("Speed of Motor");
lcd.setCursor(0,1);
lcd.print("RPM = ");
lcd.print(rpm);
lcdrefresh = millis();
   }

}
```

Appendix V

```
#include<Servo.h>//Include The servo library for the functions used

Servo esc;              //Declare the ESC as a Servo Object
int input;

voidsetup()
{
esc.attach(10);         //Attach the ESC to Digital Pin 10
Serial.begin(9600);  //Begin Reading/Open Serial Monitor
}

voidloop()
{
 input=analogRead(A0);  //Value of input is analog input on pin A0
Serial.print("Original Input Value:");  //Serial print the original input value
Serial.print(input);
Serial.print(" | ");
delay(1);
input=map(analogRead(A0), 0, 1023, 0, 180); //Map the input values from the joystick on analog
pin 0 to  correspond to max and min values for the servo output: 180 and 0
Serial.print("Mapped Input Value:");
Serial.print(input);  //Serial print the mapped input value
Serial.print("\n");
delay(1);
esc.write(input);

}
```

Appendix VI

```
clc
clear all

R = xlsread('Data.xlsx', 1, 'B9:B15')
L = xlsread('Data.xlsx', 1, 'F9:F15')

plot(R,L,'*');
hold on;

P1 = polyfit(R,L,1);
plot(R,L,'o',R,P1(1)*R+P1(2),'r-')
grid on
title('Trust vs Angular Speed')
xlabel('Angular Speed')
ylabel('Thrust')
```

BIBLIOGRAPHY

[1] SIPRI Literature Review for the Policy and Operations Evaluations Department of the Dutch Ministry of Foreign Affairs byVincentBoulanin and MaaikeVerbruggen.

[2] Himodel.com,‘Multi-copterFlightControllerKKV5.5Crossmodefirmware’, 2019.[Online]. Available: http://www.himodel.com/multi_rotors/Multicopter_Flight_Controller_KK_V5.5_Cross_ mode_firmware.html [Accessed: 18- Dec- 2018].

[3] A Survey on Civil Applications and Key Research Challenges - HazimShakhatreh, Ahmad Sawalmeh, Ala Al-Fuqaha, Zuochao Dou, EyadAlmaita, Issa Khalil, Noor Shamsiah Othman, Abdallah Khreishah, Mohsen Guizani.

[4] (Hiranya &Katupitiya et al… 2012) Hiranya Jayakody and Jay Katupitiya. Dynamic Modelling and Analysis of a Vectored Thrust Aerial Vehicle School of Mechanical and Manufacturing Engineering, *University of New South Wales, Sydney, Australia*

[5] (Makoto et al… 2011) Makoto Kumon , Jayantha Katupitiya, IkuroMizumoto. Robust Attitude Control of Vectored Thrust Aerial Vehicles, *Graduate School of Science and Technology, Kumamoto University, School of Mechanical and Manufacturing Engineering, University of New South Wales, Sydney*

[6] (Wan & Dong et al… 2010) Dong-Wan Yoo, Hyon-Dong Oh, Dae-Yeon Won and Min-JeaTahk. Dynamic Modeling and Stabilization Techniques for Tri-Rotor, Unmanned Aerial Vehicles, *Department of Aerospace Engineering, Korea Advanced Institute of Science and Technology, Daejeon 305-701, Korea*

[7] (Yogianandh et al… 2011) Yogianandh Naidoo, Riaan Stopforth and Glen Bright.Quad-Rotor Unmanned Aerial Vehicle Helicopter Modelling & Control, *School of Mechanical Engineering, University of KwaZulu Natal*

[8] (Yeob Shin et al… 2005) Jong-Yeob Shin, K.B Lim and D.D. Moerder. Attitude Control for an Aero-Vehicle Using Vector Thrusting and Variable Speed Control Moment Gyros. *National Institute of Aerospace (N.I.A.), Hampton, NASA Langley Research Center, Hampton.*

[9] (Maiti et al… 2008) DeepyamanMaiti, Ayan Acharya, Mithun Chakraborty, Amit Konar Dept. of Electronics and Telecommunication Engineering Jadavpur University Kolkata, India. Tuning PID and PID Controllers using the Integral Time Absolute Error Criterion

[10] Modeling and Analysis of Tri-Copter (VTOL) Aircraft Sai KhunSai ,HlaMyo Tun, Department of Electronic Engineering, Mandalay Technological University, Mandalay

[11] Dynamic Modeling and Stabilization Techniques for Tri-Rotor Unmanned Aerial Vehicles. Dong-Wan Yoo*, Hyon-Dong Oh*, Dae-Yeon Won* and Min-JeaTahk

[12] Classical Control Systems: Design and Implementation, Prof. Rohan Munasinghe, ISBN 978-81-8487-194-4