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ASSESSING LIFE SPAN OF VEHICLE TYRES BY ANALYZING MULTIPLE FAILURE MODES USING PARAMETRIC APPROACHES

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Abstract

In the recent decades, modelling multiple failure data which was a largely neglected analytical issue until the development of sophisticated techniques has gained a greater enthusiasm among the statisticians, which, can be advocated to address the precise nature of multiple failure type data. This study employs methodologies for the analysis of multiple failure modes for a dataset with two tyre categories namely Cross ply and Radial. Each tyre category had two types of tyres and five distinct modes of failures. Type of tyre and mode of failure are considered as explanatory variables. The lifetime (time to failure) of the tyres, which is the core response of interest, was modelled as a function of the explanatory variables and the fitted models were used to assess the validity of the prevailing warranty period. Two approaches of parametric survival regression were used for regressing the lifetime as a function of the failure mode and tyre type which were the explanatory variables considered in the study. Under the first approach, one model each was fitted for Cross ply tyres and Radial tyres treating all failure modes together, whereas in the second approach, one model for each failure mode of each tyre category was fitted. Reliability estimates for lifetime were obtained and compared across the two approaches of model fitting. A significant difference among the two approaches was not seen with respect to survival estimates. However, with respect to residual analysis, flexibility to accommodate data dependencies and provision for adjusting the shape parameter can be *regarded as improvements of the second approach, which fitted separate models for each failure mode. In conclusion, the methods deployed in this study enable better decision making with respect to warranty setting and identifying failure modes and failure timings precisely.*

Key words: Reliability, Multiple failures, warranty, Censoring, Survival

1. Introduction

Every product or process has its own modes of failures. Analysis of multiple failures has been a major field of interest within the field of engineering where scientists are interested in finding the causes and effects of failures related to components of a system by concentrating on the probability of failures and ways of detecting such failures. Out of the few studies which have been conducted during the past few decades using statistical approaches to model multiple failures, a considerable amount is related to the medical field which concerns the probability of an individual surviving up to a particular time due to a specific cause of death in an environment where multiple causes for death exist. Only few studies have been conducted to analyze the multiple failures which can occur in a particular device or a system which is made out of several components.

A tyre is a sophisticated product where product quality and customer services are highly related. In spite of the extensive attention being paid to the reliability of functional performance of tyres, an important aspect to consider is the high tendency to fail due to various reasons. Such diversities include manufacturing faults, environmental factors, care and maintenance, etc. Also an important characteristic related to tyres is that it can degrade over time whether used or not. In addition to tyre ageing, average air temperature, under or over inflation of tyres, overloading of vehicles, road hazards, improper maintenance, structural defects and improper installation are some other factors associated with tyre failures.

How long a tyre should last is a debatable fact in the tyre industry as no tyre expert could exactly estimate the lifespan of tyres. However, based on experience, a majority of the tyre manufacturers tend to grant a warranty against manufacturing and material defects for five years from the date of manufacture.

This research study is focused on assessing the reliability of tyres considering different failure modes and on gauging the warranty periods granted by a local manufacturing entity with respect to four different tyre types which are of different plies. Tyres of two-ply types are considered (Cross ply and Radial ply) based on the arrangement of the threads.

It is relatively easy to determine the cause and failure time of a product when there is only one cause of failure. However, when there are several modes of failures that can simultaneously occur in a system, determining the root cause of failure becomes cumbersome. In such instances, engineers often tend to consider only one cause which they deem to be the most critical to the product under analysis. This will not be a reasonable practice when every failure mode has a similar contribution to the failure of the product and are equally likely to occur. These different kinds of failures are often referred to as competing failure modes. Therefore, this study contains greater significance as it explains the suitable techniques and methods to use in failure analysis in the presence of multiple failure modes.

Analysis of each failure modes of tyres separately has two main advantages. Mainly it allows quantifying the ascendency of each failure mode as if that mode was the mere reason for failure. Secondly it allows manufacturers to assess the impact on tyre reliability by removing manufacturers' defects that creates early failures. Proper analysis of failure data related to tyres will be highly valuable in identifying different failure modes and the duration for failures. A comparative evaluation of the reliability of tyres would facilitate deciding future strategies in a specified planning horizon. The final outcome of the study would be a useful guideline for tyre manufacturers and vendors to enhance the product reliability and to set warranty policies.

2. Literature Review

Failure time data are typically censored as the event of interest is rarely or never observed. They are usually not normally distributed. The interest of failure data is in failure rates, failure probabilities and quantiles and not in model parameters. Such distinguishing features inherent in failure time data make it difficult to model these data using existing statistical distributions. Therefore, several distributions were identified to take in to account the form of failure time data. In this context, Weibull distribution, Lognormal distribution, Log logistic distribution and a few other distributions appeared to be viable for modelling time to event data.

The Cox's Proportional Hazard (CPH) regression model is based on the Proportional Hazard (PH) assumption, which may not hold in some survival studies. If the assumption does not hold, the standard Cox model should not be used and may entail serious bias and loss of power when estimating and making inferences. In circumstances where the PH assumption is not tenable, Collet (2003) emphasize that Accelerated Failure Time (AFT) models may prove to be fruitful.

Majority of the earlier work found in literature of multiple failure modes were resultant from the medical field. In the medical field, multiple failure modes are referred to as Competing Risks and the modelling is done by the Competing Risk model where the independence between different failure modes is not a necessity (Pintile, 2006). However, in this research study of modeling tyre failure modes, independency among failure modes was assumed.

Although Competing Risk models are inherent in medical research, its applications can be found in other researches as well. Stewart (1993) used a Competing Risk model with a flexible baseline hazard to model unemployment durations in contrast to conventional methods of analyzing unemployment data. Gamerman and West (1978) have focused on a dynamic Bayesian model for survival data in a study of analyzing contributory factors to unemployment where the model treats survival data as time series data. This study highlights the need to model time varying relationships with explanatory variables which cannot be accommodated with standard proportional hazards model.

The analysis of failure data with multiple failure modes involves complicated distributions under Accelerated life tests (ALT). Proportional Hazards (PH) models (Cox, 1972) and Proportional Mean Residual Life (PMRL) models (Chen et al., 2005) are applicable in situations of this nature. According to Peiravi and Dehqanmongabadi (2008), the PMRL model is preferred over the PH model when the concern is on the remaining lifetime of an individual at time t. However, when the main concern is on the risk of the immediate failure, PH models provides better results. Although they considered only two failure modes in their ALT, the procedure they followed can be generalized to N number of failure modes.

For highly reliable products where few or no failures are observed, making use of the survival times to assess reliability is not sensible. In such cases, failures arise from a degradation mechanism. One method of analyzing such data is to monitor the device for a specific time period and examine the changes in performance during the observed time period. But such data provides little information about proportion of survival.

A study which has a statistical approach has been carried out by Wu (2007) in the context of vehicle tyres. In his study, Log rank tests and Kaplan Meier curves have been used to compare risk factors which might lead to tyre ageing. He also makes use of Weibull analysis and Cox Proportional Hazards models in order to predict tyre failure probabilities and to explore tyre ageing relative risks. Finally, he has concluded that the three analytical models have internal links to each other and provide similar results. According to these findings, initial tyre loads, higher mileage, tyre types and manufacturing characteristics could be considered as contributory factors to tyre ageing and tyre failures. Even though some statistical methods were used in this study, comprehensive explanations on their applications and procedures used are not clearly stated. Although some articles emphasize the need of modelling multiple failure modes, only a few studies were found and most of them are based on engineering and counting processes.

Therefore, the contribution from this research study will be significant, as both censored and uncensored observations have been used in analysis and a significant proportion of quality details regarding tyres have been obtained from manufacturers. This study focuses on comparing two approaches of modelling multiple failure data, which will help to point out the nature of applicability of those approaches.

3. Methodology

3.1 Analytical Methods

This study focuses on illustrating methodologies for the analysis of multiple failure modes by way of an example related to failures of vehicle tyres. The core response of interest in this study is the lifetime of the tyres. Considered variables were tyre type, orientation of plies, different failure modes and granted warranty periods. Lifetime, which is a continuous variable, has been modeled as a function of the explanatory variables using a simple regression model.

Initially the data was analyzed descriptively using Bar charts, Survival plots and Hazard plots. The Log-Rank test and Wilcoxon test were used to test the differences between survival experiences of different groups. The distribution of the lifetime was examined using different failure time distributions such as Log Normal and Weibull. Cumulative probabilities were considered, and transformations were done according to the assumptions and properties of the above distributions. Probability plots, Anderson Darling test and Correlation Coefficient were used to assess the fit of the assumed distribution. Variables were selected using forward selection method and model comparisons were done using the likelihood ratio tests.

One of the frequent problems in estimating parameters of regression models is the failure of the likelihood maximization algorithm to converge, which is known as complete or quasi complete separation, during the analysis. Among some methods proposed in the literature, it was decided to leave the problematic variable in the model as it is not sensible to use the other suggested methods to overcome the problem of quasi complete separation in this study (Allison, 2008).

Type of tyre and mode of failure are considered as explanatory variables. The lifetime (time to failure) of the tyres, which is the core response of interest, was modeled as a function of the explanatory variables.

Censored Cox-Snell residuals were plotted in order to assess the adequacy of the fitted models. A single tyre was considered as a series of system when calculating the reliability estimates. Percentiles, mean time to failures were calculated to find the time at which a specified proportion of tyres fail and the length of time a tyre is expected to last in operation without any failure.

3.2 Description of Data

The data for the study was collected from a reputed local tyre manufacturing company. Four types of tyres (brands) were considered denoted by type A, type B, type C and type D, respectively, as the original names of the tyres cannot be divulged owing to reasons of confidentiality. The four tyre types can be further grouped into two categories based on the orientation of plies as “Cross ply” and “Radial”. Type A and type B are recognized as cross ply tyres whereas type C and type D are recognized as radial tyres.

Records of tyres which were manufactured during the period of 1st of January 2013 to 31st of August 2014 were considered. Reported claims during the same period were taken into account assuming that the absence of reporting of a failure during the study period of twenty months was due to the tyre not failing during the first twenty months of service life. This study deals with the time to observe the first failure, as the company practice is to replace the tyre when it reports a failure rather than repairing in order to maintain a competitive warranty policy.

For tyres which report claims during the study period (uncensored observations) the difference between the month of manufacture and the month in which the tyre reports a failure was computed as the time to failure of the particular tyre. For tyres which do not report any failure during the last day of the study period (31st of August 2014), the duration was computed as the lifetime of the tyre.

Examination of data revealed that type A and type B together share common failure modes which are different from the failure modes shared by type C and type D, which clearly indicates that failure modes of Cross ply tyres are different from those of Radial tyres. Thus, the data set was reproduced by pooling records of type A with type B and type C with type D. The selected failure modes and the respective codes given to identify them are presented below.

Table 1-Failure modes of tyres

Cross ply tyres		**Radial tyres**	
Defect Mode		Defect Mode	
1	Tread/Shoulder separation	**7**	Sidewall separation
2	Carcass Break up	**8**	Spot wear
3	Run under inflated	**9**	Air leakage
4	Flex break	**10**	External damages
5	Flow crack on sidewall	**11**	Through cut on sidewall

Source: Author developed

Table 2- Descriptive Statistics of data

Type of tyre		**Total Number of tyres**	**Number of failures**
Cross ply	**A**	11429	553
	B	19772	206
Radial	**C**	81709	169
	D	75070	192
Total		**187980**	**1120**

Source: Author developed

The dataset resulted in a total of 187,980 tyres where 1120 of them were reported for claims. Each tyre is at risk of five failure modes at each point of time. When a tyre failed from a particular failure mode, another four records were added considering the fact that the tyre is censored due to other four possible failure modes. Thus, each tyre will account for five records.

4. Analysis and Discussion

From the descriptive analysis it was seen that, though the production of Cross ply tyres is low compared to Radial, the number of claims reported from Cross ply tyres is comparatively high. This is mainly due to the high failure contribution of tyre type A. Break down of failure modes among the tyre types indicated that the root cause for the low reliability of tyre type A is failure mode 1.

Further in order to get a clear insight on the tendency to fail from each possible failure mode, the probabilities of failure due to each failure mode were calculated based on the raw failure counts for each tyre type.

Table 3- Classification of failures

	Failure mode					
Cross ply tyres	**1**	**2**	**3**	**4**	**5**	**Total**
A	491	0	27	20	15	**553**
B	84	69	24	20	9	**206**
Total failures	575	69	51	40	24	**759**
	Failure mode					
Radial tyres	**7**	**8**	**9**	**10**	**11**	**Total**
C	137	3	14	7	8	**169**
D	60	62	27	22	21	**192**
Total failures	197	65	41	29	29	**361**

Source: Author developed

Since the tyres considered in the study, entered into the study at different time points ranging within a time period of twenty months, indications given by the raw failure counts cannot be regarded as conclusive measures for comparison. Moreover, making use of the time at which tyres reported for failures would provide more sensible insights rather than the raw failure counts. Therefore, survival experiences and hazard rates of failure modes and tyre types were then analyzed.

In comparison of failure processes, the instantaneous failure rate (Hazard rates) depicts a more dynamic view of the process. Therefore, the Hazard rates were compared across different tyre types and it was found that there is an absence of early failures in Cross ply tyres when compared to Radial. Log Cumulative Hazard plots were drawn for tyre types separately with the aim of examining whether the assumption of proportional hazards is plausible and was found that the said assumption was not valid and hence it is not viable to use hazard models in further analysis.

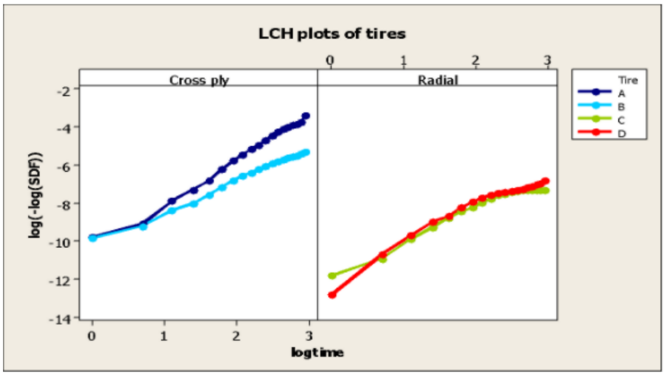

Figure 1 - Log Cumulative Hazard plots of tyres

Then Kaplan Meier estimates of the survival function were computed for different tyre types. Log Rank test and Wilcoxon test were used to test the significant difference between the survival patterns among the tyre types where some contradicting results were obtained due to the violation of the above assumption.

Since the formal non-parametric tests seem unviable, the focus moved on to parametric models. Subsequently, identification and validation of suitable failure time distributions for the data were done and was concluded that Weibull and Lognormal distributions were well-fitted for Cross ply data and Radial data respectively, which was further confirmed by the Anderson Darling goodness of fit test statistic and a high correlation coefficient compared to other survival distributions.

Two approaches have been taken in this study in order to model multiple failure modes of tyres. The first approach deals with performing a single model treating all failure modes together whereas the second approach deals with fitting separate models for each failure mode.

<u>Approach 1</u>

Selection of the most suitable model was done by examining the significance of the parameter coefficients and by comparing models using deviance. Deviance values implied that the two factors act independently. But the significant interaction terms suggest that the two factors are correlated. After a comprehensive review of the model fitted, a proper identification of the root cause could not found, so it was suspected that the reason for this contradiction would have resulted due to some dependencies present in data.

Then, methods of analyzing correlated survival data were reviewed and used. However, parametric modelling approaches for correlated survival data are still in the development stage and are not readily available in most statistical software packages. Semi-parametric approaches like Cox proportional hazard models which can be adopted using the methods like sandwich variance estimation and Generalized Estimation Equation (GEE) (Gharibvand and Liu, 2009) were also not suitable to use in this study as the assumption of proportional hazards is violated.

Under approach 1, the model which contains both the main effects and interactions was selected as the best model for both Cross ply and Radial tyres. The adequacies of the fitted models were graphically checked using the Cox-Snell residual plots and it was clearly seen that the majority of plotted points fit the fitted line adequately and are within the simultaneous confident bands.

Parameters are estimated using Maximum Likelihood method through the iterative process of modified Newton-Raphson algorithm. The reason for some large parameter estimates and extremely large standard errors was suspected as the existence of quasi complete separation. As suggested by Allison (2008) it was decided to leave the problematic variable in the model as it is, because it is not sensible to use the other suggested methods to overcome the problem of quasi complete separation in this study.

Regarding the models fitted under approach 1, reliability characteristics such as survival probabilities, warranty periods and mean time to failures (MTTF) were calculated.

Table 4–Mean time to failure (in months)

	Failure mode					Overall MTTF	Warranty Period
Tyre type	**1**	**2**	**3**	**4**	**5**		
A	38.112	very large	118.345	133.065	148.891	**68.223**	16.73 (1.39 years)
B	94.758	102.326	154.577	165.988	226.748	**125.161**	30.70 (2.56 years)
Overall MTTF	**53.281**	**121.977**	**137.264**	**150.929**	**184.261**	**89.6428**	

	Failure mode						
Tyre type	**7**	**8**	**9**	**10**	**11**	**Overall MTTF**	
C	146213	1699923	647390	1047684	925738	383746	215 (17.92 years)
D	254685	247917	417097	491902	495010	340365	191(15.89 years)

Overall MTTF	**182305**	**390864**	**505113**	**654587**	**636009**	**361152**

Source: Author developed

<u>Approach 2</u>

According to the literature related to modelling multiple failure modes, several studies have focused on conducting separate analyses for each failure mode. A major reason for this approach is the impossibility of identifying an exact distribution which fits data perfectly when all the failure modes are pooled together.

In order to perform analysis for failure modes individually, as the initial step, the most suitable distributions for each failure mode of Cross ply and Radial tyres were examined using probability plots. It was concluded that Weibull and Lognormal distributions were well fitted for each failure mode of Cross ply tyres and Radial tyres, respectively.

Then, the most suitable model was chosen by performing the likelihood ratio test followed by the examination of adequacy of the chosen model using Cox-Snell residuals. Parameters of the chosen models were estimated using maximum likelihood method. Finally, overall tyre reliabilities were calculated by combining the results of each failure mode appropriately.

Table 5– Calculated warranty periods in months for each failure mode.

Plies	Failure mode	Warranty Period
Cross Ply tyres	**1**	13.4621 (1.12 years)
	2	26.2671 (2.19 years)
	3	41.8645 (3.89 years)
	4	52.1839 (4.35 years)
	5	212.535 (17.71 years)
Radial tyres	**7**	76.5038 (6.38 years)
	8	146.293 (12.19 years)
	9	2033.25 (169.44 years)
	10	291.069 (24.25 years)
	11	1539.76 (128.31 years)

A significant difference among the two approaches was not seen with respect to survival estimates in short run. However, with respect to residual analysis, flexibility to accommodate data dependencies and provision of adjusting the shape parameter can be regarded as improvements of the second approach which fitted separate models for each failure mode.

Validating the results of the models with actual data was not possible due to non-availability of adequate, suitable failure data.

5. Conclusions and Implications

In accordance with the insights gained through the descriptive analysis, it was identified that, even though the production of Cross ply tyres is low compared to the Radial tyres, the number of claims reported from Cross ply tyres is comparatively high. It can be argued that this is mainly due to the high failure contribution of tyre type A. The breakdown of failure modes among the tyre types indicated that the root cause for the low reliability of tyre type A is failure mode 1, which manufacturers should pay immediate attention to.

According to the reliability figures at two-year time period, it was identified that the failure mode 1 of Cross ply tyres is the most critical mode among others, which manufacturers should pay attention to. When considering the reliabilities of each type of tyre at five years period it was depicted that tyre type A has the lowest survival probability. One can argue that this is due to the comparatively higher number of claims reported from tyre A. Moreover, it should be noticed that this may be due to nonsensical extrapolation as the study tries to estimate survival probabilities for a lengthy period of five years using data relevant only to twenty months.

With reference to the calculated warranty periods it can be concluded that for Cross ply tyres, the prevailing warranty period of five years granted by the manufactures is invalid, forcing the manufacturers to identify root causes prevalent in the manufacturing process. For Radial tyres, the prevailing warranty period can be extended as the calculated warranty periods are greater than five years. Such a move will attract more consumers.

The comparison done regarding the two approaches revealed that neither approach is significantly better than the other with respect to short term estimates. However, a clear classification of the two approaches would be possible if more claim data are incorporated in the analysis as it was identified that the estimates given by the two approaches depends on the failure frequency and the time period of interest.

Bootstrapping could have been used in the validation procedure in order to overcome the issue of lack of failure data. Thereby an assessment of the fitted models can be made by the results obtained for the test data. Further research can also be carried out on penalized maximum likelihood estimation to overcome the convergence issue. Incorporation of Expectation Maximization (EM) algorithm can also solve the issue of incomplete data. By incorporating these suggestions, the scope of the research analysis can be broadened while improving the stability of the research outcomes.

According to the insights gained by literature related to tyre ageing, the failure modes considered in this study can be applied to any type of tyre regardless of the manufacturer, as they are some of the common modes of failures that could be seen in any type of tyre. In essence, even though the study is titled as a case study and deals with data obtained by a specific tyre manufacturer, it can be argued that the results can be applied in general. Therefore, the whole tyre industry can make use of the estimations done in this study to improve the quality of the products and make more economically and strategically useful decisions by incorporating the stated suggestions and overcoming the drawbacks which are identified in this study.

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