INVESTIGATION ON THE EQUIVALENT MODULUS OF SUBGRADE REACTION OF LAYERED SOIL

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This dissertation was submitted to the Department of Civil Engineering of the University of Moratuwa in partial fulfilment of the requirements for the Master of Engineering in Foundation Engineering and Earth retaining systems (2016/2017)

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January 2020

DECLARATION

I declare that this is my own work and this dissertation does not incorporate without acknowledgement of any material previously submitted for a Degree or Diploma in any other University or institute of higher learning and to the best of my knowledge and belief it does not contain any material previously published or written by another person except where the acknowledgement is made in the text.

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Prof. Saman Thilakasiri Date

Dean,

Faculty of Engineering

SLIIT

ABSTRACT

'Modulus of subgrade reaction' is the ratio between the pressure applied on the soil and the corresponding settlement. There is no theoretical relationship found to obtain equivalent subgrade modulus of layered soil. Top dense sand layer thickness, bottom loose sand layer thickness, strip footing width and thickness are changed and equivalent modulus of subgrade reactions are obtained by equivalent spring theory and weighted average method. These, equivalent subgrade modulus are separately applied in Heteryni method equations in order find vertical settlement, bending moment and shearing force along the medium length footings. PLAXIS 3D numerical models are developed for same footing parameters and soil properties to compare the Heteryni method outputs.

Equivalent subgrade modulus using equivalent spring method is constant with top soil layer thickness for a given footing width and footing depth. Weighted average method equivalent subgrade module is non linearly increasing with top dense sand layer thickness for a given footing and bottom loose sand layer thickness. Equivalent subgrade module for thinner footing depth is always greater than the thicker footing for a given footing width and soil profile in both spring theory and weighted average method. Settlement along footing obtained by equivalent spring method equivalent subgrade modulus applied in Heteryni method equation is highly varying from weighted average method equivalent subgrade module applied in Heteryni method equation and PLAXIS 3D model settlement output. Equivalent spring method is considered as unsuitable to calculate the equivalent modulus of subgrade reaction for layered soil stratum. Settlement difference between PLAXIS 3D method and weighted average method equivalent subgrade module applied in Heteryni method equation shows up to 45 percentages and this difference cannot be negligible.

This study will shed a light in the theoretical relationship of equivalent subgrade module research field as this would be the first attempt to check the behavior and suitability of equivalent subgrade modulus of layered soil stratum.

***Keywords**: equivalent subgrade modulus, layered soil, strip footing, PLAXIS 3D, beams on elastic foundation, Hetenyni method, finite element method*

ACKNOWLEDGEMENTS

This master thesis work is done in order to meet the requirements of the two year M.Eng in Foundation Engineering and Earth Retaining Systems course. This thesis is carried out at the Department of Civil Engineering at University of Moratuwa.

I hereby thanking all the academic and administrative staffs for their supports in accomplishing this task. I specially thank Dr.L.I.N. De Silva for coordinating and arranging all the academic matters to begin my Master studies at University of Moratuwa.

I am grateful to Professor Thilakasiri for his supervision and letting me work on this thesis. I have to thank my supervisor Professor Thilakasiri to spend his valuable time with me on clarifying matters on the whole process of writing my thesis and to help me on technical matters.

Finally, I thank all those who helped me in various ways to complete this thesis successfully.

30.01.2020 **T.Mathuwanthy**

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CHAPTER ONE

1.0 INTRODUCTION

1.1 Background

'Modulus of subgrade reaction' is an important term that is frequently used in structural analysis and design of both shallow and deep foundations. The ratio between the pressure applied on the soil and the corresponding settlement is termed modulus of subgrade reaction. Modulus of subgrade reaction of a homogeneous soil shall be found using several theoretical methods such as, Parry's method, Terzaghi and Peck method, Bowle's method and Vesic's method. None of the methods give any suggestions to find the equivalent subgrade modulus of layered soil medium. Elasticity theories proposed by various researchers to find subgrade modulus are primarily focused on homogenous medium. But in most practical situations ground soil profile is not homogeneous, it is a combination of different soil layers. Subgrade modulus of layered soil can be determined by Plate-load test. Time consumption and difficult to load the plate uniformly are some problems in the Plate-load test (Joseph E. Bowles 1997). Therefore, it is important to develop an appropriate theoretical relationship between subgrade modulus of individual homogeneous soil layers and equivalent subgrade modulus of that soil layers.

This research work is one of the attempts to find a suitable theoretical relationship for equivalent subgrade modulus of layered soil from individual subgrade modulus. Homogeneous modulus of subgrade reaction obtained from Vesic method are used in equivalent spring theory and weighted average method equations to find the equivalent modulus of subgrade reaction of layered soil stratum. This equivalent modulus of subgrade reaction are separately applied in Heternyi method to obtain the settlement curves along strip footing. Settlement curves from PLAXIS 3D numerical model and Heternyi method are compared to estimate the accuracy of the equivalent subgrade modulus.

1.2 Scope

If the research outcome of the equivalent subgrade modulus could be validated with PLAXIS 3D, then this could be used in the foundation engineering. Huge uncertainties associated with the site plate load test on determining the equivalent modulus of subgrade reactions could be eliminated by using the research outcome. The scope of this research is to aid the accuracy on the foundation design results which will contribute to much more economical outcomes.

1.3 Objectives of the research

Still there hasn't been much direct theoretical or empirical equations are developed to find the equivalent modulus of subgrade reaction of layered soil. Primary aim of this research is to find an appropriate relationship from the obtained equivalent subgrade modulus through weighted average method and equivalent spring theory.

The results will be validated with PLAXIS 3D numerical model. This equivalent subgrade modulus of layered soil can be used to find the vertical settlement, bending moment and shearing force of strip footings resting on layered soil.

1.4 Contents of the report

This report consists of seven chapters begins with research introduction. Introduction and motivations behind carrying out of this study are in Chapter one. Further it explains the background of the selected study as well. Chapter two elaborates homogeneous subgrades modulus of individual soils, beam on elastic foundation and numerical modelling of PLAXIS 3D of previous research attempts powered this research field. Chapter three describes the methodology followed; the dimension selection of footing and soil layers, material properties of soils, PLAXIS 3D model inputs, finite element meshing, validation of PLAXIS 3D model, homogeneous subgrade modulus of soil layers using Vesic method, equivalent modulus of subgrade reaction using equivalent spring theory and weighted average method. Chapter four summarizes the results obtained and discussion on results. Chapter five is the conclusion part of the present study and recommendations for future works. Chapter six and seven are references and appendix respectively.

1.5 Input variable selection

Both soil and foundation properties are affecting stiffness of the foundation. Following soil and footing properties are selected for this study.

01. Soil layers: Dense sand and loose sand are selected for the study. Young modulus, Poison ratio, permeability, cohesion and angle of friction are the quantifiable physical properties which may differ between these loose and dense mediums. These physical property values will be chosen to match the Sri Lankan typical geology.

02. Width of the footing (B): Three different footing widths are chosen to study the size effects in the numerical model. Strip footing widths 1.0m, 1.5m and 2.0m are selected.

03. Thickness of the footing (t): Two different footing thicknesses 0.4m and 0.6m are chosen to study the footing thickness effects in the numerical model.

04. Column clear spacing to footing width ratio (S/B): S/B of 4.0 is chosen as the arbitrary combination in this study.

05. Soil layer Thickness: Three different top layer thicknesses are assumed in this study as a function of foundation width (B). 1.0B, 2.0B and 3.0B are the selected top soil layer thicknesses from ground level. Bottom layer thicknesses will be chosen as 1.0B, 2.0B and 3.0B against each top layer thickness which will result in nine combinations. So, these nine combinations could be studied for each foundation width.

Following assumptions are made during this input variable selection.

1. Soil layers and footing are horizontal
2. Footing base is resting on the ground level (no embedment)
3. Water table is not present
4. Hard layer appears below the bottom soil layer

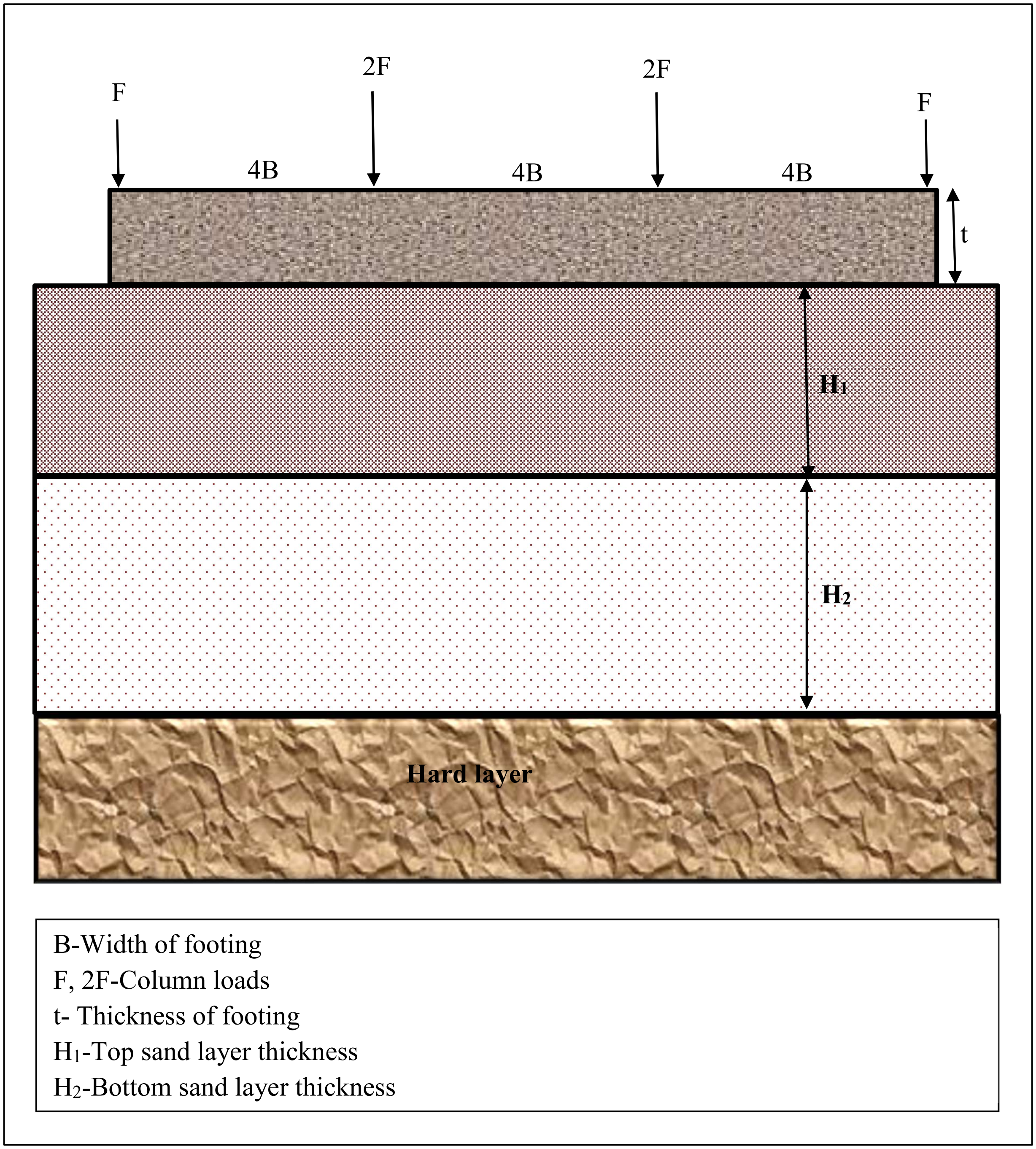


Figure 1. 1 : Schematic sketch of the strip footing

CHAPTER TWO
LITERAURE REVIEW

2.1 Subsoil profile of Sri Lanka

Since Colombo is the capital city of Sri Lanka, major construction projects are in Colombo district. Type of soil distribution and some subsoil cross sections of Colombo district are considered in this study. Various soil stratums such as sand, sand stone, residual soil, peat and alluvium soil are distributed as non-uniform layers in various depths in Colombo (Cooray 1967). Therefore, studies on layered soil medium will be most important to design the foundations in Sri Lanka.

2.2 Properties of Soil

Following design parameters are obtained from 'Publications of the Geotechnical Control Office', Hong Kong (1982)

Table 2. 1 : Soil deformation parameters (Geotechnical control office 1982)

Soil Type	Young's Modulus (E) (MPa)	Poisson's Ratio (ν)
Loose sand	5-20	0.30-0.40
Medium dense sand	16-40	0.20-0.35
Dense sand	30-100	0.15-0.30
Soft clay	1-4 (2-6)	0.10-0.30
Firm clay	3-8 (5-12)	
Stiff clay	5-15 (10-20)	

Table 2. 2 : Design parameters in Sandy soils (Geotechnical control office 1982)

Average SPT	Layer description	Shear strength parameters (c'(kPa), ϕ' (˚)	Submerged Density (kN/m^3)	Elastic Modulus E (kN/m^2)
0-1	Very loose sand	c' = 0, ϕ' = 23	8.0	4000
3-5	Loose sand	c' = 0, ϕ' = 24	8.0	5000
6-8	Loose sand	c' = 0, ϕ' = 26	8.0	7000
10-14	Medium dense sand	c' = 0, ϕ' = 28	8.5	12000
15-20	Medium dense sand	c' = 0, ϕ' = 30	8.5	18000
20-30	Dense sand	c' = 0, ϕ' = 33	9.0	20000
30-50	Very dense sand	c' = 0, ϕ' = 38	9.0	25000
> 50	Very dense sand	c' = 0, ϕ' = 43	9.0	> 25000

2.3 Modulus of subgrade reaction

The ratio between the pressure applied on the soil and the corresponding settlement is termed the modulus of subgrade reaction. It is noted from the theoretical background that this parameter plays an important role in the evaluation of deflections and stresses in pavement slabs and plates resting on soil. The magnitude of subgrade modulus depends on several factors such as width, shape, position and depth of the foundation. It was found that a higher value of modulus of subgrade will lower the maximum deflection and bending moment.

The modulus of subgrade reaction can be determined by various mathematical equations and both field tests and laboratory tests. The most common tests used are Standard Penetration Test, Cone Penetration Test, plate load tests and tri-axial tests. The subgrade modulus values will vary over the entire area which makes the soil investigation by plate load test more complex and expensive (Ranjitha and Sathyanarayanan Sridha 2017). The subgrade reaction from the plate load test should be adjusted because the subgrade reaction is a function of:

- Soil elastic properties, both the initial response and the long-term response due to soil consolidation from the sustained loading.
- Loading intensity that will influence the long-term consolidation settlement.
- Amount of surface area loaded and load shape over which the load is applied. Wider and larger area loadings will involve consolidation of the deeper soil layers.
- Stiffness of the slab, which will influence the distribution of the soil bearing pressure

Field tests are cumbersome, expensive, and time-consuming. Theoretical equations are limited for homogeneous soils. Therefore, it is important to develop theoretical relationship for layered soil.

Vesic proposed following equation for modulus of subgrade reaction of a homogeneous soil.

$$k_s = \frac{0.65Es}{B(1-v^2)}\left(\frac{E_s B^4}{E_b I}\right)^{\frac{1}{12}} \quad (1)$$

Here,

k_s - Modulus of subgrade reaction of soil

B - Width of the footing

E_s - Elastic modulus of soil

E_b - Elastic modulus of footing

I - Moment of inertia of footing

v - Poisson's ratio of soil

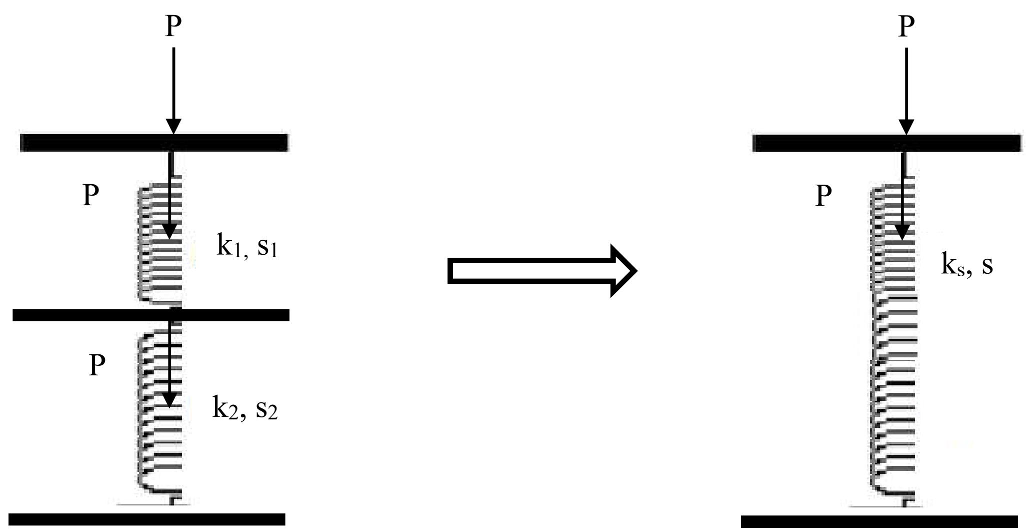
Following table is used to estimate the subgrade modulus values (Joseph E. Bowles 1997).

Table 2. 3 : Range of modulus of subgrade reaction k_s (Joseph E. Bowles 1997)

Soil	k_s(kN/m^3)
Loose sand	4800 - 16000
Medium dense sand	9600 - 80000
Dense sand	64000 - 128000
Clayey medium dense sand	32000 - 80000
Silty medium dense sand	24000 - 48000
Clayey soil, qa <= 200 kPa	12000 - 24000
Clayey soil, 200 kPa < qa <= 800 kPa	24000 - 48000
Clayey soil, qa > 800 kPa	> 48000

2.4 Equivalent spring theory of layered soil

The concept of spring constant was first introduced by Wrinkler in 1867. He modeled flexible foundation, such as raft, to stand on an independent discrete spring elements or supports. In 1955, Terzaghi, in his paper ‘Evaluation of coefficients of subgrade reaction’ proposed a method to estimate the magnitude of the spring constants. His approach, also known as subgrade reaction model, was then became popular and commonly used in the design of shallow foundation. In layered soils with different elastic parameters, an equivalent model must be developed in order to derive a representative modulus of subgrade reaction. To do this the elastic settlement of the layered soils induced by the foundation pressure must first be calculated (Tjie-Liong 2001).



P-Vertical point load
k_1-Spring constant of top soil
k_2-Spring constant of bottom soil
s_1-Settlement of top soil due to P
s_2-Settlement of bottom soil due to P
k_s-Equivalent sprint constant of layered soil
s-Total settlement of layered soil due to P

Figure 2. 1 : Soil spring diagram of double layered soil

Subgrade modulus of top soil k_1,

$$\boldsymbol{k_1} = \frac{\boldsymbol{P}}{\boldsymbol{s_1}} \tag{2}$$

Subgrade modulus of bottom soil k_2,

$$k_2 = \frac{P}{s_2} \tag{3}$$

Total settlement of layered soil,

$$s = s_1 + s_2 \tag{4}$$

Replace settlements in terms of P and k,

$$\frac{P}{k_s} = \frac{P}{k_1} + \frac{P}{k_2} \tag{5}$$

Equivalent subgrade modulus of doubled layered soil can be estimated from following equation.

$$k_s = \frac{k_1 k_2}{k_1 + k_2} \tag{6}$$

2.5 Beams on elastic foundation theory

Different researchers developed various mathematical tools to solve the problem of a footing on an elastic foundation. Analysis of a beam on an elastic foundation is a classical problem first introduced by Winkler in the 19th century and later developed by many other investigators, most notably by Heternyi(1921).

To study the behavior of subgrade under an application of load, Winkler introduced a simplified assumption in 1867. His hypothesis assumes the subgrade to be a dense liquid represented by a bed of closely-spaced discrete springs. A loaded beam or slab resting on subgrade is supported by localized forces, each of which is proportional to the deflection of the spring at that point. By distributing these forces over a unit area, the sub grade support is represented as a unit pressure q, which is equal to a constant times the deflection δ

$$q = k_s \delta \tag{7}$$

In this expression Winkler assumed that the subgrade modulus k_s is constant at every point, independent of the deflection, and the same at all points within the area of consideration. This theory thus assumes a linear relationship between pressure and deflection (Siddiqi, et al., 1970).

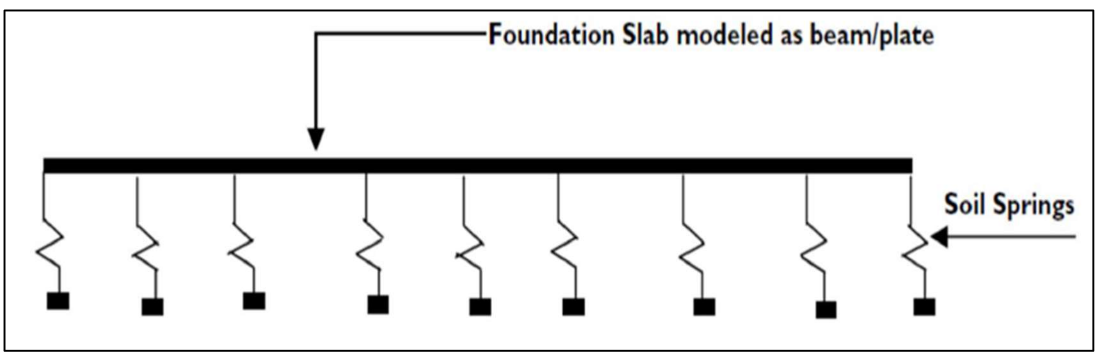


Figure 2. 2 : Winkler foundation consisting of independent springs

Heteryni (1921) introduced flexibility of beam λ. Here λ includes the flexural rigidity of the beam as well as the elasticity of the supporting medium, and is an important factor influencing the shape of the elastic line. For this reasons the factor λ called the characteristic of the system and, since its dimension is length^{-1}, the term *1/λ* is frequently referred to as the characteristic length.

Heternyi proposed that the rigidity criteria λL as,

$$\lambda L = \left(\frac{k_s{'} L^4}{4EI}\right)^{\frac{1}{4}} \tag{8}$$

Here,
$k_s' = k_s B$
k_s - modulus of subgrade reaction of soil
B - width of the footing
L - total length of foundation member
E - elastic modulus of footing material
I - moment of inertia of footing

Based on λL footings are categorized in three types. If $\lambda L < \pi/4$ member is short beam, if $\pi/4 < \lambda L < \pi$ member is medium length beam and if $\lambda L > \pi$ member is long beam. The characteristics of medium length beam is that a force acting at one end of the beam has a finite, and not negligible, effect at other end.

The solutions for a finite beam with free ends and a concentrated force P at any point proposed by Heternyi are attached in annex.

2.6 PLAXIS 3D numerical analysis

Several analytical and empirical models have evolved in soil mechanics to describe soil behaviour and recently, numerical methods have become commonplace, one such being the Finite Element Method which is now widely used because of its inherent versatility. At present numerical analysis program has wide-spread applications in Geotechnical Engineering. There are various Finite Element Method software packages available such as PLAXIS, ABAQUS, GEOSTUDIO, FLAC etc. These finite element software packages are worldwide used for numerical modeling and analysis of any geotechnical problems (Anitha and Niranjana 2016).

PLAXIS 3D is a three-dimensional finite element program especially developed for the analysis of foundation structures in geotechnical engineering. It is important in any analysis to adopt a consistent systems of units. In PLAXIS 3D the basic units comprise a unit for length, force and time. In a dynamic analysis, the time is usually measured in seconds rather than the default unit days. In PLAXIS 'time' and 'dynamic time' are different parameters. In the case where a dynamic analysis and a consolidation analysis are involved, the unit of time can be left as days whereas the dynamic time is in seconds. The generation of a three dimensional finite element model in PLAXIS 3D is based on the creation of a geometry model. The geometry model involves a composition of volumes, surfaces, lines and points (PLAXIS 3D Manual, 2017).

Stresses computed in PLAXIS 3D are based on the Cartesian coordinate system. In all of the output data, compressive stresses and forces, including pore pressures are taken to be negative, whereas tensile stresses and forces are taken to be positive. Following figure shows the positive stress direction.

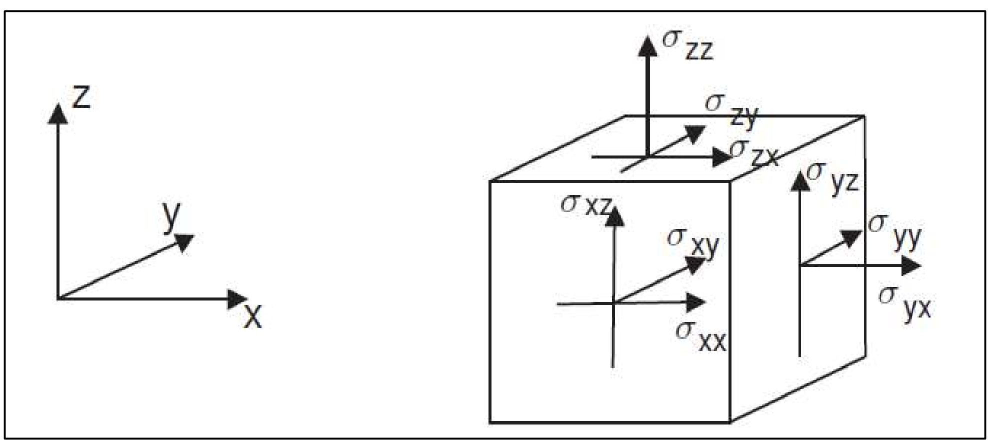


Figure 2. 3 : Coordinate system and indication of positive stress components (PLAXIS 3D Manual, 2017)

The size of the footing affects selection of horizontal and the vertical boundaries. The boundaries should not affect the full development of the failure mechanisms. A set of general fixities is automatically applied to the boundaries of the geometry model for the selected calculation phase (PLAXIS 3D Manual, 2017).

Footings are defined as plate elements. The plate elements are based on Mindlin's plate theory. This theory allows for plate deflection due to shearing as well as bending. In addition, the element can change length when an axial force is applied (PLAXIS 3D Manual, 2017).

When the geometry model is fully defined the geometry has to be divided into finite elements in order to perform finite element calculations. The geometry model is discretized and transformed to a finite element mesh in the mesh mode. A composition of finite elements is called a mesh. The mesh should be sufficiently fine to obtain accurate numerical results. On the other hand, very fine meshes should be avoided since this will lead to excessive calculation times. The geometric configuration cannot be modified in this mode. The mesh should be regenerated whenever the geometry of the project is modified. PLAXIS 3D allows for a fully automatic mesh generation procedure, in which the geometry is divided into volume elements and compatible structure elements, if applicable (PLAXIS 3D Manual, 2017).

In staged construction parts of the geometry model can be activated or deactivated and properties can be modified. The project is calculated in the staged construction mode (PLAXIS 3D Manual, 2017).

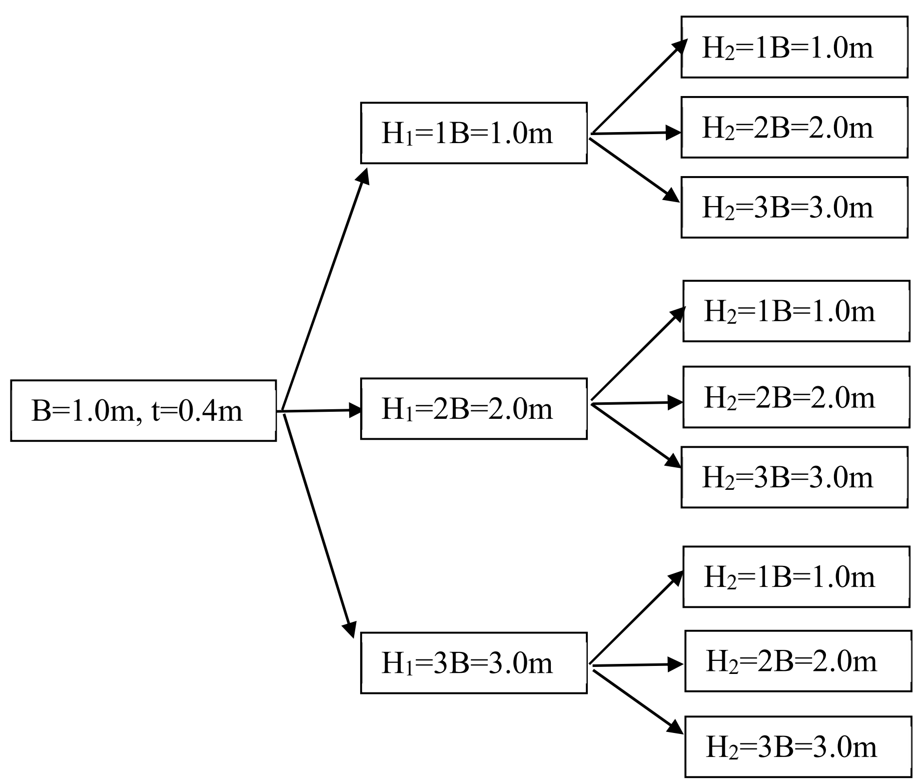
CHAPTER THREE
METHODOLOGY

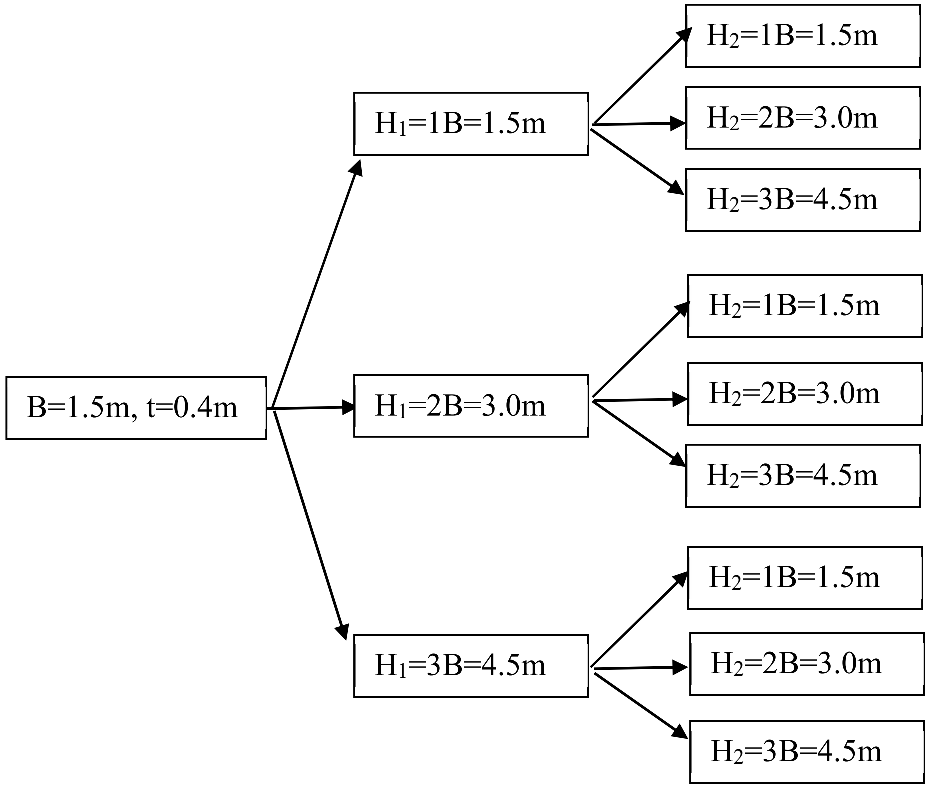
3.1 Dimension selection of footing and soil layers

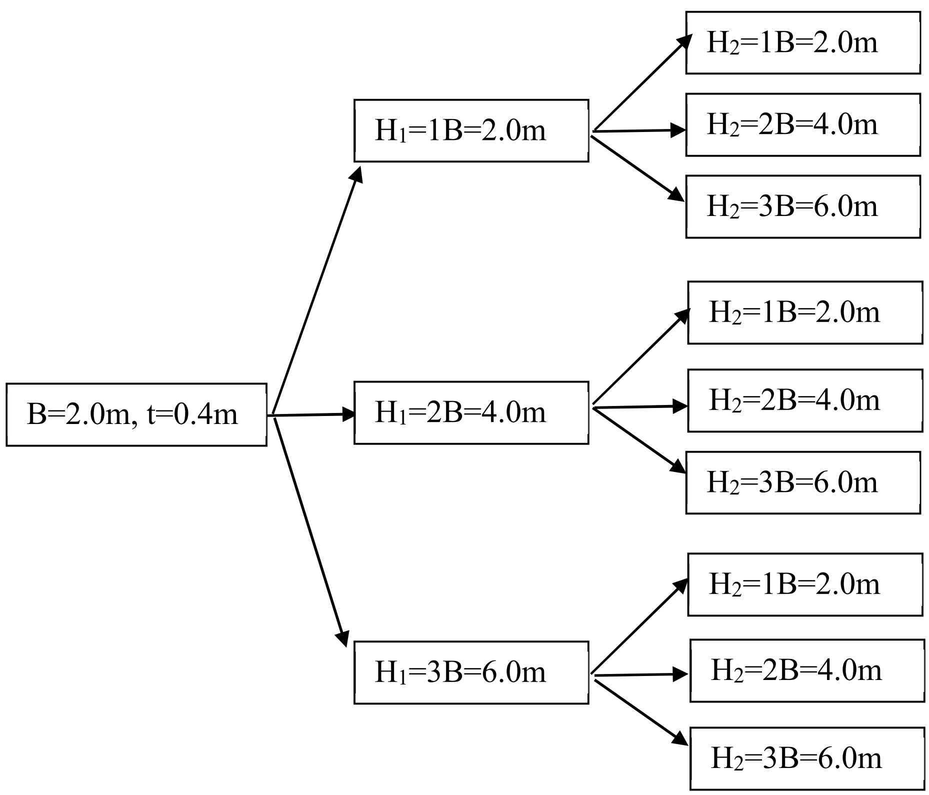
To investigate the equivalent subgrade modulus of layered soil, strip footing resting on a dense sand over loose sand layered soil combination was selected. Column spacing and soil layer thickness were selected in terms of footing width B. Size effect of footing also might influence the outcome which could be investigated by changing the footing width and depth.

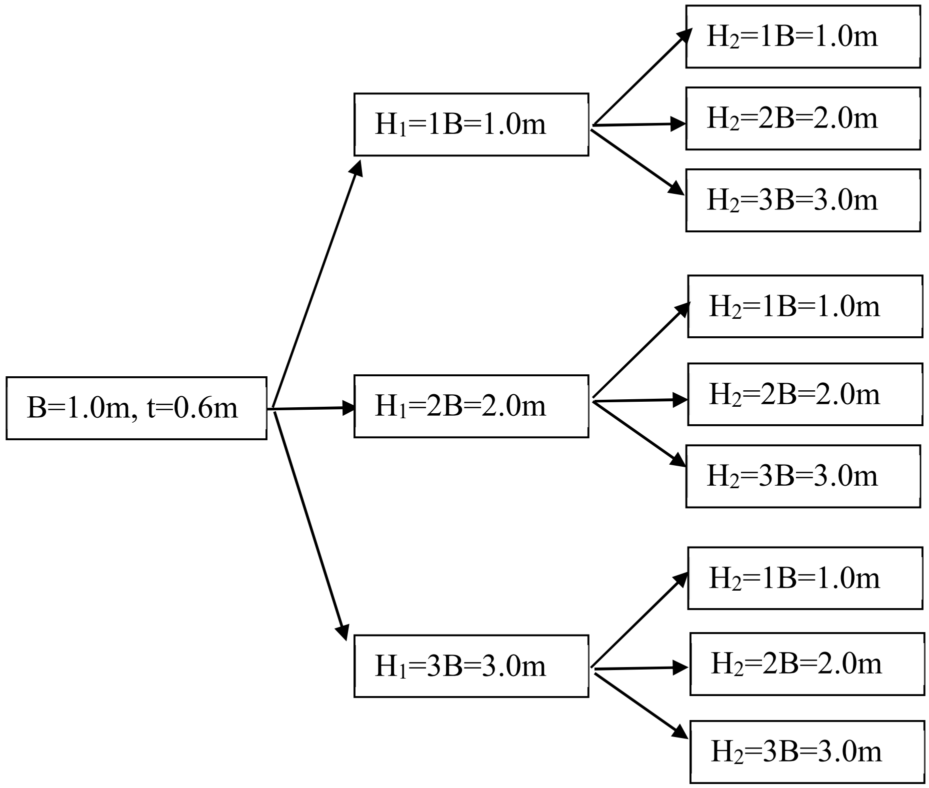
Most common practical strip footing widths of 1.0m, 1.5m and 2.0m and footing thickness 0.4m and 0.6m were selected for this study. Constant column clear spacing of 4B was used. For 1.0m, 1.5m and 2.0m footing widths 4.0m, 6.0m and 8.0m column clear spacing were selected respectively. Top layer dense sand thickness was selected as 1B, 2B and 3B. For each top layer thickness bottom layer loose sand thickness was used as 1B, 2B and 3B thicknesses. For all three footing widths 27 combinations of layered soil were obtained for a particular thickness of footing. In same way for all two footing depths totally 54 cases were used in this study. 40 kPa allowable bearing pressure was considered to find concentrated points loads.

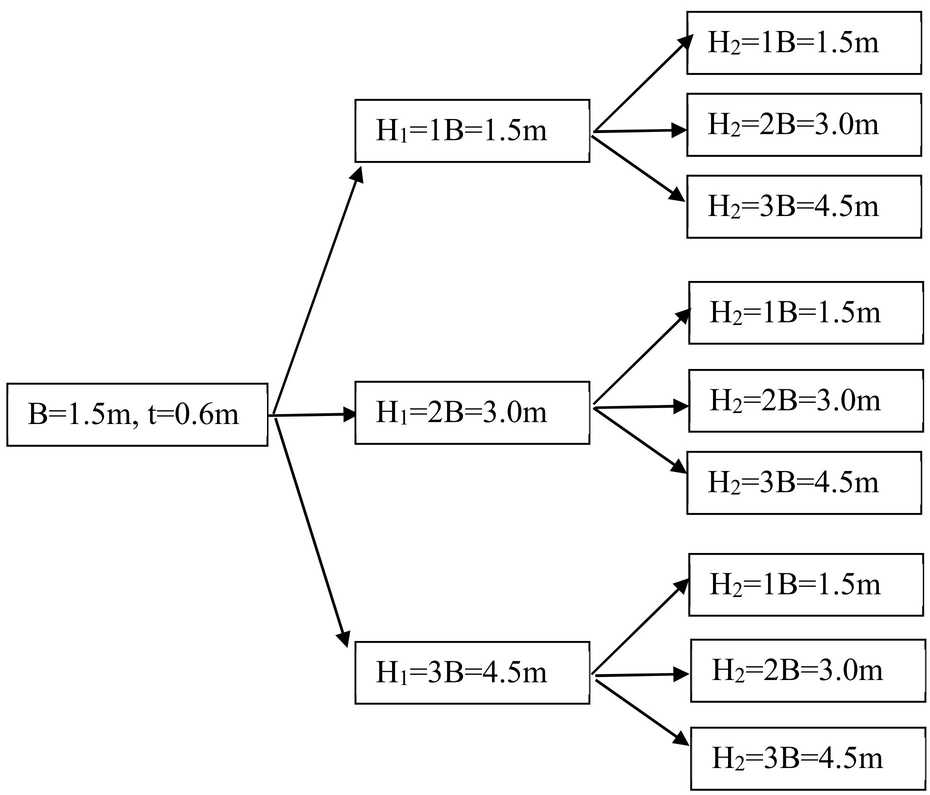
Following flow charts describe the selection of footing width B, footing thickness t, top soil layer thickness H_1 and bottom soil layer thickness H_2 in this study.

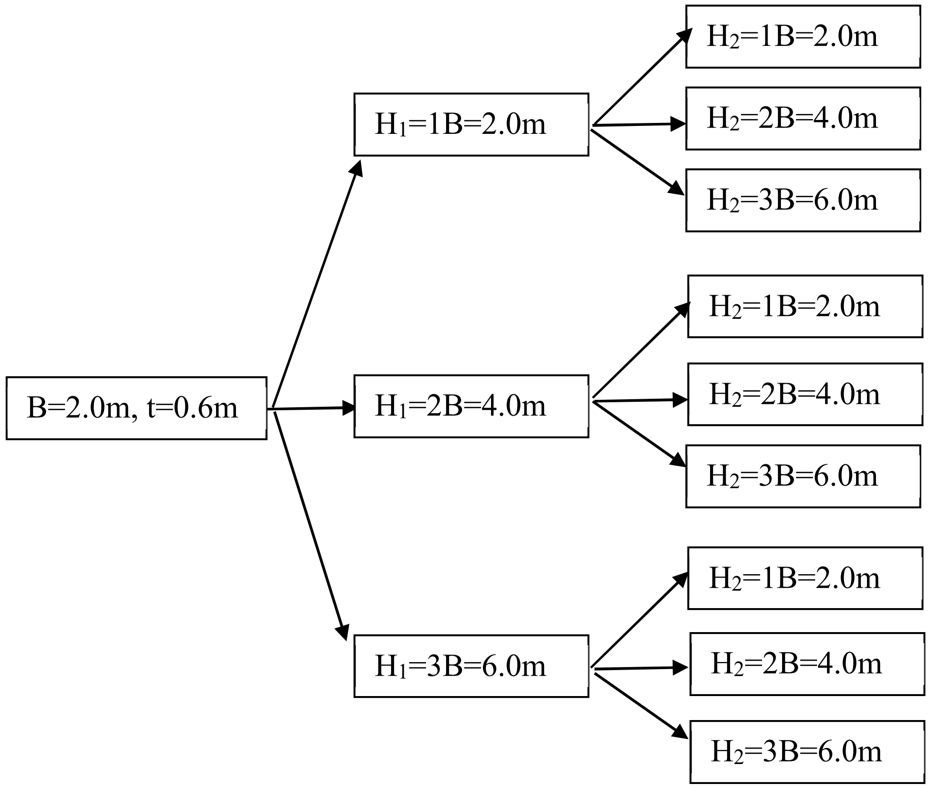



B=1.5m, t=0.4m
H_1=1B=1.5m
H_1=2B=3.0m
H_1=3B=4.5m
H_2=1B=1.5m
H_2=2B=3.0m
H_2=3B=4.5m
H_2=1B=1.5m
H_2=2B=3.0m
H_2=3B=4.5m
H_2=1B=1.5m
H_2=2B=3.0m
H_2=3B=4.5m


B=2.0m, t=0.4m
H_1=1B=2.0m
H_1=2B=4.0m
H_1=3B=6.0m
H_2=1B=2.0m
H_2=2B=4.0m
H_2=3B=6.0m
H_2=1B=2.0m
H_2=2B=4.0m
H_2=3B=6.0m
H_2=1B=2.0m
H_2=2B=4.0m
H_2=3B=6.0m


B=1.0m, t=0.6m
H_1=1B=1.0m
H_1=2B=2.0m
H_1=3B=3.0m
H_2=1B=1.0m
H_2=2B=2.0m
H_2=3B=3.0m
H_2=1B=1.0m
H_2=2B=2.0m
H_2=3B=3.0m
H_2=1B=1.0m
H_2=2B=2.0m
H_2=3B=3.0m


B=1.5m, t=0.6m
H_1=1B=1.5m
H_1=2B=3.0m
H_1=3B=4.5m
H_2=1B=1.5m
H_2=2B=3.0m
H_2=3B=4.5m
H_2=1B=1.5m
H_2=2B=3.0m
H_2=3B=4.5m
H_2=1B=1.5m
H_2=2B=3.0m
H_2=3B=4.5m



3.2 Material properties

Following material properties of dense sand, loose sand and footing were selected for this study. A concrete footing was assumed.

Table 3. 1 : Selected material properties for the present study

Material	E’ (kPa)	v	γ (kN/m^3)	γ_{sat} (kN/m^3)	c’(kPa)	Ø’ (̊)	k_v (m/s)	k_h (m/s)
Dense sand	20000	0.28	18	19	0	33	10^{-4}	10^{-3}
Loose sand	5000	0.35	17	18	0	24	10^{-3}	10^{-2}
Concrete	$26*10^6$	0.15	24	-	-	-	-	-

Here,

E’- Elastic modulus

v- Poisson’s ratio

γ – Unit weight of the material

γ_{sat} – Saturated unit weight of the material

c’ - Cohesion

Ø’ - Angle of friction

k_v - Vertical permeability

k_h - Horizontal permeability

3.3 PLAXIS 3D model

For Global coarseness very fine mesh was used. Drainage condition was assumed as drained. Mohr–Coulomb constitutive is assumed for dense and loose sands. Plate element with concrete properties was defined for strip footing.

Hard layer was assumed at the end of loose sand layer and vertical boundary was extended up to hard layer. Horizontal boundaries were extended two times vertical boundary depth from each ends of the beam. Roller supports for side boundaries and fixed support for bottom boundary was applied as default.

All geometrical features, material properties, loadings and meshing were done based on PLAXIS 3D manual. Finally, modal was analyzed and vertical settlement of footing was obtained.

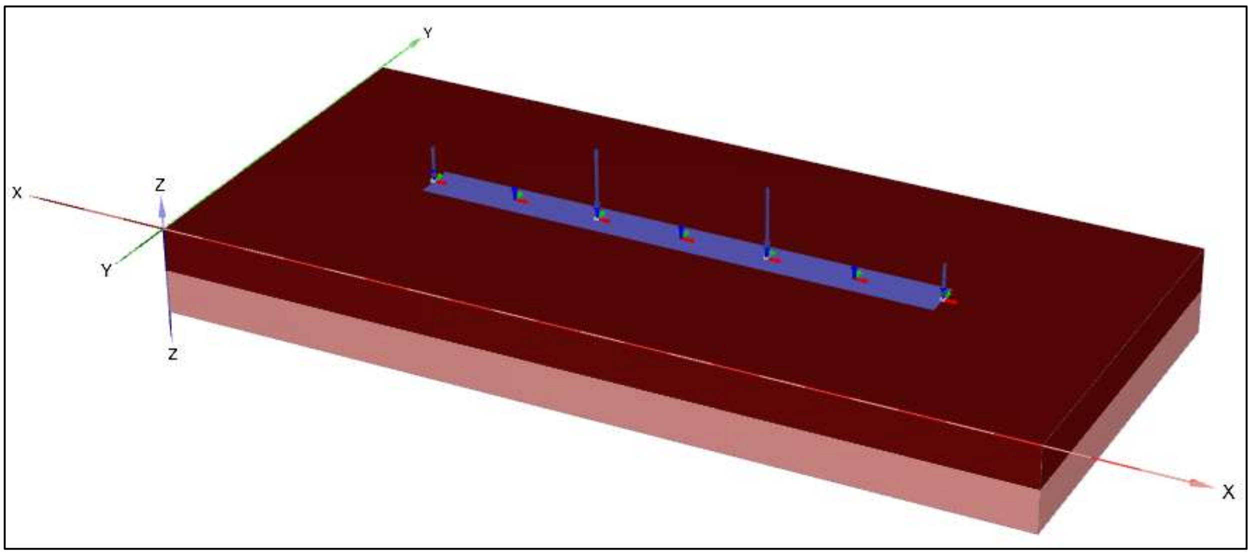


Figure 3. 1 : PLAXIS 3D inputs of strip footing
(for B=1m, t=0.4m, H_1=1.0m, H_2=1.0m, L=12.0m, F=80kN and 2F=160kN)

3.4 Validation of PLAXIS 3D model

Validation with previous case study

To ensure the accuracy of PLAXIS 3D numerical model settlement output, a previous case study called ‘A case study on settlement of oil storage tank foundations’ (Ramasamy and Kalaiselvan 1998) was used. In this case study soil at the site consists of alternating layers of cohesive and cohesionless soils. Tanks were load tested and immediate settlements were observed along the periphery on tank shell base. PLAXIS 3D numerical models were modelled and loaded similar as previous case study. PLAXIS 3D numerical model settlements were compared with the observed immediate settlements in case study.

An average representative soil profile as shown in Figure 3.2 was obtained for the site. It can be seen that the subsoil consists alternating layers of clay and silt of varying thickness up to 18.0m, the maximum depth of exploration.

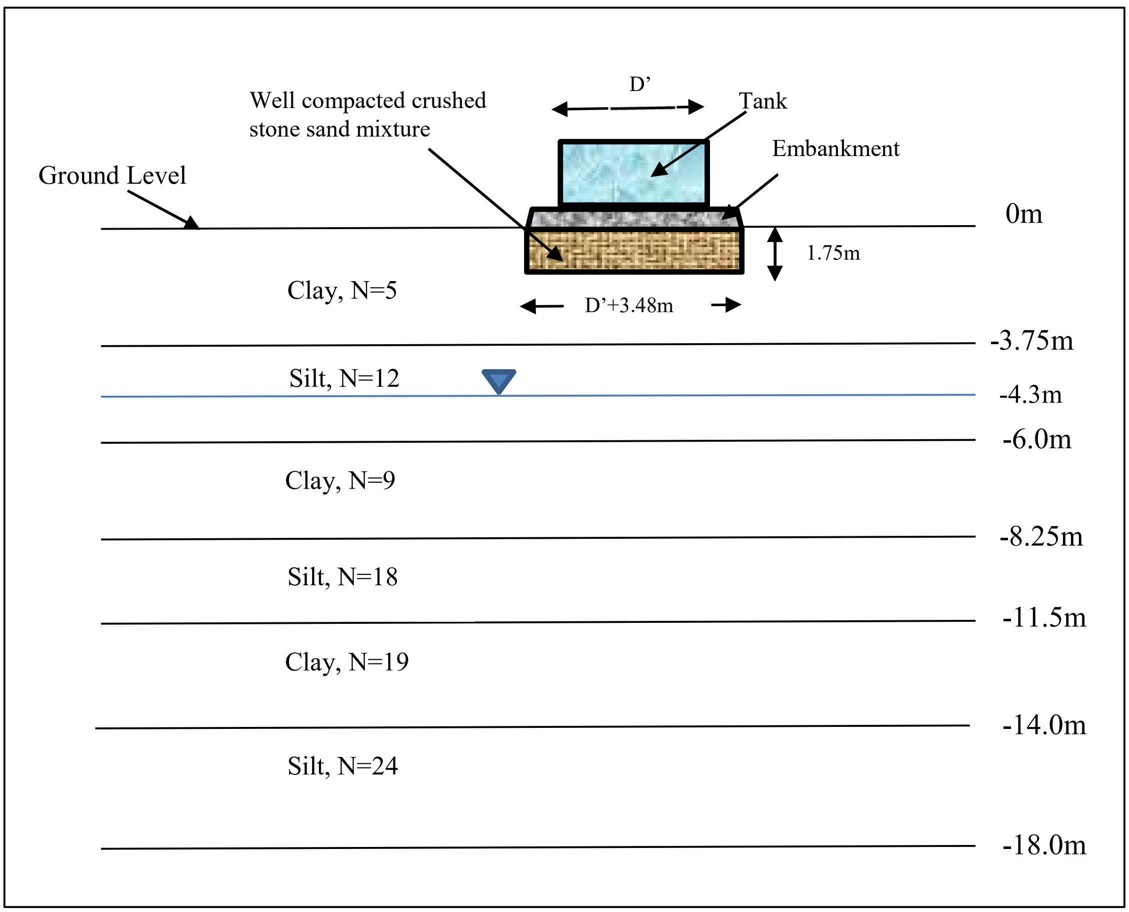


Figure 3. 2 : Sub soil profile and foundation of oil storage tank

To find out the soil strength parameters by the method of the SPT correction, following corrections were applied for Site SPT values. Correlation factors were obtained from page159 Table 3-3 of 'Foundation analysis and design' (Joseph E. Bowles 1997).

Overburden correction

Overburden correction factor CN, was obtained by the following expression (Joseph E. Bowles 1997).

$$C_N = \sqrt{\frac{95.76}{\rho_0{}'}} \tag{9}$$

Where Po$^{/}$ is the effective overburden pressure in kPa.

The following unit weights for the different layers were assumed to calculate the effective overburden in site.

For Clay layer	γ_{wet}	=	17.0 kN/m^3
	γ_{sat}	=	18.0 kN/m^3
For Silt Layer	γ_{wet}	=	16.0 kN/m^3
	γ_{sat}	=	17.0 kN/m^3

At 1.875m depth,

Po$^{/}$	=	1.875x17
	=	31.875kN/m^2
C_N	=	$(95.76/31.875)^{0.5}$
	=	1.733

Similarly, C_N was calculated at middle of each and every layer and tabulated in Table 3.2.

Energy correction factor (η_1)

Energy correction factor η_1, was obtained by the following expression (Joseph E. Bowles 1997)

$$\eta_1 = \frac{E_r}{70} \tag{10}$$

Energy ratio of the SPT set up Er was assumed as 55%,

η_1	=	55/70
	=	0.785

Rod length correction factor (η_2)

Table 3. 2 : Rod length correction factor η_2 for SPT correlation (Joseph E. Bowles 1997)

L (m)	η_2
> 10	1.00
6-10	0.95
4-6	0.85
0-4	0.75

Let assumed the rod length above the ground surface is 2.0m,

At 1.875m depth $L = 3.875m$

$\eta_2 = 0.75$

Similarly η_2 was calculated at middle of each and every layer and tabulated in Table 3.3.

Sampler correction factor (η_3)

The Sampler is assumed as without liner.

$\eta_3 = 1.00$

Borehole Diameter correction factor (η_4)

The borehole diameter is assumed as within 60 – 120mm

$\eta_4 = 1.00$

Calculation of N_{70}'

Corrected SPT N value is referred to as N_{70}' since the energy ratio is converted to that corresponding to 70%. N_{70}' was calculated from following equation and tabulated in Table 3.2.

$$N_{70}' = N_{field} C_N \eta_1 \eta_2 \eta_3 \eta_4 \quad (11)$$

At 1.875m depth, $N_{70}' = 5x1.733x0.79x1.0x1.0x1.0$

$= 7$

Similarly, N_{70}' was calculated at middle of each and every layer and tabulated in Table 3.3.

Empirical values of ϕ based on SPT N_{70}' were obtained from page163 Table 3-4 of 'Foundation analysis and design' (Joseph E. Bowles 1997). Consistency of cohesive soils were obtained from page165 Table 3-5 of 'Foundation analysis and design' (Joseph E. Bowles 1997).

Table 3. 3 : Corrected SPT N_{70}' values and soil parameters

Depth (m)	**Soil Type**	**P_0' (kPa)**	**C_N**	η_1	η_2	η_3	η_4	**N_{Field}'**	**N_{70}'**	ϕ	**c_u (kPa)**
1.875	Clay	31.875	1.733	0.785	0.75	1.00	1.00	5	5	-	25
4.875	Silt	82.725	1.075	0.785	0.95	1.00	1.00	12	10	31	-
7.125	Clay	122.1	0.885	0.785	0.95	1.00	1.00	9	6	-	50
9.875	Silt	169.975	0.750	0.785	1.00	1.00	1.00	18	11	32	-
12.75	Clay	220.1	0.659	0.785	1.00	1.00	1.00	19	10	-	50
16	Silt	276.6	0.588	0.785	1.00	1.00	1.00	24	11	32	-

Calculation of Poison ratio of subsoil layers (μ)

As per the values mentioned in the page123 Table 2-7 'Foundation analysis and design' (Joseph E. Bowles 1997) Poisson ratio of 0.4 was used for unsaturated clay and 0.5 was used for saturated clay. Poisson ratio of 0.3 was used for the silt in PLAXIS 3D model.

Calculation of Elastic modulus of subsoil layers (Es)

Following equation was used to find the Elastic modulus of Silt (Joseph E. Bowles 1997).

$$E_s = 300(N_{55} + 6) \tag{12}$$

Energy correction factor η_1 is only varying to SPT N_{55} and SPT N_{70}
Energy correction factor η_1 for SPT N_{55},

$$\eta_1 = 1.00$$

Table 3. 4 : Corrected SPT N_{55} values

Depth (m)	**Soil Type**	**P_0' (kPa)**	**C_N**	η_1	η_2	η_3	η_4	**N_{Field}'**	**N_{55}**
1.875	Clay	31.875	1.733	1.0	0.75	1.00	1.00	5	6
4.875	Silt	82.725	1.075	1.0	0.95	1.00	1.00	12	12
7.125	Clay	122.1	0.885	1.0	0.95	1.00	1.00	9	8
9.875	Silt	169.975	0.750	1.0	1.00	1.00	1.00	18	14
12.75	Clay	220.1	0.659	1.0	1.00	1.00	1.00	19	13
16	Silt	276.6	0.588	1.0	1.00	1.00	1.00	24	14

Elastic modulus of Silt at 4.875m depth,

E_s = 300(12+6)

= 5400 kPa

Elastic modulus of Silt at 9.875m and 16m depth,

E_s = 300(14+6)

= 6000 kPa

Following equation was used to find the Elastic modulus of Clay (Joseph E. Bowles 1997).

$$E_s = (100\ to\ 500)su \tag{13}$$

Average of 100 and 500 was used.

$$E_s = 300su \tag{14}$$

Elastic modulus of Clay at 1.875m depth,

E_s = 300x25

= 7500 kPa

Elastic modulus of Clay at 7.175m and 12.75m depth,

E_s = 300x50

= 15000 kPa

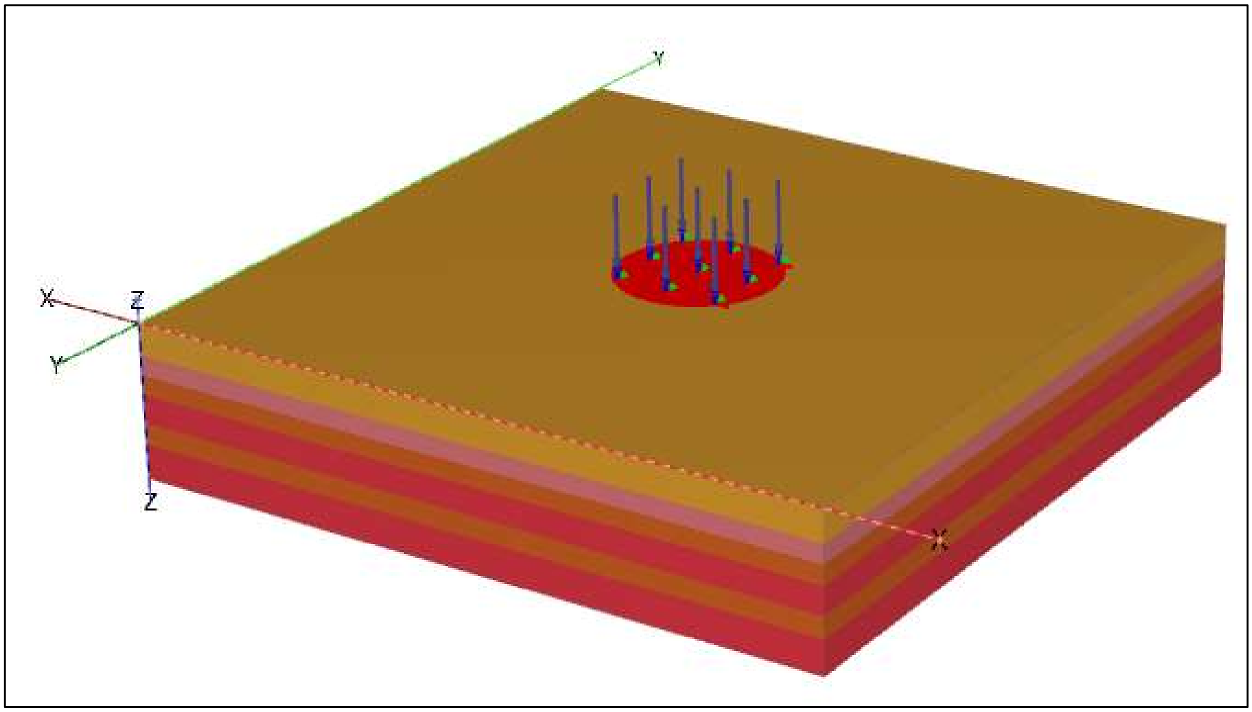


Figure 3. 3 : PLAXIS 3D inputs of oil storage tank foundation (for D'=17.0m, q=68kN/m^2)

Table 3. 5 : Oil storage tank foundation actual settlement and PLAXIS 3D model settlement

Tank diameter D' (m)	Load intensity q (kN/m^2)	Observed settlement (mm)	Settlement from PLAXIS 3D model (mm)
17.0	68	40	38
14.0	63	39	42
12.6	55	40	38
9.0	34	26	25

Immediate settlement obtained by PLAXIS 3D numerical model and actual immediate settlement observed in the previous study are almost same. Therefore, it is assumed that, PLAXIS 3D models give correct settlement outputs.

3.5 Equivalent modulus of subgrade reaction

Equation (1) was used to find the modulus of subgrade reaction of dense sand and loose sand. One sample calculation is done below.

Consider footing width B = 1.0m and footing depth d = 0.4m,

For dense sand,

Elastic modulus of dense sand E_s	=	20000 kN/m^2
Elastic modulus of footing E_b	=	$26x10^6$ kN/m^2
Poisson's ratio of soil v	=	0.28
Second moment of inertia I	=	$(1/12)*1*0.4^3$
	=	$5.333x10^{-3}m^4$
Modulus of subgrade reaction k_1	=	$(0.65/1)*(20000*1/(26x10^6 * 5.333x10^{-3}))^{(1/12)}*$ $(20000/(1-0.28^2))$
	=	12004 kN/m^3

For loose sand,

Elastic modulus of loose sand E_s	=	5000 kN/m^2
Elastic modulus of footing E_b	=	$26x10^6$ kN/m^2
Poisson's ratio of soil v	=	0.35
Second moment of inertia I	=	$(1/12)*1*0.4^3$
	=	$5.333x10^{-3}m^4$
Modulus of subgrade reaction k_2	=	$(0.65/1)*(5000*1/(26x10^6 * 5.333x10^{-3}))^{(1/12)}*$ $(5000/(1-0.35^2))$
	=	2808 kN/m^3

Equivalent subgrade modulus by equivalent spring theory using Equation (6),

k_{s1}	=	$(k_1*k_2)/(k_1+k_2)$
	=	(12004*2808)/(12004+2808)
	=	2276 kN/m^3

Consider,

Top soil layer thickness H_1	=	1.0m
Bottom soil layer thickness H_2	=	1.0m

Equivalent subgrade modulus by weighted average method,

k_{s2}	=	$(k_1H_1+k_2H_2)/(H_1+H_2)$
	=	$\{(12004*1)+(2808*1)\}/(1+1)$
	=	7406 kN/m^3

3.6 Theory of beam on elastic foundation Hetenyi method

To find the equivalent subgrade modulus of dense sand over loose sand layered soil, Vesic method homogeneous subgrade reaction was used in equivalent spring theory. This equivalent subgrade modulus was applied in Hetenyi method elastic beam theory and settlement curve along strip footing was obtained. For all four concentrated point loads, method of superposition was used and resultant settlement was obtained. Rigidity criteria λL was selected in a range of 1.0 to 5.0 in order to satisfy medium length beam criteria. One sample calculation is done below.

Consider footing width B as 1.0m, depth d as 0.4m, top soil layer thickness of 1.0B and bottom soil layer thickness of 1.0B. Therefore,

Top soil layer thickness H_1	=	1.0m
Bottom soil layer thickness H_2	=	1.0m
Length of beam L	=	12B
	=	12.0m
Column clear spacing s	=	4.0B
	=	4.0m.
Second moment of inertia I	=	$(1/12)*1*0.4^3$
	=	$5.333\text{x}10^{-3}$ m^4
Elastic modulus of dense sand	=	20000 kN/m^2
Elastic modulus of loose sand	=	5000 kN/m^2
Elastic modulus of concrete	=	$26*10^6$
Equivalent subgrade modulus of soil	=	2276 kN/m^3
For 40 kPa allowable bearing pressure, Point load at end of beam F	=	40*2B*B
	=	40*2*1*1
	=	80kN.

Point load at middle column 2F = 160kN

Equivalent subgrade modulus is 2276 kN/m^3 and use in equation (08),

At (x=0)

λL = $((2276*1*12^4)/(4*26*10^6*5.333x10^{-3}))^{(1/4)}$

= 3.04

(1.0 < λL< 5.0) Hetenyi method can apply.

Settlement due to 1st point load from left end, use equation (16) with [a=0 m, b=12 m],

y_1(x=0) = 17.89 mm

Settlement due to 2nd point load from left end, use equation (15) with [a=4 m, b=8 m],

y_2(x=0) = 7.01 mm

Settlement due to 3rd point load from left end use equation (15) with [a=8 m, b=4 m],

y_3(x=0) = -2.40 mm

Settlement due to 4th point load from left end, use equation (15) with [a=12 m, b=0 m],

y_4(x=0) = -1.88 mm

Total settlement at x=0 ,

y(x=0) = 17.89 + 7.01 -2.40 -1.88

= 20.62 mm

Similarly, settlements on various points of the footing can be calculated.

Vertical settlement along footing, obtained by PLAXIS 3D numerical model and Heternyi method are plotted in the same graph to find the settlement difference. Equivalent subgrade reaction of layered soil and settlement difference between PLAXIS 3D and Heternyi method are analyzed using graphic tools.

CHAPTER FOUR

RESULTS AND DISCUSSION

4.1 Calculations

Equivalent subgrade modulus obtained by equivalent spring theory and weighted average method are listed in Table 4.1 and Table 4.2.

Table 4. 1: Equivalent subgrade modulus for 0.4m footing thickness

Thickness of footing t (m)	Width of footing B(m)	Top soil layer thickness H1(m)	Bottom soil layer thickness H2 (m)	Vertical column load at x=0 F(kN)	Vertical column load at x=4B 2F(kN)	Subgrade modules of dense sand k1 (kN/m^3) by Vesic method	Subgrade modules of loose sand k2 (KN/m^3) by Vesic method	Equivalent subgrade module by equivalent spring theory ks1 (kN/m3)	Equivalent subgrade module by weighted average of soil layers ks2 (kN/m3)
0.4	1.0	1.0	1.0	80.0	160.0	12004	2808	2276	7406
	1.0	1.0	2.0	80.0	160.0	12004	2808	2276	5873
	1.0	1.0	3.0	80.0	160.0	12004	2808	2276	5107
	1.0	2.0	1.0	80.0	160.0	12004	2808	2276	8939
	1.0	2.0	2.0	80.0	160.0	12004	2808	2276	7406
	1.0	2.0	3.0	80.0	160.0	12004	2808	2276	6486
	1.0	3.0	1.0	80.0	160.0	12004	2808	2276	9705
	1.0	3.0	2.0	80.0	160.0	12004	2808	2276	8326
	1.0	3.0	3.0	80.0	160.0	12004	2808	2276	7406
	1.5	1.5	1.5	180.0	360.0	8856	2072	1679	5464
	1.5	1.5	3.0	180.0	360.0	8856	2072	1679	4333
	1.5	1.5	4.5	180.0	360.0	8856	2072	1679	3768
	1.5	3.0	1.5	180.0	360.0	8856	2072	1679	6595
	1.5	3.0	3.0	180.0	360.0	8856	2072	1679	5464
	1.5	3.0	4.5	180.0	360.0	8856	2072	1679	4786
	1.5	4.5	1.5	180.0	360.0	8856	2072	1679	7160
	1.5	4.5	3.0	180.0	360.0	8856	2072	1679	6142
	1.5	4.5	4.5	180.0	360.0	8856	2072	1679	5464
	2.0	2.0	2.0	320.0	640.0	7138	1670	1353	4404
	2.0	2.0	4.0	320.0	640.0	7138	1670	1353	3492
	2.0	2.0	6.0	320.0	640.0	7138	1670	1353	3037
	2.0	4.0	2.0	320.0	640.0	7138	1670	1353	5315
	2.0	4.0	4.0	320.0	640.0	7138	1670	1353	4404
	2.0	4.0	6.0	320.0	640.0	7138	1670	1353	3857
	2.0	6.0	2.0	320.0	640.0	7138	1670	1353	5771
	2.0	6.0	4.0	320.0	640.0	7138	1670	1353	4950
	2.0	6.0	6.0	320.0	640.0	7138	1670	1353	4404

Table 4. 2: Equivalent subgrade modulus for 0.6m footing

Thickness of footing t (m)	Width of footing B(m)	Top soil layer thickness H1(m)	Bottom soil layer thickness H2 (m)	Vertical column load at x=0 F(kN)	Vertical column load at x=4B 2F(kN)	Subgrade modules of dense sand k1 (kN/m^3) by vesic method	Subgrade modules of loose sand k2 (KN/m^3) by vesic method	Equivalent subgrade module by equivalent spring theory ks1 (kN/m3)	Equivalent subgrade module by weighted average of soil layers ks2 (kN/m3)
0.6	1.0	1.0	1.0	80.0	160.0	10413	2447	1981	6430
	1.0	1.0	2.0	80.0	160.0	10413	2447	1981	5102
	1.0	1.0	3.0	80.0	160.0	10413	2447	1981	4438
	1.0	2.0	1.0	80.0	160.0	10413	2447	1981	7757
	1.0	2.0	2.0	80.0	160.0	10413	2447	1981	6430
	1.0	2.0	3.0	80.0	160.0	10413	2447	1981	5633
	1.0	3.0	1.0	80.0	160.0	10413	2447	1981	8421
	1.0	3.0	2.0	80.0	160.0	10413	2447	1981	7226
	1.0	3.0	3.0	80.0	160.0	10413	2447	1981	6430
	1.5	1.5	1.5	180.0	360.0	7682	1805	1462	4744
	1.5	1.5	3.0	180.0	360.0	7682	1805	1462	3764
	1.5	1.5	4.5	180.0	360.0	7682	1805	1462	3274
	1.5	3.0	1.5	180.0	360.0	7682	1805	1462	5723
	1.5	3.0	3.0	180.0	360.0	7682	1805	1462	4744
	1.5	3.0	4.5	180.0	360.0	7682	1805	1462	4156
	1.5	4.5	1.5	180.0	360.0	7682	1805	1462	6213
	1.5	4.5	3.0	180.0	360.0	7682	1805	1462	5332
	1.5	4.5	4.5	180.0	360.0	7682	1805	1462	4744
	2.0	2.0	2.0	320.0	640.0	6192	1455	1178	3823
	2.0	2.0	4.0	320.0	640.0	6192	1455	1178	3034
	2.0	2.0	6.0	320.0	640.0	6192	1455	1178	2639
	2.0	4.0	2.0	320.0	640.0	6192	1455	1178	4613
	2.0	4.0	4.0	320.0	640.0	6192	1455	1178	3823
	2.0	4.0	6.0	320.0	640.0	6192	1455	1178	3349
	2.0	6.0	2.0	320.0	640.0	6192	1455	1178	5007
	2.0	6.0	4.0	320.0	640.0	6192	1455	1178	4297
	2.0	6.0	6.0	320.0	640.0	6192	1455	1178	3823

4.2 Result analysis

4.2.1 Equivalent subgrade modulus

Equivalent subgrade modulus with top soil layer thickness

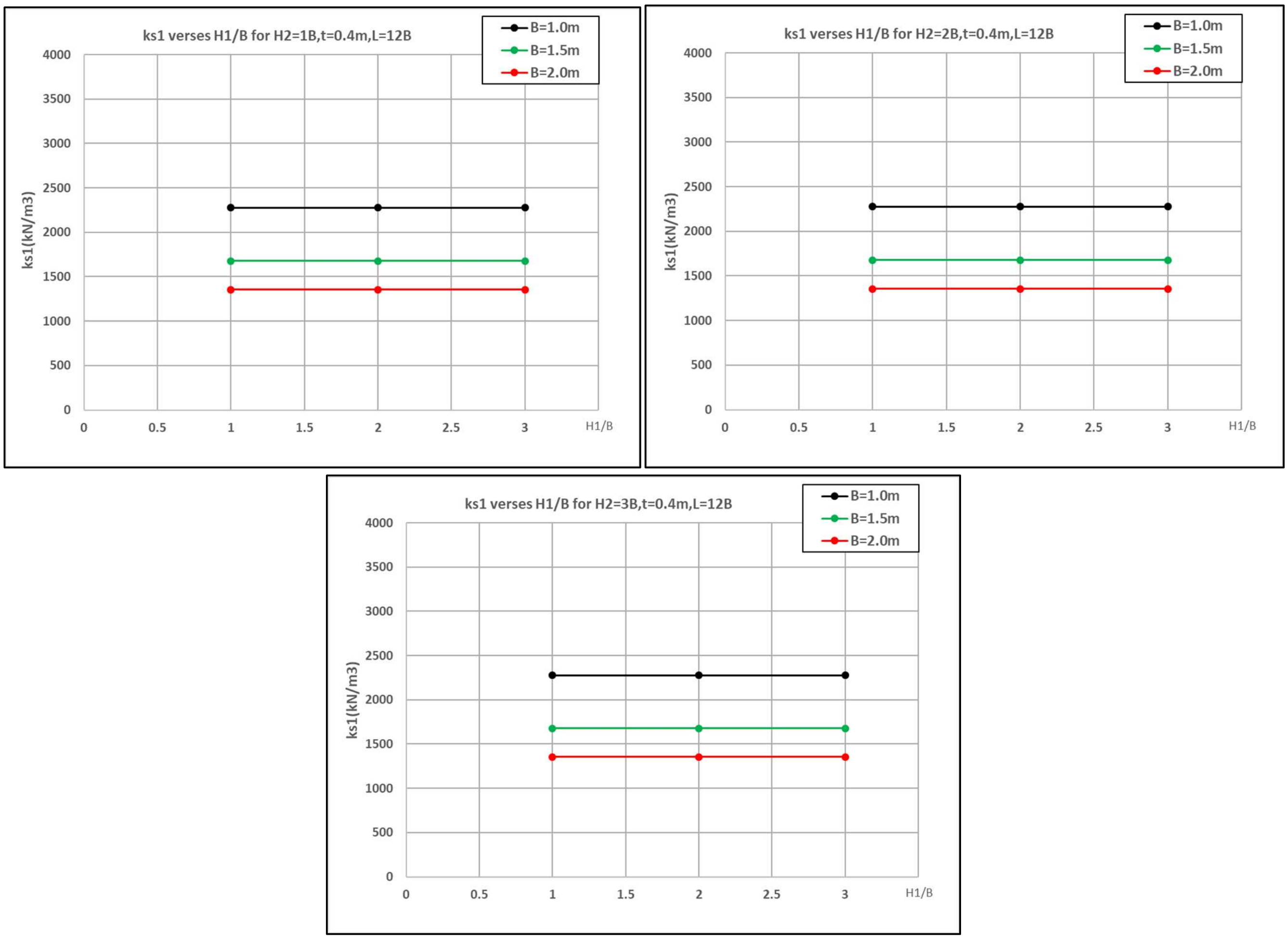


Figure 4. 1: Equivalent spring theory equivalent subgrade modulus k_{s1} verses H_1/B for t=0.4m

Equivalent subgrade modulus using equivalent spring theory verses top soil layer thickness graph is constant for a given footing width and footing thickness. From equation (6), equivalent subgrade modulus depends on top and bottom soil layer homogeneous subgrade modulus. From equation (1), homogeneous subgrade modulus is a function of footing width, footing thickness, Young's modulus and Poisson ratio of footing and top soil layer Young's modulus. Hence subgrade modulus is not a function of top soil layer thickness or bottom soil layer thickness, k_{s1} verses top soil layer thickness curve is constant for a given footing and same graphs are obtained while changing the bottom soil layer thicknesses.

When footing width increases, k_{s1} becomes smaller for given footing thickness. For 1.0m, 1.5m and 2.0m footing widths and a constant footing thickness of 0.4m, k_{s1} values 2276 kN/m^3, 1679 kN/m^3 and 1353 kN/m^3 are obtained as shown in the Figure 4.1.

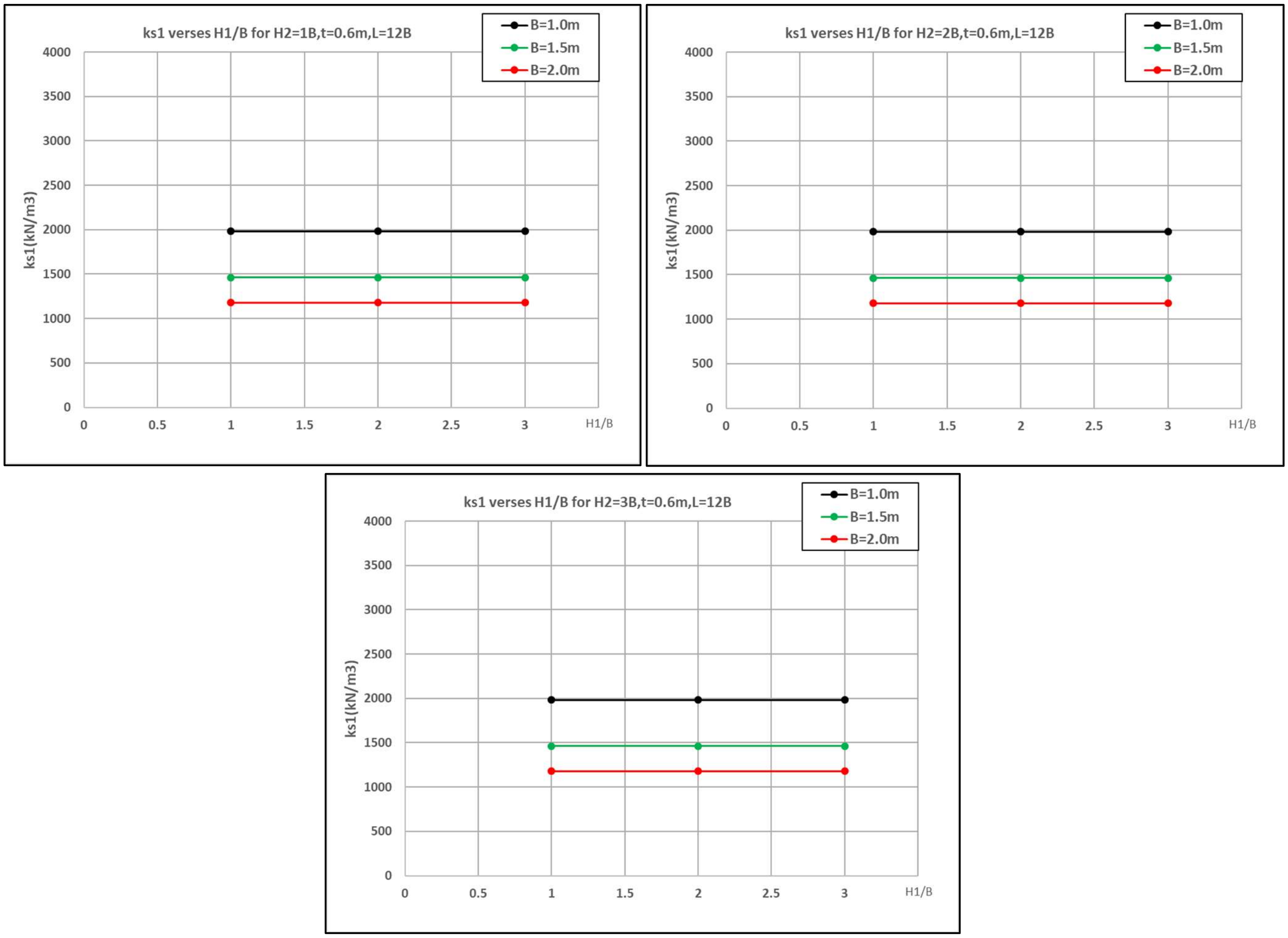


Figure 4. 2: Equivalent spring theory equivalent subgrade modulus k_{s1} verses H_1/B for t=0.6m

As described under Figure 4.1, k_{s1} verses top soil layer thickness curve is constant for a given footing width and footing thickness in Figure 4.2 also. Same graphs are obtained while changing the bottom soil layer thicknesses.

For 1.0m, 1.5m and 2.0m footing widths and a constant footing thickness of 0.6m, k_{s1} values 1981 kN/m^3, 1462 kN/m^3 and 1178 kN/m^3 are obtained as shown in the Figure 4.2. while comparing Figure 4.1 and Figure 4.2 a lesser k_{s1} is obtained for a higher footing thickness with a constant footing width.

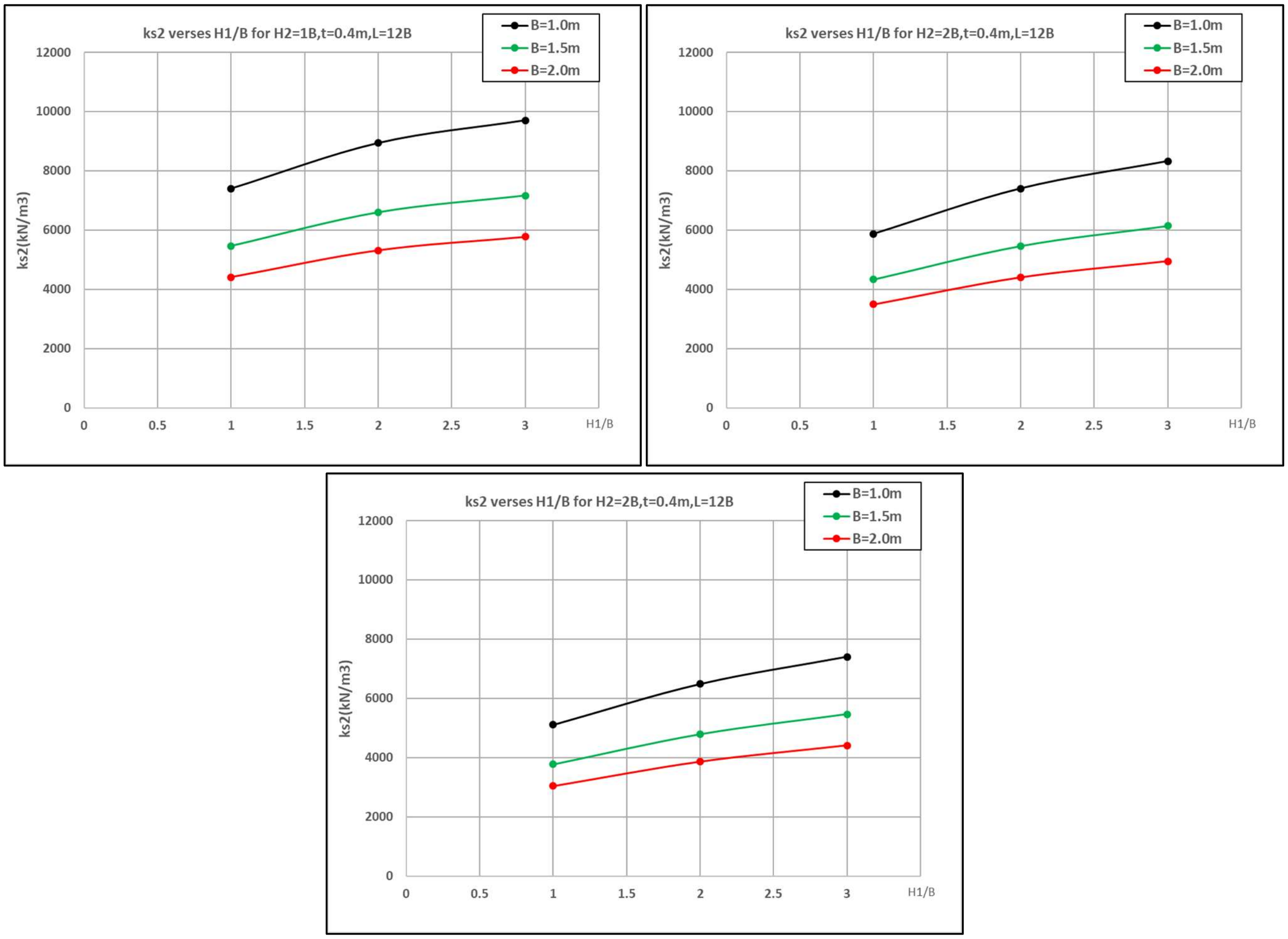


Figure 4. 3: Weighted average method equivalent subgrade modulus k_{s2} verses H_1/B for t=0.4m

Weighted average method equivalent subgrade modulus is non linearly increasing with top soil layer thickness for a given footing and bottom soil layer thickness. K_{s2} increment for H_1/B segment 1.0 to 2.0 is higher than 2.0 to 3.0 segment in all curves. In this study top soil layer is dense sand with higher subgrade modulus and bottom soil layer is loose sand with lesser subgrade modulus. Hence for a given footing and bottom loose sand layer thickness equivalent subgrade modulus is increasing with a decreasing gradient with top dense sand layer thickness.

From equation (1) homogeneous subgrade modulus is inversely proportional to footing width. Therefore, when footing width increases, k_{s2} becomes smaller for a given footing depth and soil profile.

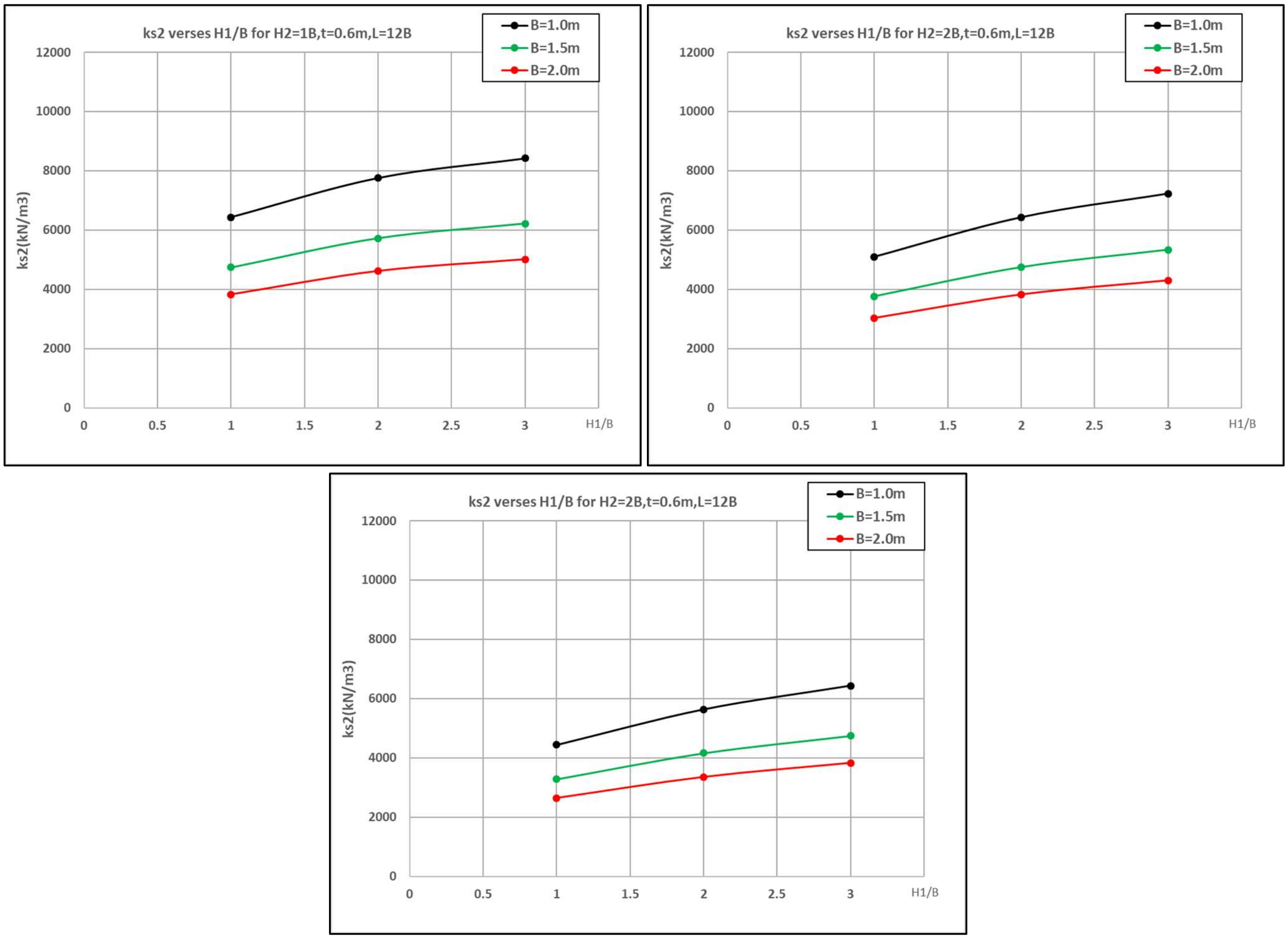


Figure 4. 4: Weighted average method equivalent subgrade modulus k_{s2} verses H_1/B for d=0.6m

Hence footing thickness is the only difference in both cases, Figure 4.4 shows similar behavior and graph pattern as in Figure 4.3. Lesser k_{s1} is obtained for a higher footing thickness with similar footing width and soil profile.

Footing thickness effects

To find out footing thickness effects, equivalent spring theory method and weighted average method equivalent subgrade modulus are separately plotted with top soil layer thicknesses for different footing thicknesses.

Table 4. 3: Equivalent subgrade modulus k_{s1} with H_1/B for various footing thicknesses(t)

H_1/B	H_2	k_{s1} (kN/m^3)	
B=1.0m	(m)	t=0.4m	t=0.6m
1.0	1.0	2276	1981
2.0	1.0	2276	1981
3.0	1.0	2276	1981
1.0	2.0	2276	1981
2.0	2.0	2276	1981
3.0	2.0	2276	1981
1.0	3.0	2276	1981
2.0	3.0	2276	1981
3.0	3.0	2276	1981

H_1/B	H_2	k_{s1} (kN/m^3)	
B=1.5m	(m)	t=0.4m	t=0.6m
1.0	1.5	1679	1462
2.0	1.5	1679	1462
3.0	1.5	1679	1462
1.0	3.0	1679	1462
2.0	3.0	1679	1462
3.0	3.0	1679	1462
1.0	4.5	1679	1462
2.0	4.5	1679	1462
3.0	4.5	1679	1462

H_1/B	H_2	k_{s1} (kN/m^3)	
B=2.0m	(m)	t=0.4m	t=0.6m
1.0	2.0	1353	1178
2.0	2.0	1353	1178
3.0	2.0	1353	1178
1.0	4.0	1353	1178
2.0	4.0	1353	1178
3.0	4.0	1353	1178
1.0	6.0	1353	1178
2.0	6.0	1353	1178
3.0	6.0	1353	1178

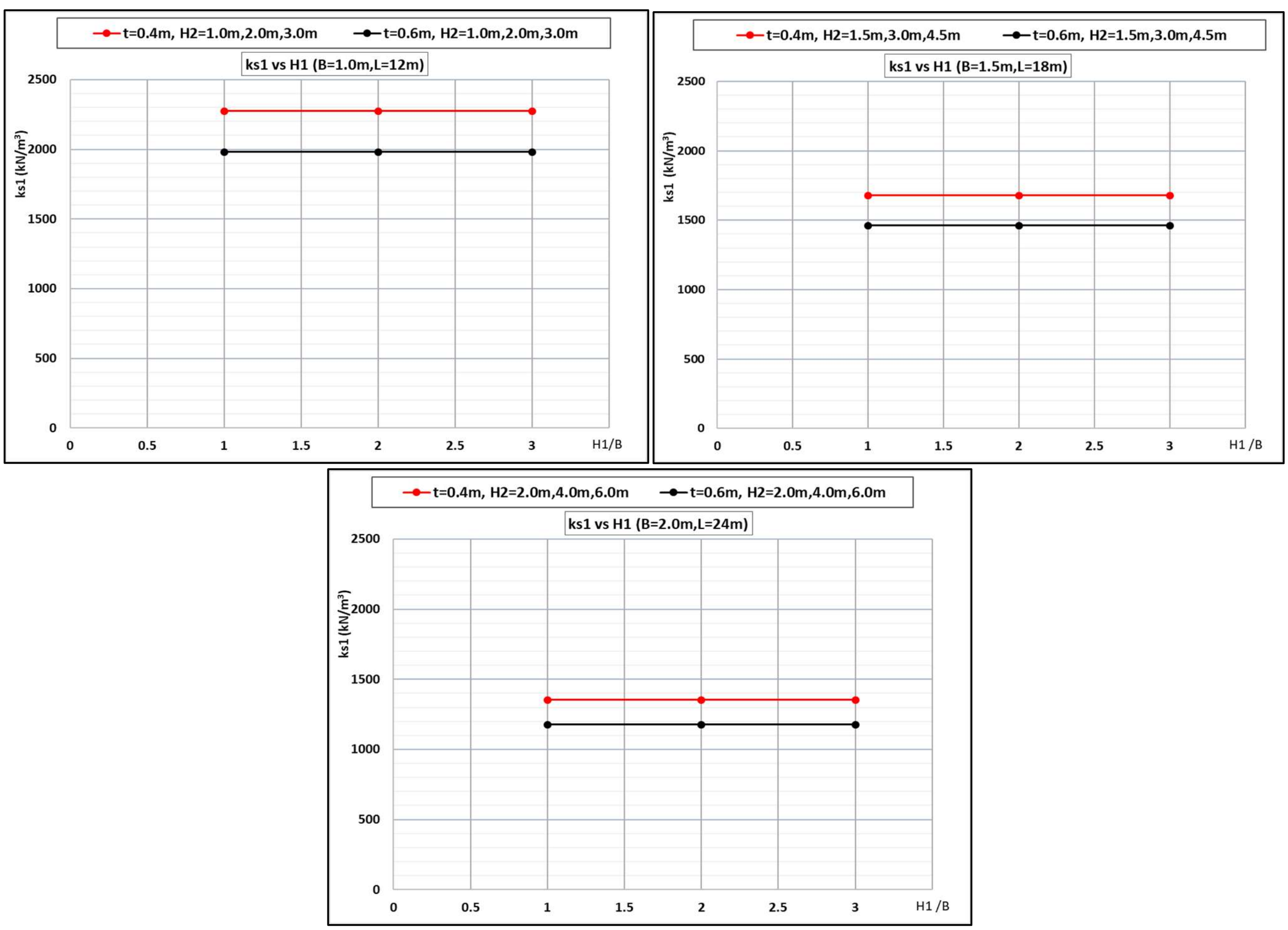


Figure 4. 5: Equivalent subgrade modulus k_{s1} with H_1/B for various footing thicknesses

Table 4. 4: Equivalent subgrade modulus k_{s2} with H_1/B for B=1.0m and various footing thicknesses (t)

H_1/B	H_2 (m)	k_{s2} (kN/m^3) t=0.4m	k_{s2} (kN/m^3) t=0.6m
1.0	1.0	7406	6430
2.0	1.0	8939	7757
3.0	1.0	9705	8421

H_1/B	H_2 (m)	k_{s2} (kN/m^3) t=0.4m	k_{s2} (kN/m^3) t=0.6m
1.0	2.0	5873	5102
2.0	2.0	7406	6430
3.0	2.0	8326	7226

H_1/B	H_2 (m)	k_{s2} (kN/m^3) t=0.4m	k_{s2} (kN/m^3) t=0.6m
1.0	3.0	5107	4438
2.0	3.0	6486	5633
3.0	3.0	7406	6430

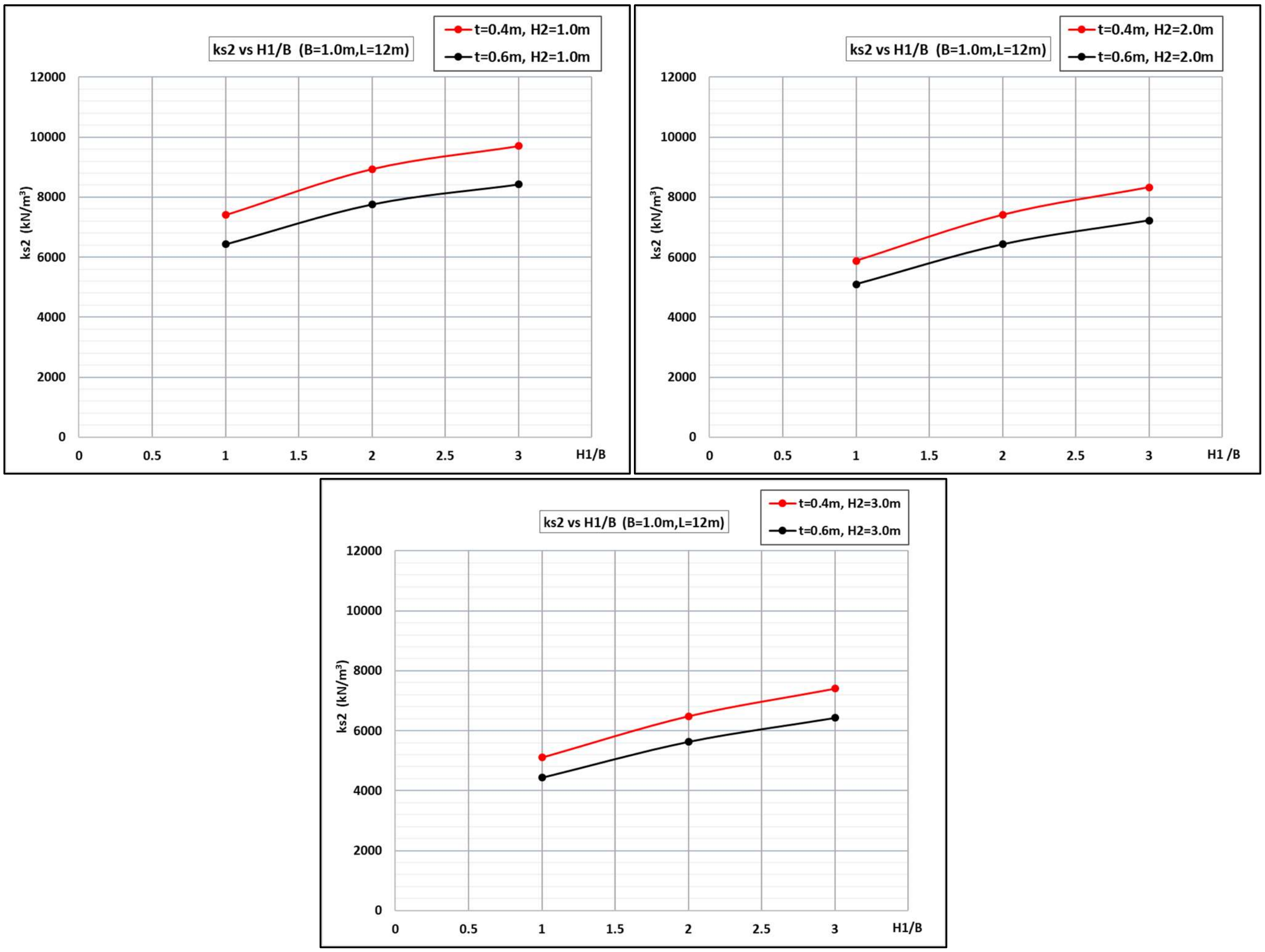


Figure 4. 6: Equivalent subgrade modulus k_{s2} with H_1/B for B=1.0m and various footing thicknesses(t)

Table 4. 5: Equivalent subgrade modulus k_{s2} with H_1/B for B=1.5m and various footing thicknesses(t)

H_1/B	H_2 (m)	k_{s2} (kN/m^3) t=0.4m	k_{s2} (kN/m^3) t=0.6m
1.0	1.5	5464	4744
2.0	1.5	6595	5723
3.0	1.5	7160	6213

H_1/B	H_2 (m)	k_{s2} (kN/m^3) t=0.4m	k_{s2} (kN/m^3) t=0.6m
1.0	3	4333	3764
2.0	3	5464	4744
3.0	3	6142	5332

H_1/B	H_2 (m)	k_{s2} (kN/m^3) t=0.4m	k_{s2} (kN/m^3) t=0.6m
1.0	4.5	3768	3274
2.0	4.5	4786	4156
3.0	4.5	5464	4744

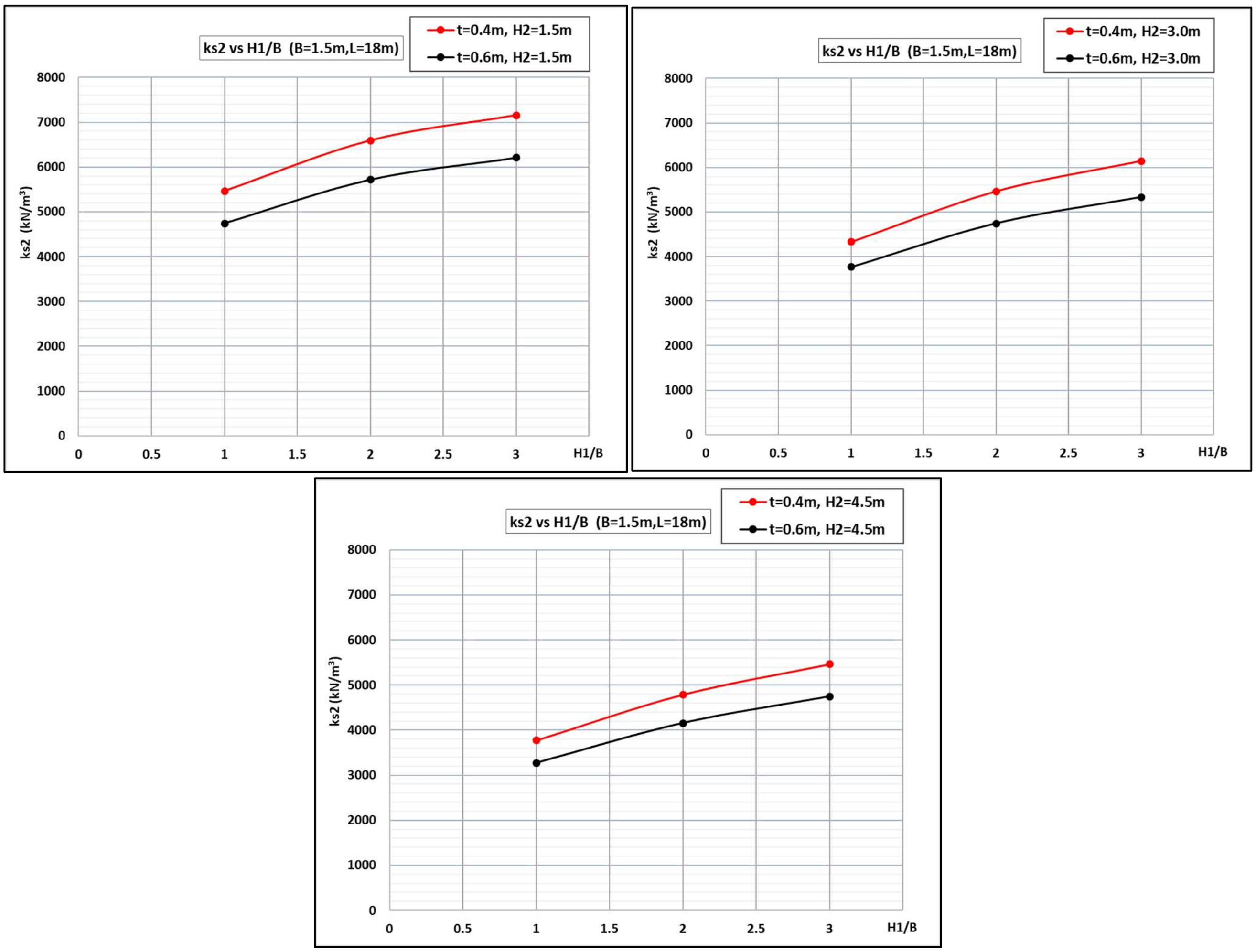


Figure 4. 7: Equivalent subgrade modulus k_{s2} with H_1/B for B=1.5m and various footing thicknesses(t)

Table 4. 6: Equivalent subgrade modulus k_{s2} with H_1/B for B=2.0m and various footing thicknesses (t)

H_1/B	H_2 (m)	k_{s2} (kN/m^3) t=0.4m	k_{s2} (kN/m^3) t=0.6m
1.0	2.0	4404	3823
2.0	2.0	5315	4613
3.0	2.0	5771	5007

H_1/B	H_2 (m)	k_{s2} (kN/m^3) t=0.4m	k_{s2} (kN/m^3) t=0.6m
1.0	4.0	3492	3034
2.0	4.0	4404	3823
3.0	4.0	4950	4297

H_1/B	H_2 (m)	k_{s2} (kN/m^3) t=0.4m	k_{s2} (kN/m^3) t=0.6m
1.0	6.0	3037	2639
2.0	6.0	3857	3349
3.0	6.0	4404	3823

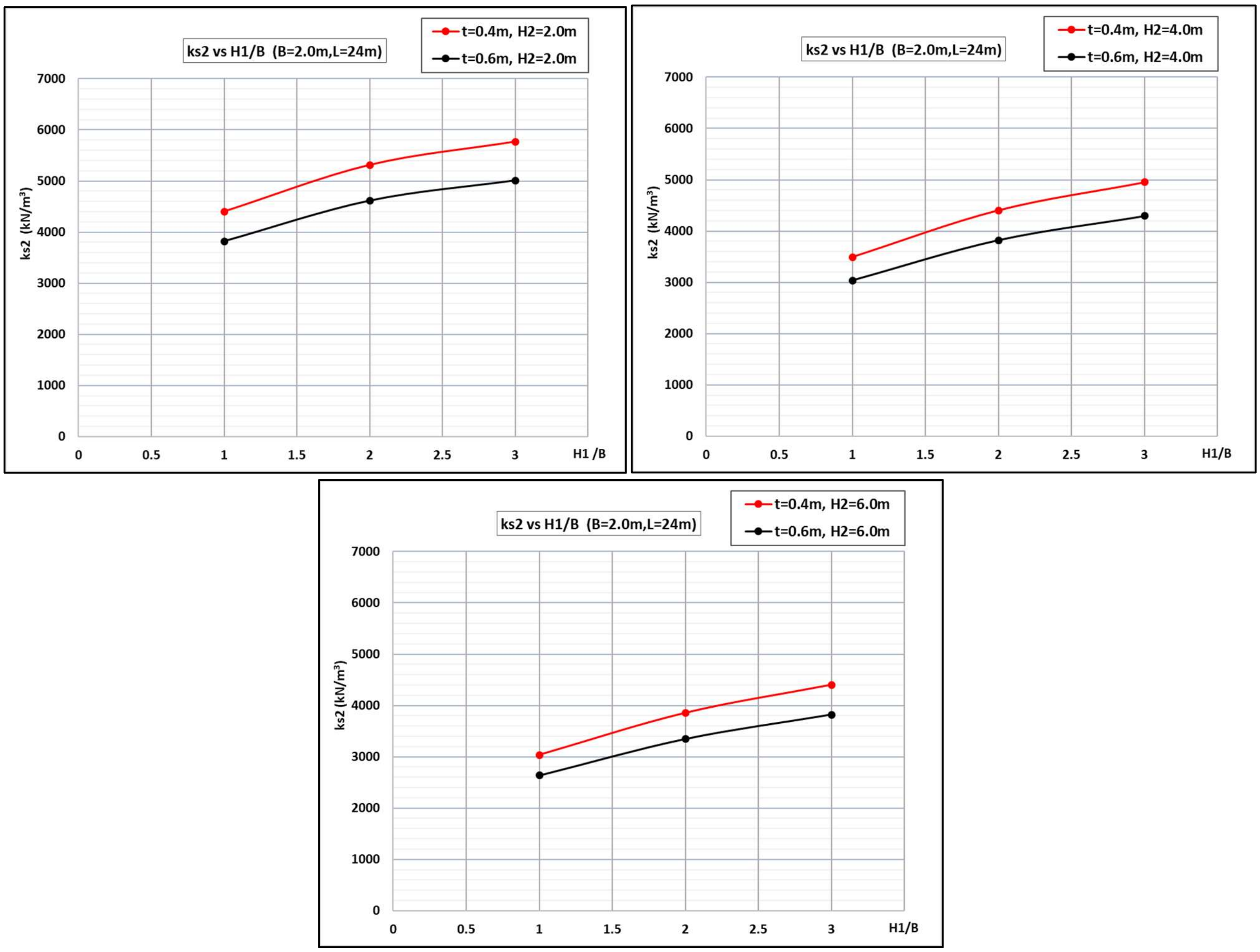


Figure 4. 8: Equivalent subgrade modulus k_{s2} with H_1/B for B=2.0m and various footing thicknesses (t)

Equivalent subgrade modulus using equivalent spring theory verses top soil layer thickness graph is constant for a given footing and various bottom soil layer thicknesses. Equivalent subgrade modulus for the thinner foundation depth is always greater than the thicker one at every H_1 as elaborated in Figure 4.5 for a given footing width.

Figure 4.6 shows change in the weighted average method equivalent subgrade modulus for constant loose sand thicknesses and footing width, but dense sand thickness and footing thickness are variables. As explained in Figure 4.6, k_{s2} value increases non-linearly with dense sand thickness for both footing thicknesses of 0.4m and 0.6m. Equivalent subgrade modulus obtained by weighted average method for the thinner foundation depth is always greater than the thicker one at every H_1 as elaborated in Figure 4.6, Figure 4.7 and Figure 4.8.

Equivalent subgrade modulus for the thinner foundation depth is always greater than the thicker one at every H_1 for both equivalent spring method and weighted average method. This is because footing thickness is non-linearly proportional to the moment of inertia of the footing about the horizontal axis. As per Vesic method moment of inertia of the footing is inversely proportional to the subgrade modulus in a uniform medium. Therefore, equivalent subgrade modulus is higher for thinner footings and this Figures approve this argument.

4.2.2 Settlement analysis

Settlement with top soil layer thickness

Equivalent modulus of subgrade reaction obtained by equivalent spring theory and weighted average methods are separately used in Heteryni method settlement equation to find the settlement profile along footing. PLAXIS 3D numerical is used to model the strip footing resting on doubled layered soil system with same input parameters and vertical settlement along footing is obtained. To analysis the vertical settlement with top soil layer thickness settlement verses top soil thickness to footing width ratio is plotted.

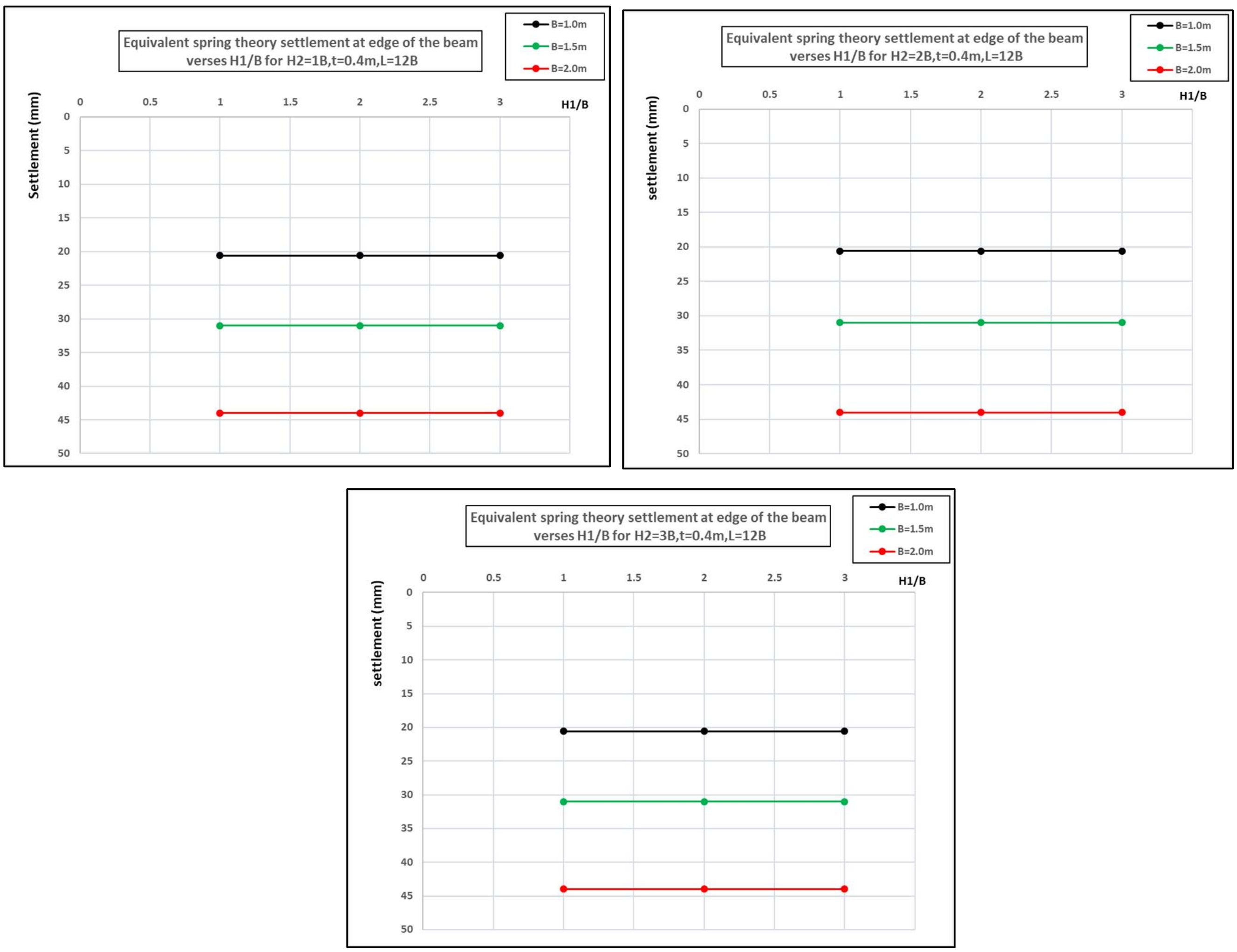


Figure 4. 9 : Equivalent spring theory settlement at edge of the footing verses H_1/B for t=0.4m

From Figure 4.13, vertical settlement at edge of the footing using equivalent spring method verses top soil layer thickness graph is constant for a given footing width and footing thickness. This is because equivalent subgrade modulus obtained by equivalent spring theory does not depend on top or bottom soil layer thicknesses. Higher footing width gives lower equivalent subgrade modulus for a given footing thickness. Hence footing with higher width gives higher settlement for a given footing thickness.

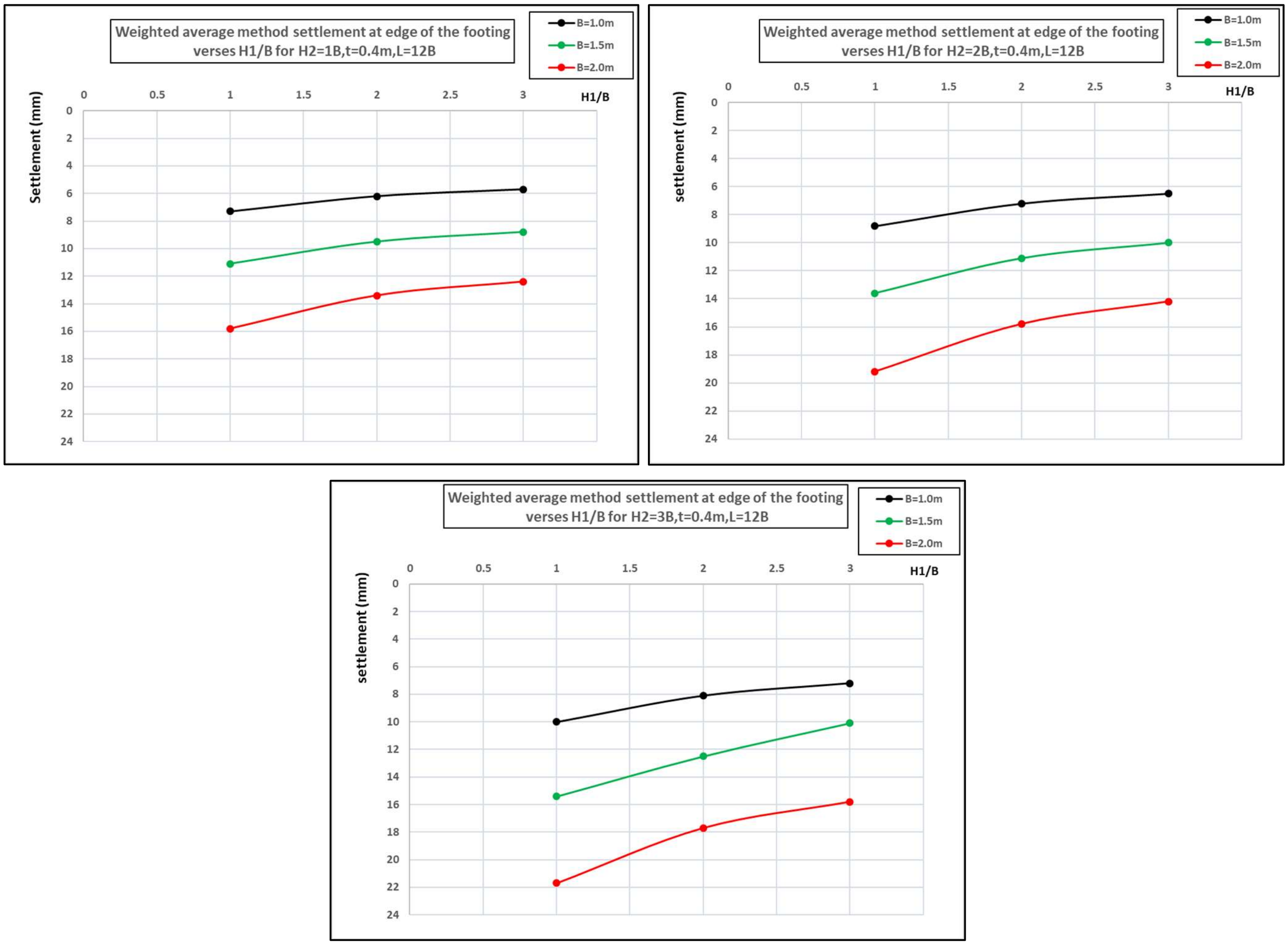


Figure 4. 10: Weighted average method settlement at edge of the footing verses H_1/B for t=0.4m

Settlement obtained from weighted average method equivalent subgrade modulus applied in Heteryni method settlement equation is plotted with top soil layer thickness in Figure 4.15. Vertical settlement is non linearly decreasing with top soil layer thickness for a given footing and bottom soil layer thickness. Here dense sand is used as top soil and loose sand is used as bottom soil. When top dense sand layer thickness increases for a constant bottom loose sand layer, equivalent subgrade modulus increases. Hence settlement decreases with top soil layer thickness for a given footing and bottom soil layer thickness.

Since larger footing width gives lower equivalent subgrade modulus, larger footing width shows higher vertical settlement for a given footing thickness and soil profile.

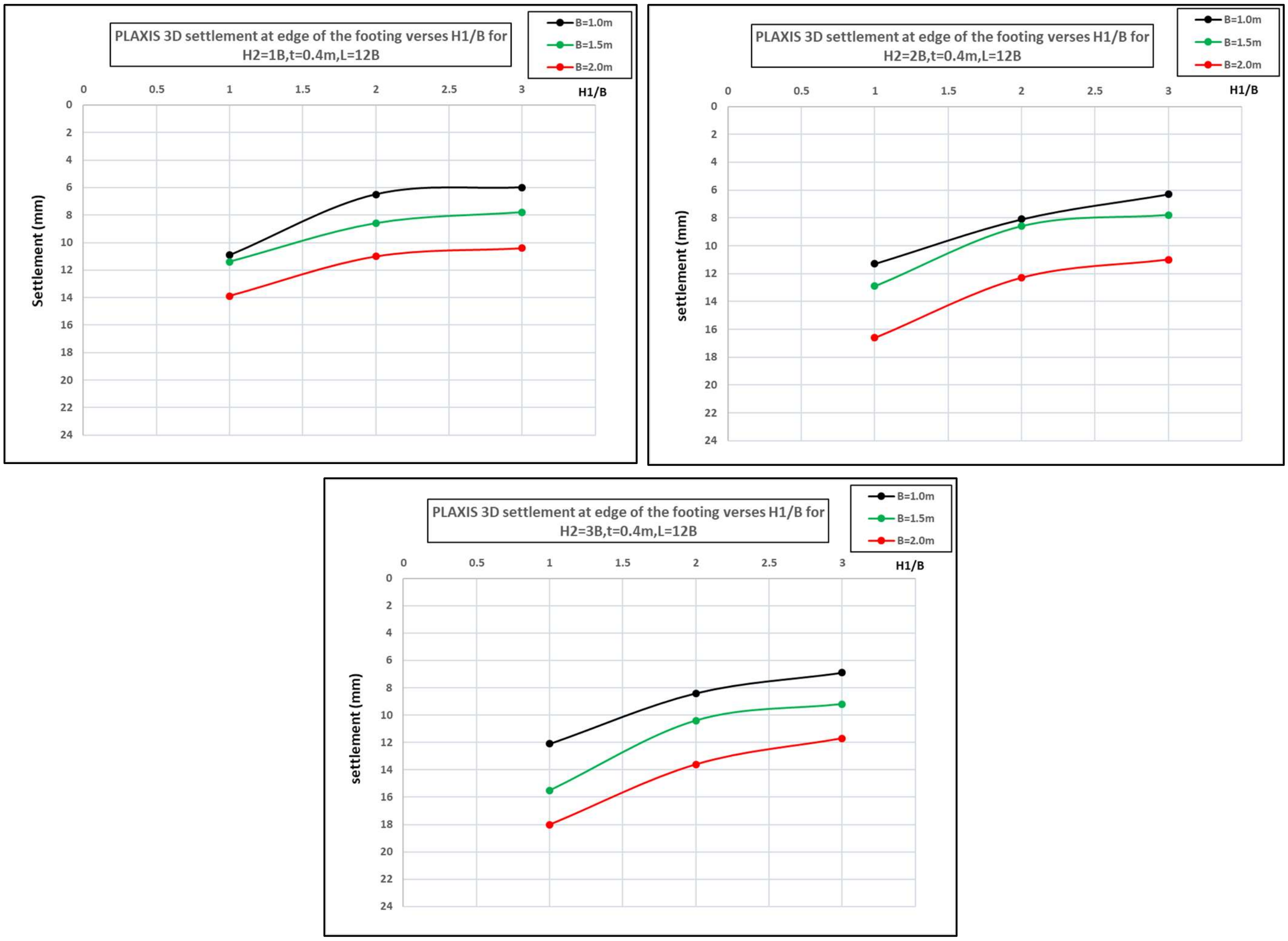


Figure 4. 11: PLAXIS 3D settlement at edge of the footing verses H_1/B for t=0.4m

PLAXIS 3D numerical model settlement is plotted with top soil layer thickness in Figure 4.17. Vertical settlement is non linearly decreasing with top soil layer thickness for a given footing and bottom soil layer thickness. Here dense sand is used as top soil and loose sand is used as bottom soil. When top dense sand layer thickness increases for a constant bottom loose sand layer equivalent subgrade modulus increases. Hence settlement decreases with top soil layer thickness for a given footing and bottom soil layer thickness.

Since larger footing width gives lower equivalent subgrade modulus, larger footing width shows higher vertical settlement for a given footing thickness and soil profile.

Settlement profile along footing

Vertical settlement profile obtained from equivalent spring theory, weighted average method and PLAXIS 3D methods are plotted below in Figure 4.19.

Table 4. 7 : Settlement along footing (for B=1.5m,L=18.0m,H_1=3.0m,H_2=3.0m)

B=1.5m, L=18.0m, H_1=3.0m, H_2=3.0m, F=180kN, 2F=360kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	31.0	32.0	11.1	11.3	10.4	10.9
3	23.7	27.7	7.5	8.4	10	10.4
6	22.1	26.1	6.9	7.8	9.7	10.1
9	20.7	25.2	6.2	7.4	9.5	9.8
12	22.1	26.1	6.9	7.8	9.7	10.1
15	23.7	27.7	7.5	8.4	10	10.4
18	31.0	32.0	11.1	11.3	10.4	10.9

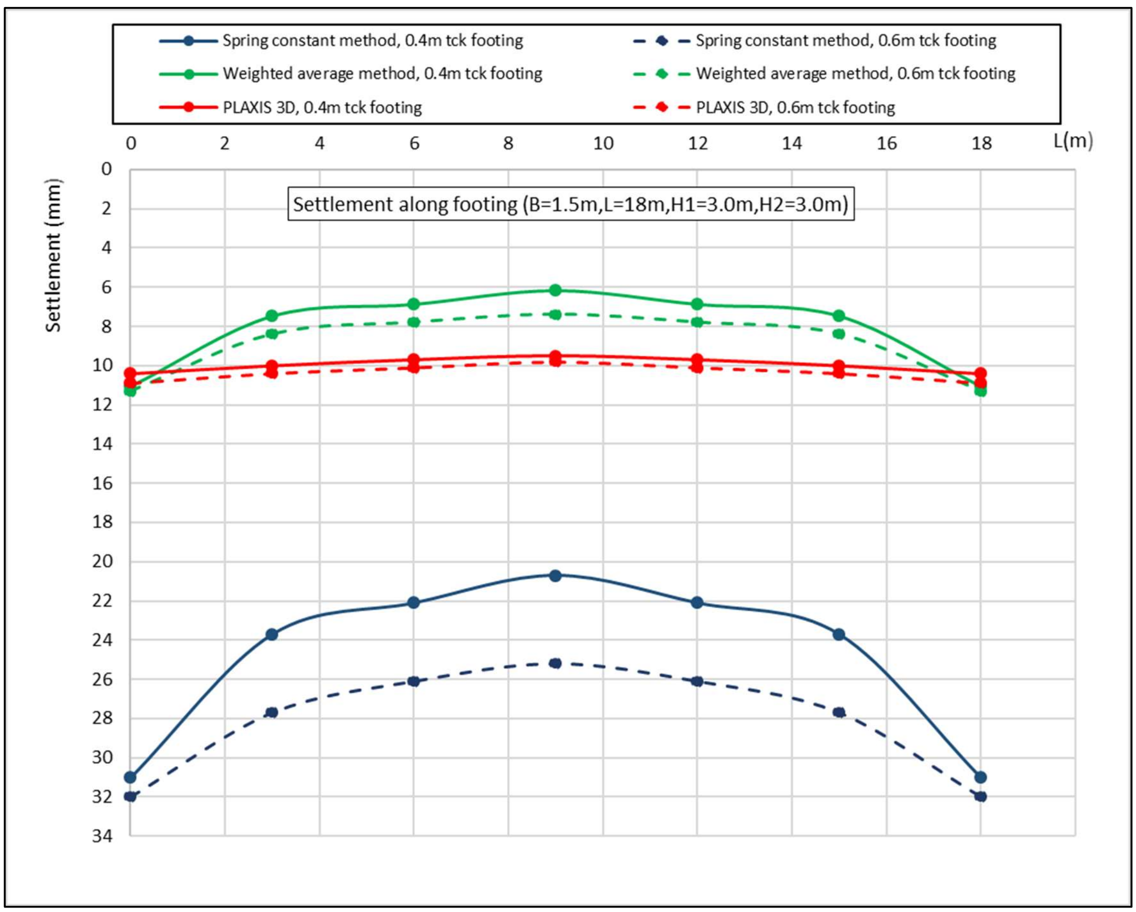


Figure 4. 12: Vertical settlement along footing (for B=1.5m,L=18.0m,H_1=3.0m,H_2=3.0m)

In all three methods higher settlement is at free ends of footing and lesser settlement is at the middle of footing. Settlement obtained for thicker footing is always higher than thinner footing along the beam for all three methods.

Equivalent spring method equivalent subgrade modulus applied in Heteryni method settlement equation shows significant higher settlement profile than PLAXIS 3D model settlement along beam. Weighted average method equivalent subgrade modulus applied in Heteryni method settlement equation shows lesser settlement than PLAXIS 3D model settlement from 0.5m to 17.5m length of footing for a 18m length footing. Settlement from weighted average method equivalent subgrade modulus applied in Heteryni method slightly exceeds at edges of footing.

Since settlement along footing obtained by equivalent spring method equivalent subgrade modulus used in Heteryni method is highly varying from other two methods, equivalent spring method is considered unsuitable to calculate the equivalent modulus of subgrade reaction of layered soil stratum. PLAXIS 3D model gives conservative settlement than weighted average method equivalent subgrade modulus used in Heteryni method.

Settlement difference between PLAXIS 3D and weighted average method

Difference between PLAXIS 3D settlement and Heternyi method settlement obtained by weighted average method equivalent subgrade modulus is plotted as percentage of PLAXIS 3D settlement along footing in Figure 4.20 to Figure 4.22.

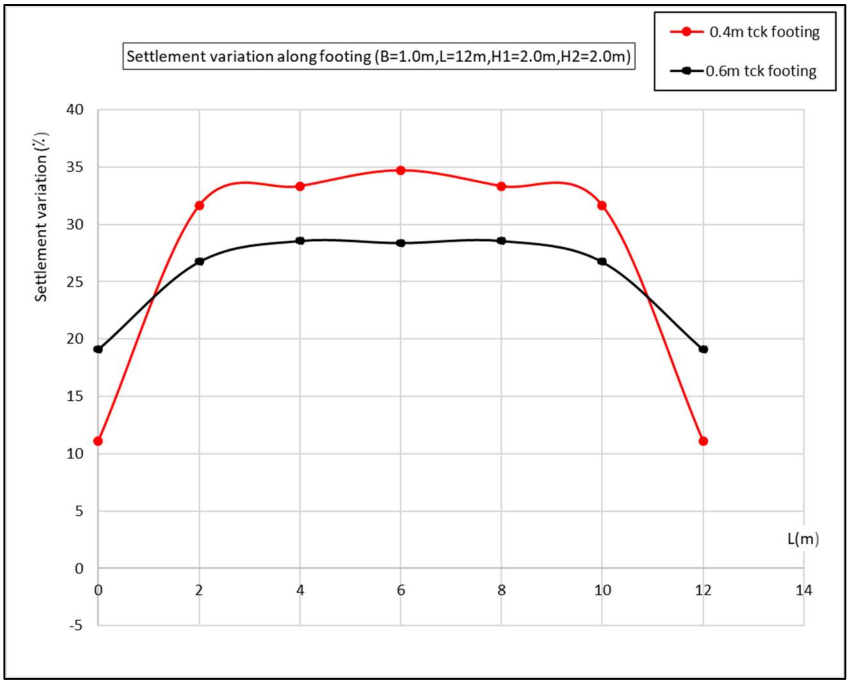


Figure 4. 13: Settlement difference between PLAXIS 3D and weighted average equivalent subgrade reaction applied in Heteryni equation along footing (for B=1.0m, L=12.0m, H_1=2.0m, H_2=2.0m)

Throughout the beam settlement different is positive. It means settlement obtained from weighted average method equivalent subgrade modulus applied in Heteryni method is lesser than PLAXIS 3D settlement along beam for 1.0m footing width.

Settlement difference graph shows smaller value of 11 percentages at edges of the footing and maximum value of 35 percentages at middle of footing for 0.4m thick footing. Settlement difference graph for 0.6 m thickness footing shows smaller value of 11 percentages at edges of the footing and maximum value of 29 percentages at middle of footing. Settlement difference is greater than 10 percentages and cannot be neglected along footing for both 0.4m and 0.6m footing thicknesses.

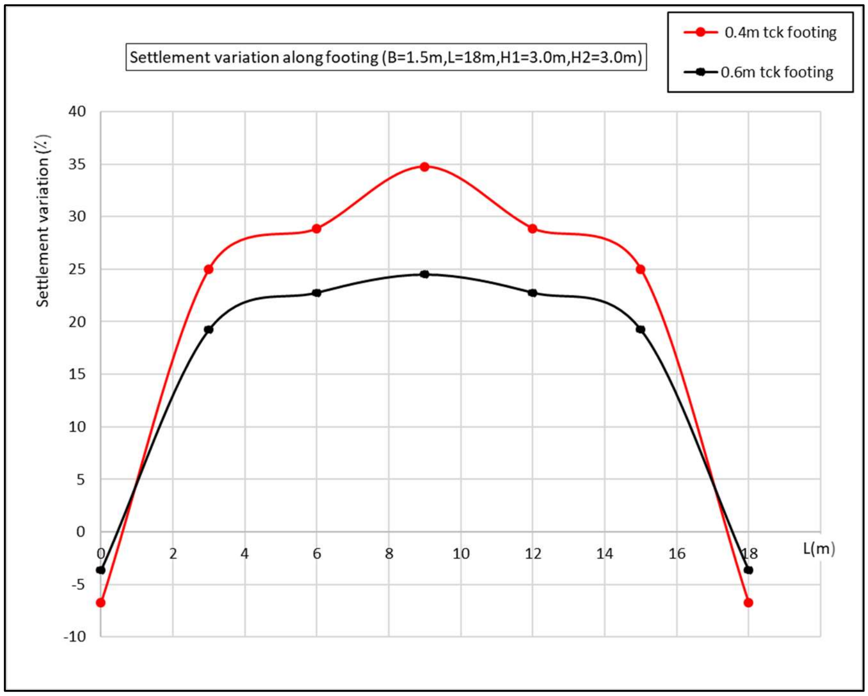


Figure 4. 14: Settlement difference between PLAXIS 3D and weighted average equivalent subgrade reaction applied in Heteryni equation along footing (for B=1.5m, L=18.0m, H_1=3.0m, H_2=3.0m)

From both ends of footing approximately for a 0.5m distance, settlement different is negative. It means settlement obtained from weighted average method equivalent subgrade modulus applied in Heteryni method is higher than PLAXIS 3D settlement. In remain middle length of beam PLAXIS 3D settlement is dominant and conservative.

Settlement difference graph shows (-)7 percentages at edges of the footing and 35 percentages at middle of footing for 0.4m thick footing. Settlement difference graph for 0.6 m thickness footing shows (-)4 percentages at edges of the footing and 24 percentages at middle of footing.

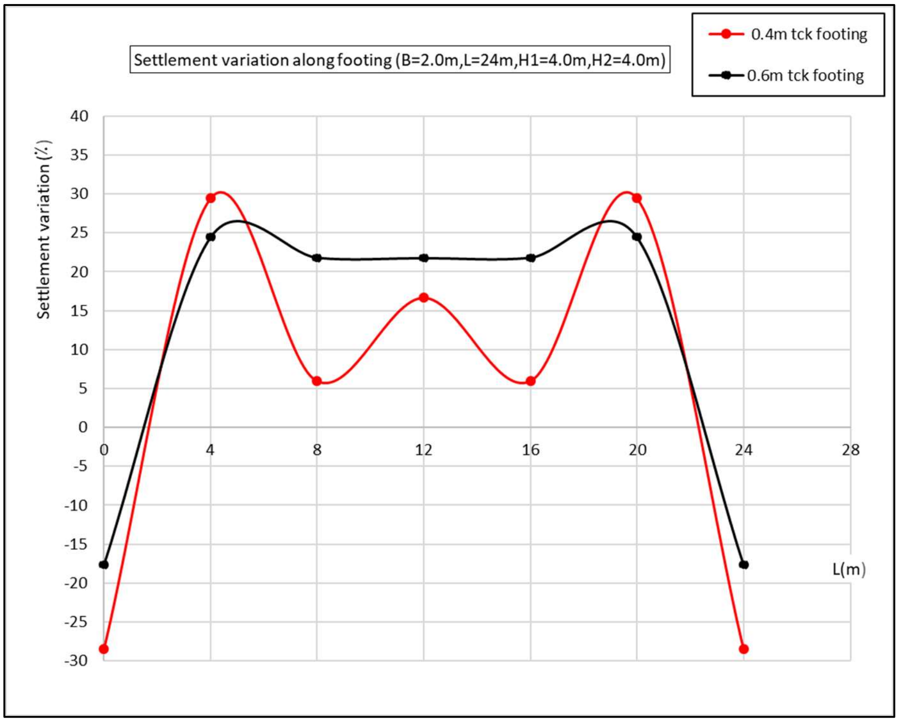


Figure 4. 15: Settlement difference between PLAXIS 3D and weighted average equivalent subgrade reaction applied in Heteryni equation along footing (for B=2.0m, L=24.0m, H_1=4.0m, H_2=4.0m)

From both edges of footing approximately for 2.0m distance, settlement different is negative. It means settlement obtained from weighted average method equivalent subgrade modulus applied in Heteryni method is higher than PLAXIS 3D settlement. In remain middle length of beam PLAXIS 3D settlement is dominant and conservative.

Settlement difference graph shows (-)28 percentages at edge of the footing and 29 percentages at 4.0m from distance from edge for 0.4m thick footing. Settlement difference graph for 0.6 m thickness footing shows (-)18 percentages at edge of the footing and 24 percentages at 4.0m from distance from edge of the footing. Weighted average method equivalent subgrade modulus applied in Heteryni method shows significant settlement difference than PLAXIS 3D model along the footing for 2.0m width footing.

When consider above three cases together Settlement difference graphs show maximum (-)28 percentages at edge of the footing and maximum 35 percentages in the middle of the footing. Validation in methodology proofs PLAXIS 3D gives correct settlement output and PLAXIS 3D settlement is dominant in more than 80 percentage length of footing. Therefore, PLAXIS 3D settlement can be suggested to use in strip footing settlement calculations from all above three methods.

4.2.3 Bending moment and shearing force analysis

Weighted average method equivalent subgrade modulus is used in Heteryni method bending moment equation. Heteryni method bending moment and PLAXIS 3D bending moment along footing are drawn in same graph.

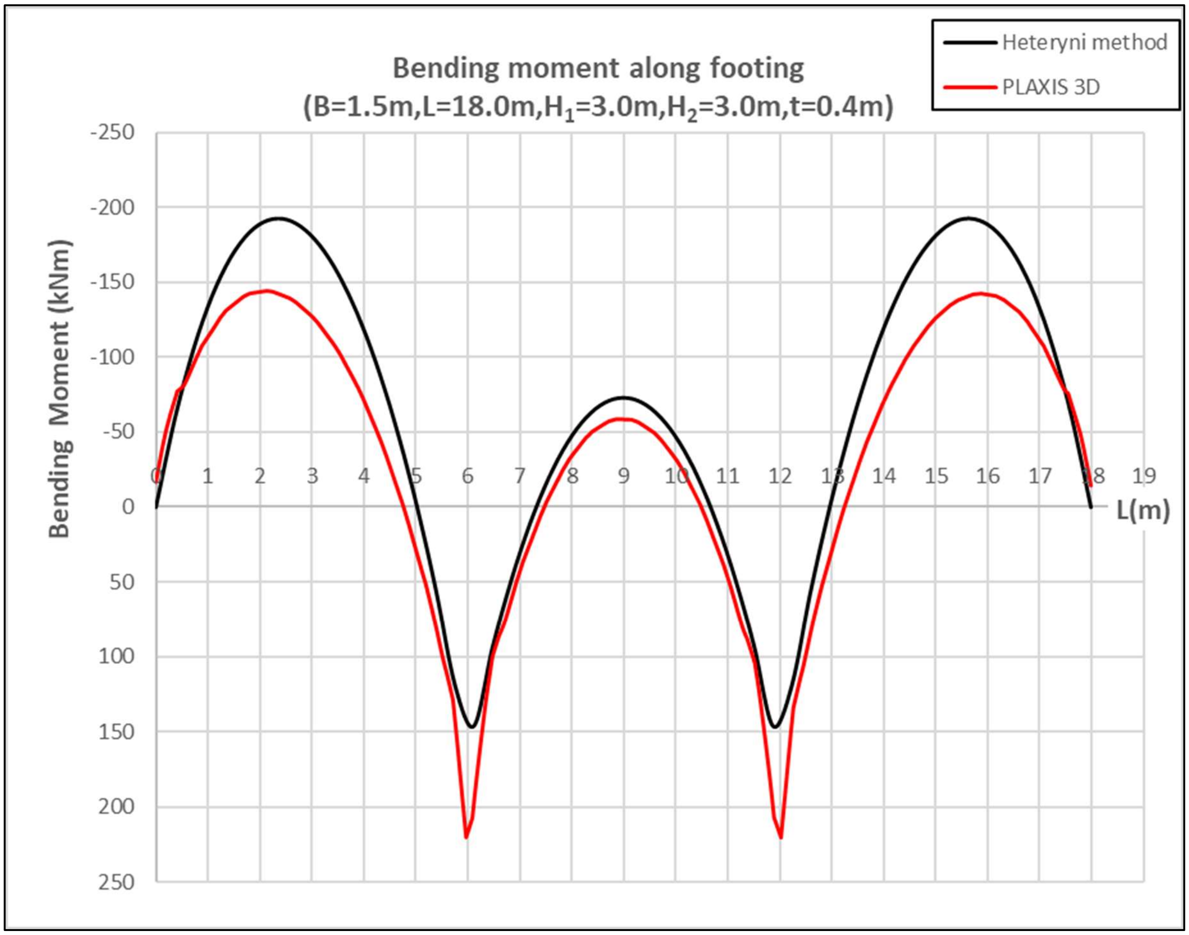


Figure 4. 16: Bending moment along footing (for B=1.5m, t=0.4m, L=18.0m, H_1=3.0m, H_2=3.0m)

Both method bending moment curves are showing similar pattern with different peak values. In this graph weighted average method equivalent subgrade modulus applied in Heteryni method shows 192kNm peak value at 2.2m and 146kNm at 6.0m. PLAXIS 3D shows 143kNm peak value at 2.2m and 208kNm peak value at 6.0m. Difference between two methods are 34 percentages and 30 percentages respectively. These differences are not negligible.

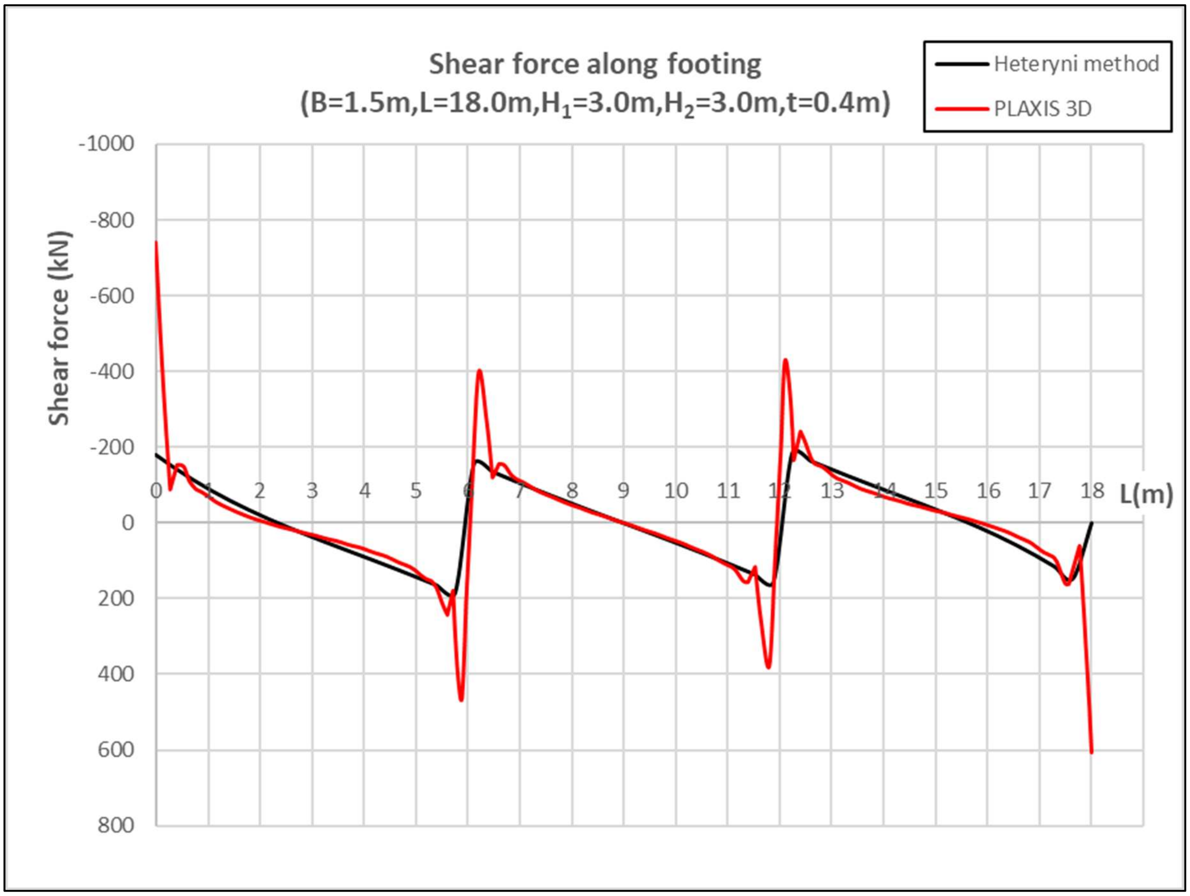


Figure 4. 17: Shear force along footing (for B=1.5m, t=0.4m, L=18.0m, H_1=3.0m, H_2=3.0m)

Weighted average method equivalent subgrade modulus is used in Heteryni method shear equation. Heteryni method and PLAXIS 3D shearing force along footing are drawn in same graph. Both graphs show same pattern and values at the mid spans. At the support points PLAXIS 3D Shear force shows higher value than weighted average equivalent subgrade modulus applied in Heteryni method equation. These differences cannot be negligible.

4.3 Discussion on results

Equivalent subgrade modulus derived using spring method is constant with top soil layer thickness for a given footing width and footing depth. Spring method equivalent subgrade modulus depends on top and bottom soil layer homogeneous subgrade modulus only. Homogeneous subgrade modulus is a function of footing width, footing thickness, footing material, and top soil layer material. Equivalent spring method equivalent subgrade modulus is not a function of top soil layer thickness or bottom soil layer thickness. Therefore, ks1 verses top soil layer thickness curve is constant for a given footing and same graph is obtained while changing the bottom soil layer thickness.

Weighted average method equivalent subgrade modulus is non linearly increasing with top soil layer thickness for a given footing and bottom soil layer thickness. In this study top soil layer is dense sand with a higher homogeneous subgrade modulus and bottom soil layer is loose sand with lesser subgrade modulus. For a given footing and bottom loose sand layer thickness, weighted average method equivalent subgrade modulus is non-linearly increasing with top dense sand layer thickness.

Equivalent subgrade modulus for thinner foundation thickness is always greater than the thicker footing for a given footing width and soil profile. As per Vesic method equation, moment of inertia of the footing is inversely proportional to the homogeneous subgrade modulus. When footing thickness increases moment of inertia increases for strip footing with rectangular cross section. Hence thinner footings always have greater equivalent subgrade modulus than thicker footings for given footing width and soil strata.

At a particular point on footing, vertical settlement using equivalent spring method verses top soil layer thickness graph is constant for a given footing width and footing depth. This is because equivalent subgrade modulus obtained by equivalent spring theory does not depend on top or bottom soil layer thicknesses. Larger footing width gives lower equivalent subgrade modulus for a given footing thickness and soil profile. Hence footing with larger width gives constant higher value settlement curve with top soil layer thickness for a given footing thickness.

At a particular point on footing, settlement obtained from weighted average method equivalent subgrade modulus applied in Heteryni method settlement equation is non linearly decreasing with top soil layer thickness for a given footing and bottom soil layer thickness. Here dense sand is used as top soil and loose sand is used as bottom soil. Hence settlement decreases with top soil layer thickness for a given footing and bottom soil layer thickness.

Settlement along footing obtained by spring method equivalent subgrade modulus applied in Heteryni method equation is highly varying from weighted average method equivalent subgrade modulus applied in Heteryni method equation and PLAXIS 3D model settlement output. Hence equivalent spring method is considered as unsuitable to calculate the equivalent modulus of subgrade reaction for layered soil stratum. PLAXIS 3D model gives conservative settlement output than settlement obtained from weighted average method equivalent subgrade modulus applied in Heteryni method. In all three methods higher settlement is in the free ends of footing and lesser settlement is in the middle of footing. For all three methods, settlement obtained for thicker foundation is always higher than thinner foundation along footing.

Settlement difference between PLAXIS 3D method and weighted average method equivalent subgrade modulus applied in Heteryni method settlement equation graphs show mode value up to 30 percentages at edge of the footing and up to 45 percentages at middle of the footing. These settlement differences cannot be neglected.

CHAPTER FIVE

CONCLUSION AND RECOMMENDATIONS

5.1 Conclusion

Primary aim of this research is to find an appropriate relationship from the obtained equivalent subgrade modulus through weighted average method and equivalent spring theory. Equivalent spring method is not suitable to find the equivalent subgrade modulus of layered soil. Weighted average method gives same settlement, bending moment and shear force pattern as PLAXIS 3D outputs. Settlement difference between PLAXIS 3D method and weighted average method equivalent subgrade modulus applied in Heteryni method equation shows up to 45 percentages and this difference cannot be negligible. Bending moment and shear force magnitudes also show significant difference in values while showing the similar pattern as PLAXIS 3D output. Therefore, further investigations should be done in future to minimize the differences.

5.2 Recommendations

The present study is limited to dense sand over loose sand doubled layer soil combination. It can be extended to dense sand, loose sand, stiff clay and soft clay multi-layer soil combinations. This study is restricted to strip footings resting on homogeneous horizontal dense sand over homogeneous horizontal loose sand. Footings resting on slopes can be considered. In this study footing embedment depth is considered as zero which means footing is resting on top of the ground. Footing resting on various depths into the soil also can be considered. Effect of ground water table and other shape of foundations can be considered.

CHAPTER SIX

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CHAPTER SEVEN
APPENDIX

7.1 Hetenyi method equations

The solutions for a finite beam with free ends and a concentrated force P at any point proposed by Hetenyi are below.

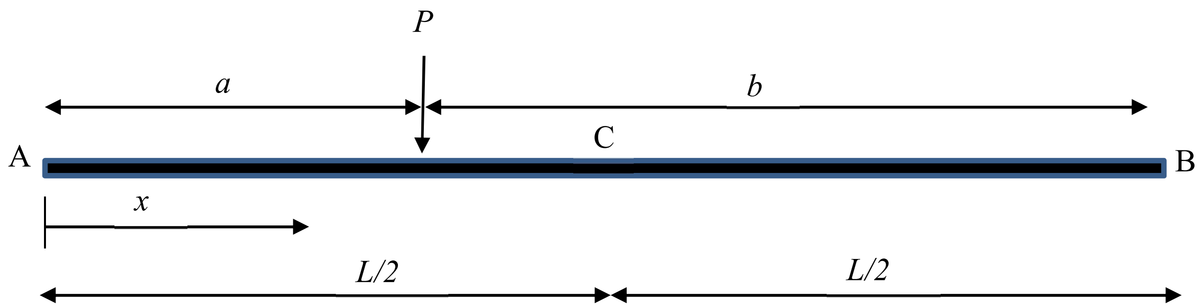


Figure 7. 1 : A finite beam subjected to a concentrated load

Deflection y at any point $x < a$,

$$y = \frac{P\lambda}{k_s'\{(\sinh\lambda L)^2 - (\sin\lambda L)^2\}}\{2\cosh\lambda x\cos\lambda x\,(\sinh\lambda L\cos\lambda a\cosh\lambda b - \sin\lambda L\cosh\lambda a\cos\lambda b) + (\cosh\lambda x\sin\lambda x + \sinh\lambda x\cos\lambda x)\big[\sinh\lambda L\,(\sin\lambda a\cos\lambda b - \cos\lambda a\sin\lambda b) + \big(\sin\lambda L\,(\sinh\lambda a\cos\lambda b - \cosh\lambda a\sin\lambda b)\big)\big]\} \tag{15}$$

Deflection y_c at the point of application of the load $x=a$,

$$y_c = \frac{P\lambda}{k_s'\{(\sinh\lambda L)^2 - (\sin\lambda L)^2\}}\{((\cosh\lambda a)^2 + (\cos\lambda a)^2)(\sinh\lambda b\cosh\lambda b - \sin\lambda b\cos\lambda b) + ((\cosh\lambda b)^2 + (\cos\lambda b)^2)(\sinh\lambda a\cosh\lambda a - \sin\lambda a\cos\lambda a)\} \tag{16}$$

Bending moment M at any point $x < a$,

$$M = \frac{P}{2\lambda\{(\sinh\lambda L)^2 - (\sin\lambda L)^2\}}\{2\sinh\lambda x\sin\lambda x\,(\sinh\lambda L\cos\lambda a\cosh\lambda b - \sin\lambda L\cosh\lambda a\cos\lambda b) + (\cosh\lambda x\sin\lambda x - \sinh\lambda x\cos\lambda x)\big[\sinh\lambda L\,(\sin\lambda a\cosh\lambda b - \cos\lambda a\sinh\lambda b) + \big(\sin\lambda L\,(\sinh\lambda a\cos\lambda b - \cosh\lambda a\sin\lambda b)\big)\big]\} \tag{17}$$

Bending moment M_c at the point of application of the load *x=a,*

$$M_c = \frac{P}{4\lambda\{(\sinh\lambda L)^2 - (\sin\lambda L)^2\}}\{((\cosh\lambda a)^2 - (\cos\lambda a)^2)(\sinh 2\lambda b - \sin 2\lambda b) + ((\cosh\lambda b)^2 - (\cos\lambda b)^2)(\sinh 2\lambda a - \sin 2\lambda a)\} \quad (18)$$

Shearing force Q at any point $x < a$,

$$Q = \frac{P}{\{(\sinh\lambda L)^2 - (\sin\lambda L)^2\}}\{(\cosh\lambda x\sin\lambda x + \sinh\lambda x\cos\lambda x)(\sinh\lambda L\cos\lambda a\cosh\lambda b - \sin\lambda L\cosh\lambda a\cos\lambda b) + \sinh\lambda x\sin\lambda x\,[\sinh\lambda L\,(\sin\lambda a\cosh\lambda b - \cos\lambda a\sinh\lambda b) + \sin\lambda L\,(\sinh\lambda a\cos\lambda b - \cosh\lambda a\sin\lambda b)]\} \quad (19)$$

Shearing force Q_c at the point of application of the load *x=a*,

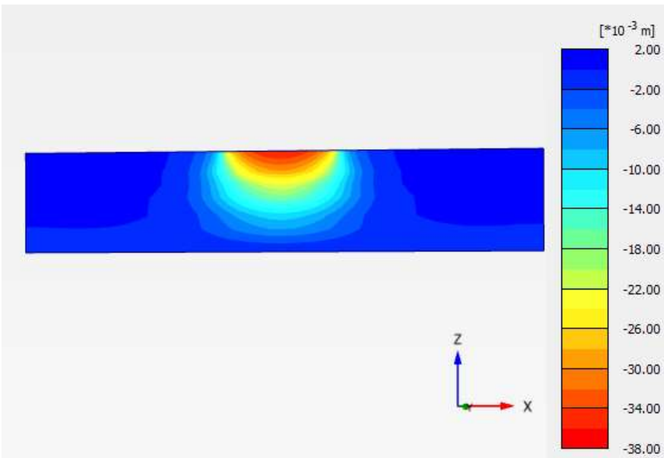
$$Q_c = \frac{P}{4\{(\sinh\lambda L)^2 - (\sin\lambda L)^2\}}\{4\sinh\lambda L\sinh\lambda a\,\cosh\lambda b - 4\sin\lambda L\sin\lambda a\cos\lambda b - \sinh 2\lambda a\,\sin 2\lambda b + \sin 2\lambda a\sinh 2\lambda b\} \quad (20)$$

***Note*:**

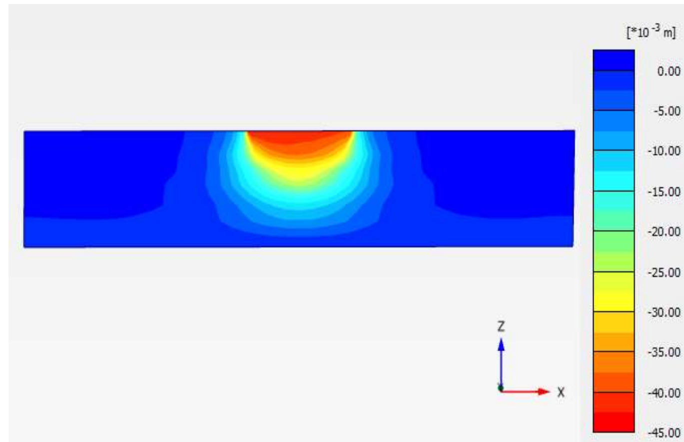
Above formulas are for the A-C portion of the beam, where x < a. Same formulas can be used for the B-C section, where x < b , by measuring x from B and replacing b by a.

7.2 PLAXIS 3D settlement outputs for validation

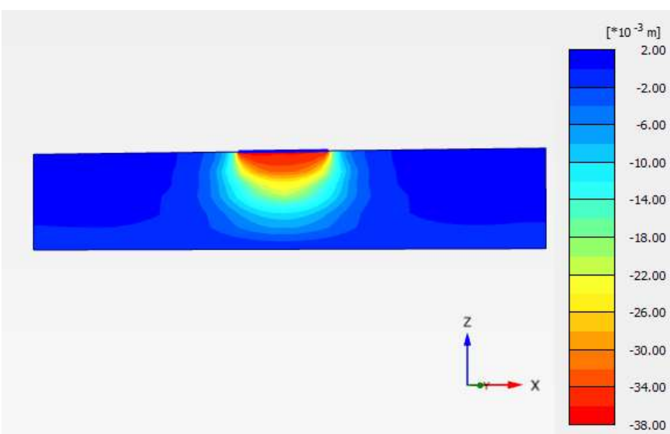
Oil storage tank foundation



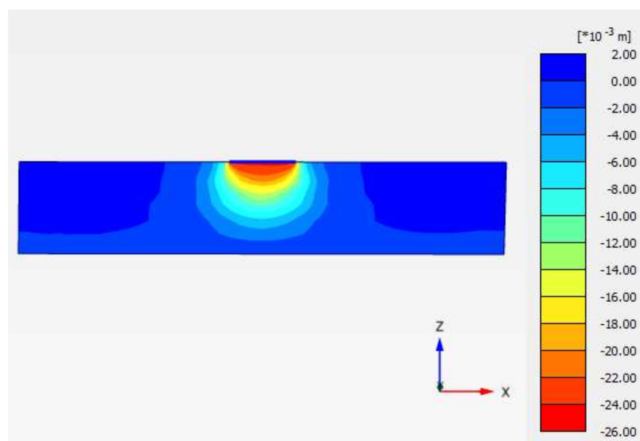
Settlement of oil storage tank foundation for D’=17.0m, q=149.46kN/m^2



Settlement of oil storage tank foundation for D’=14.0m, q=149.46kN/m^2



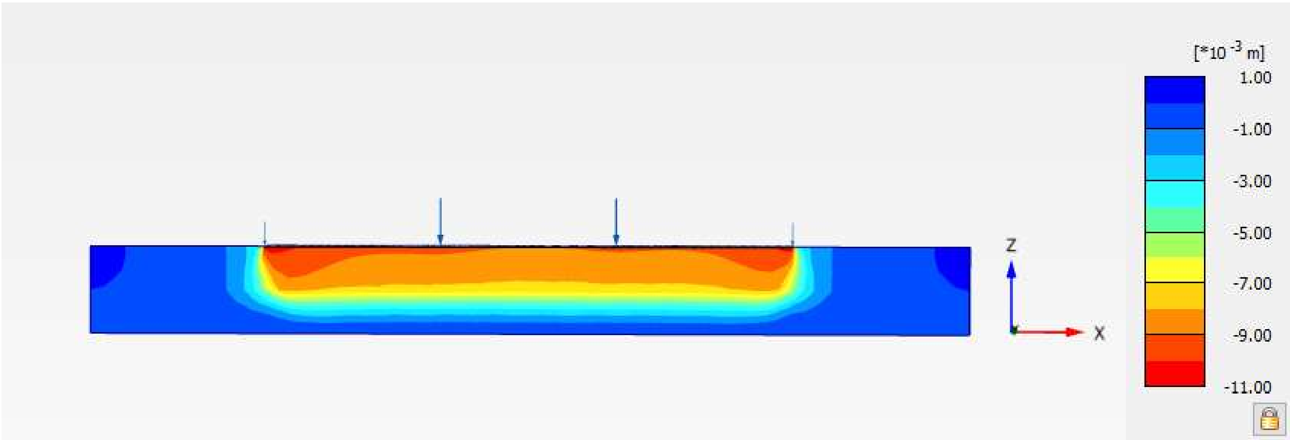
Settlement of oil storage tank foundation for D’=12.6m, q=149.46kN/m^2



Settlement of oil storage tank foundation for D’=9.0m, q=34kN/m^2

7.3 PLAXIS 3D settlement, shear and bending moment outputs for strip footings

B=1.0m, t=0.4m, L=12.0m, H_1=1.0m, H_2=1.0m, F=80kN, 2F=160kN strip footing



Total displacements u_z

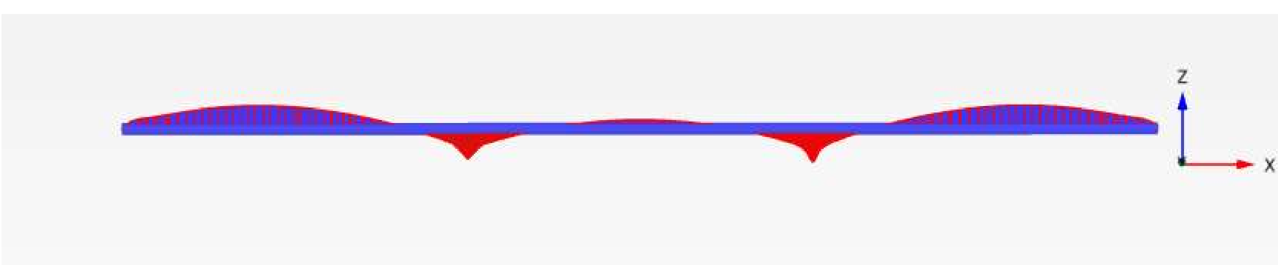
Maximum value = 0.01274×10^{-3} m

Minimum value = -0.01072 m

Shear forces Q_{13} (scaled up 0.500×10^{-3} times)

Maximum value = 369.4 kN/m

Minimum value = -567.6 kN/m

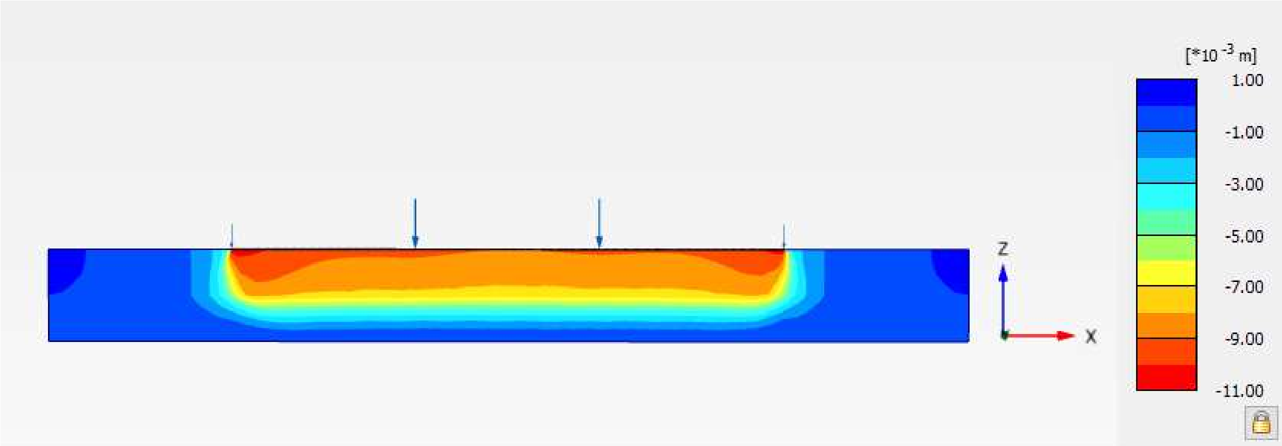


Bending moments M_{11} (scaled up 5.00×10^{-3} times)

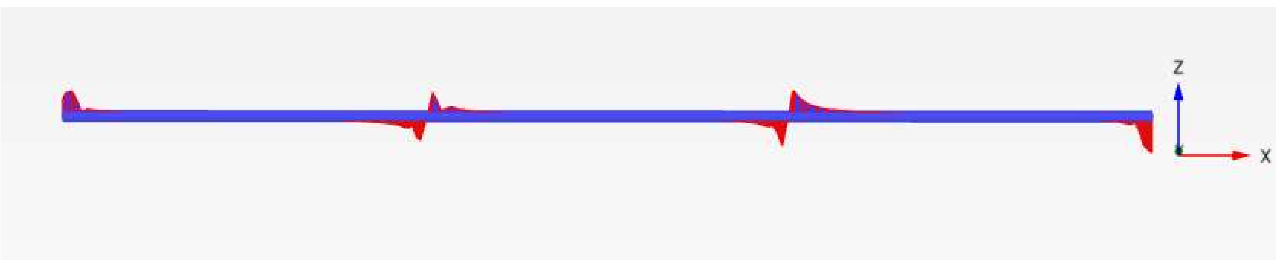
Maximum value = 54.02 kN m/m

Minimum value = -79.15 kN m/m

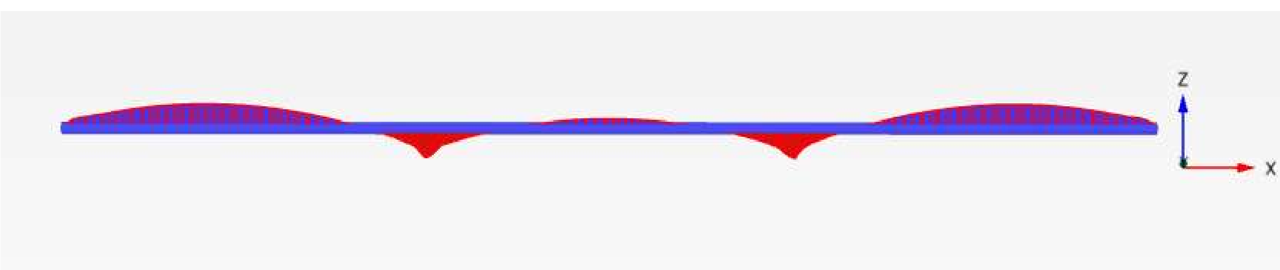
B=1.0m, t=0.4m, L=12.0m, H_1=1.0m, H_2=2.0m, F=80kN, 2F=160kN strip footing



Total displacements u_z
Maximum value = 0.01274*10 -3 m
Minimum value = -0.01072 m

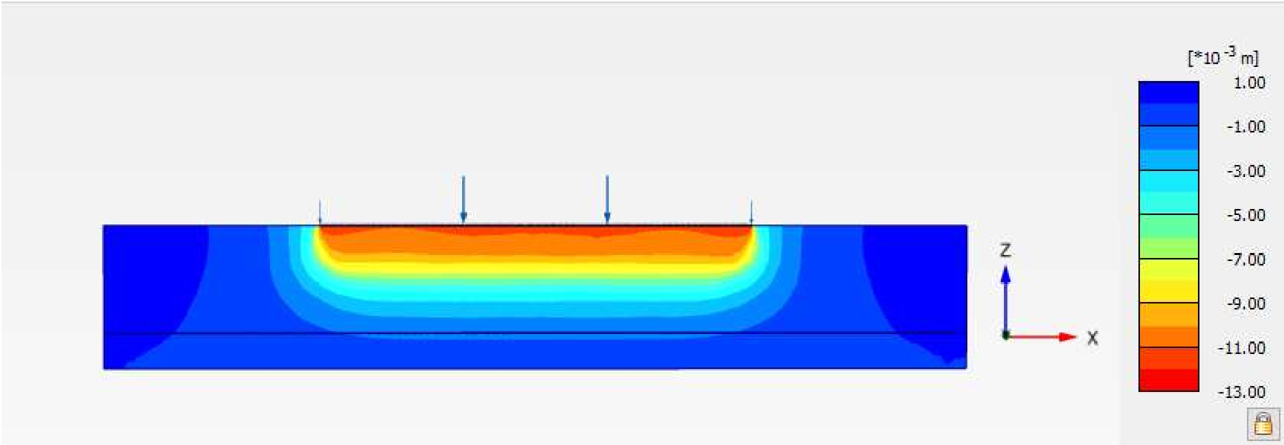


Shear forces Q_{13} (scaled up 1.00*10 -3 times)
Maximum value = 281.5 kN/m
Minimum value = -403.3 kN/m



Bending moments M_{11} (scaled up 5.00*10 -3 times)
Maximum value = 54.00 kN m/m
Minimum value = -67.48 kN m/m

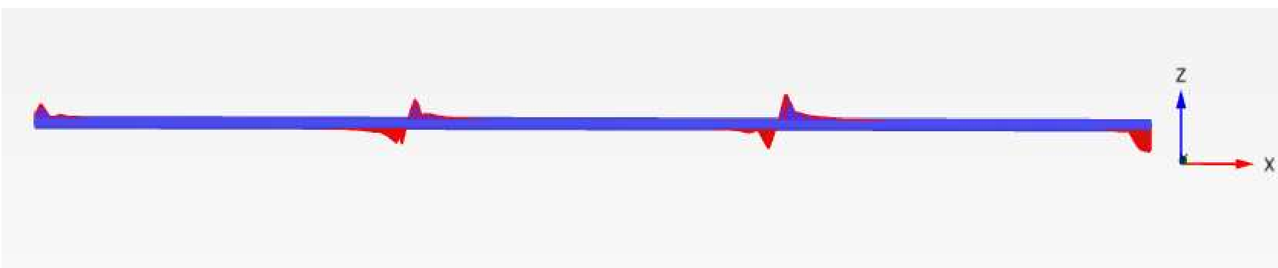
B=1.0m, t=0.4m, L=12.0m, H_1=1.0m, H_2=3.0m, F=80kN, 2F=160kN strip footing



Total displacements u_z

Maximum value = $0.1505*10^{-3}$ m

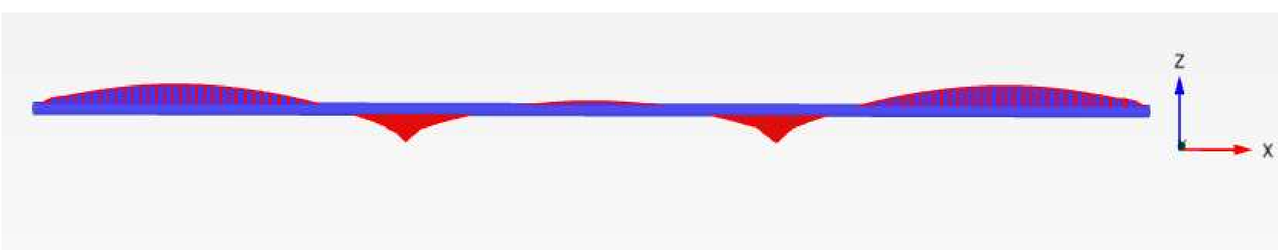
Minimum value = -0.01208 m



Shear forces Q_{13} (scaled up $1.00*10^{-3}$ times)

Maximum value = 317.9 kN/m

Minimum value = -289.5 kN/m

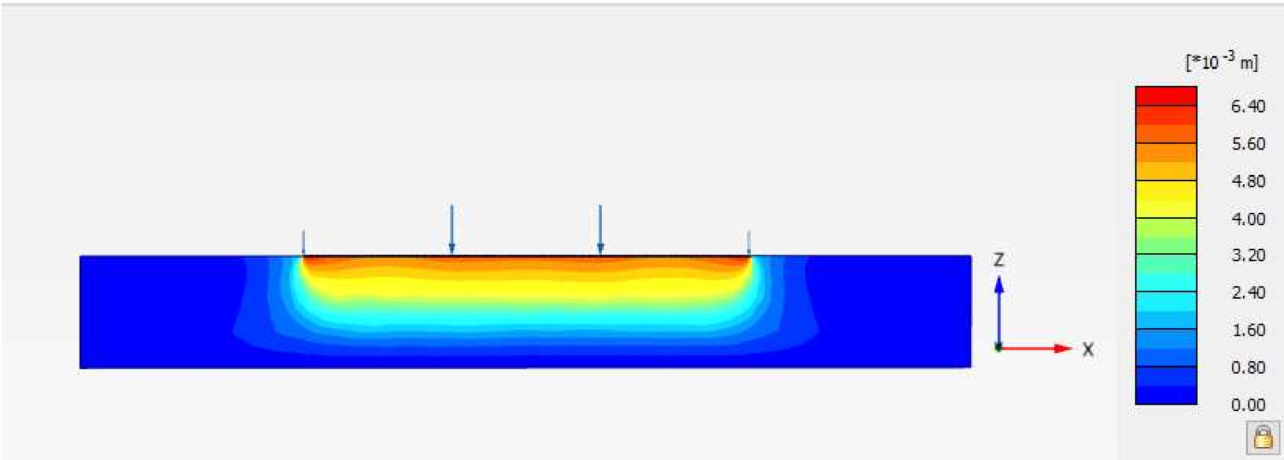


Bending moments M_{11} (scaled up $5.00*10^{-3}$ times)

Maximum value = 53.73 kN m/m

Minimum value = -69.74 kN m/m

B=1.0m, t=0.4m, L=12.0m, H_1=2.0m, H_2=1.0m, F=80kN, 2F=160kN strip footing



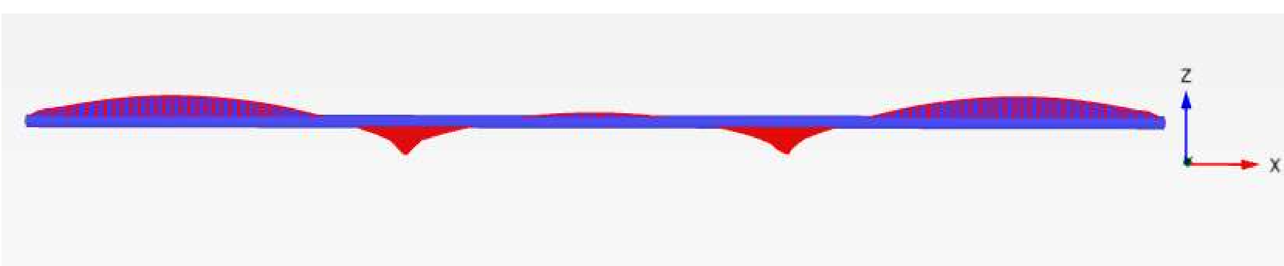
Total displacements |u|

Maximum value = $6.565*10^{-3}$ m

Shear forces Q_{13} (scaled up $1.00*10^{-3}$ times)

Maximum value = 347.2 kN/m

Minimum value = -319.5 kN/m

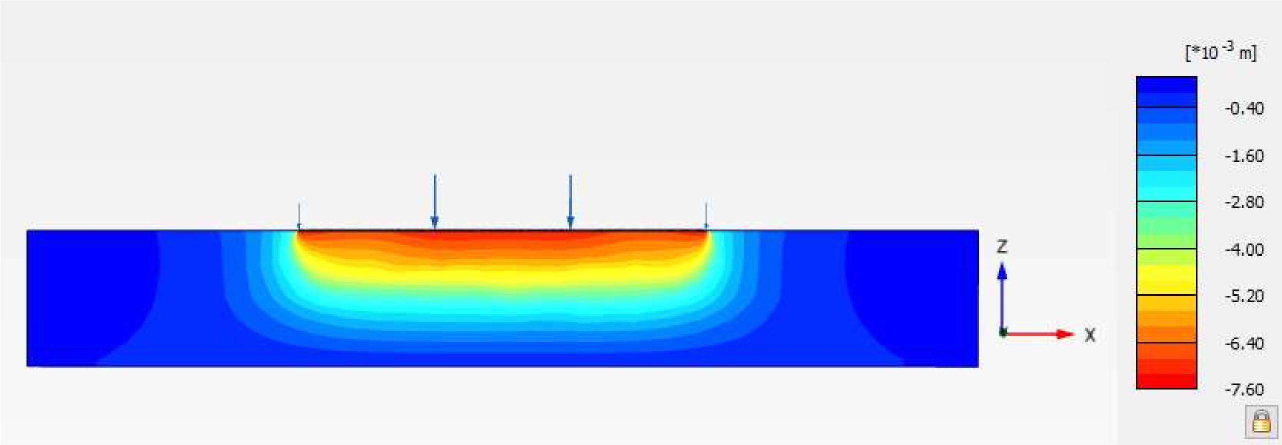


Bending moments M_{11} (scaled up $5.00*10^{-3}$ times)

Maximum value = 53.42 kN m/m

Minimum value = -69.48 kN m/m

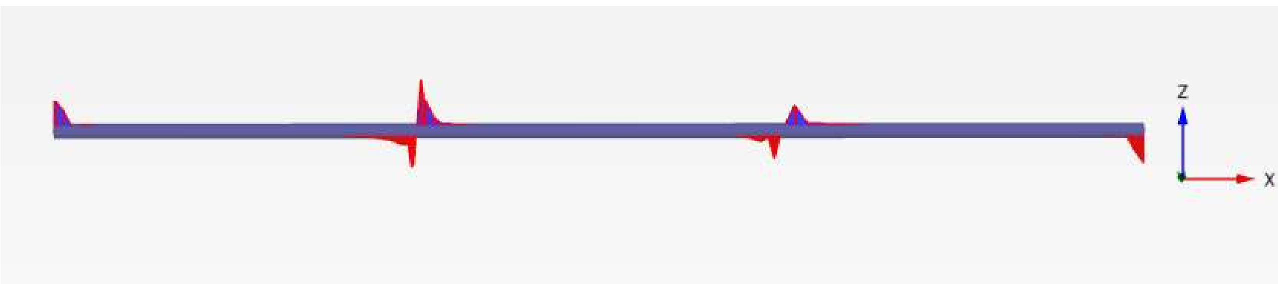
B=1.0m, t=0.4m, L=12.0m, H_1=2.0m, H_2=2.0m, F=80kN, 2F=160kN strip footing



Total displacements u_z

Maximum value = $0.05974*10^{-3}$ m

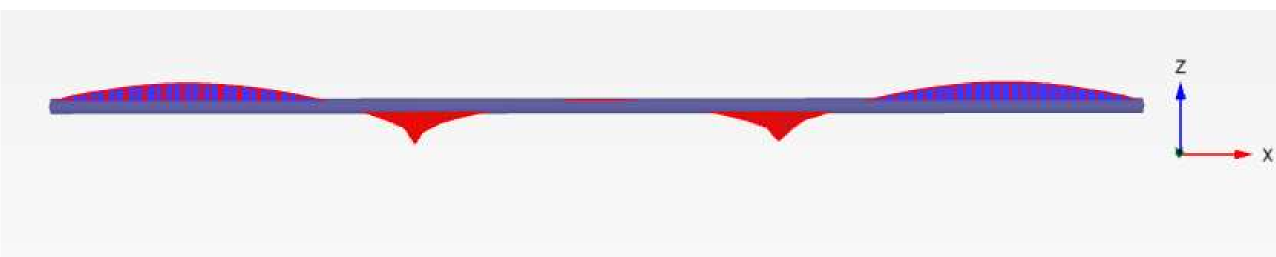
Minimum value = $-7.390*10^{-3}$ m



Shear forces Q_{13} (scaled up $1.00*10^{-3}$ times)

Maximum value = 563.6 kN/m

Minimum value = -399.9 kN/m

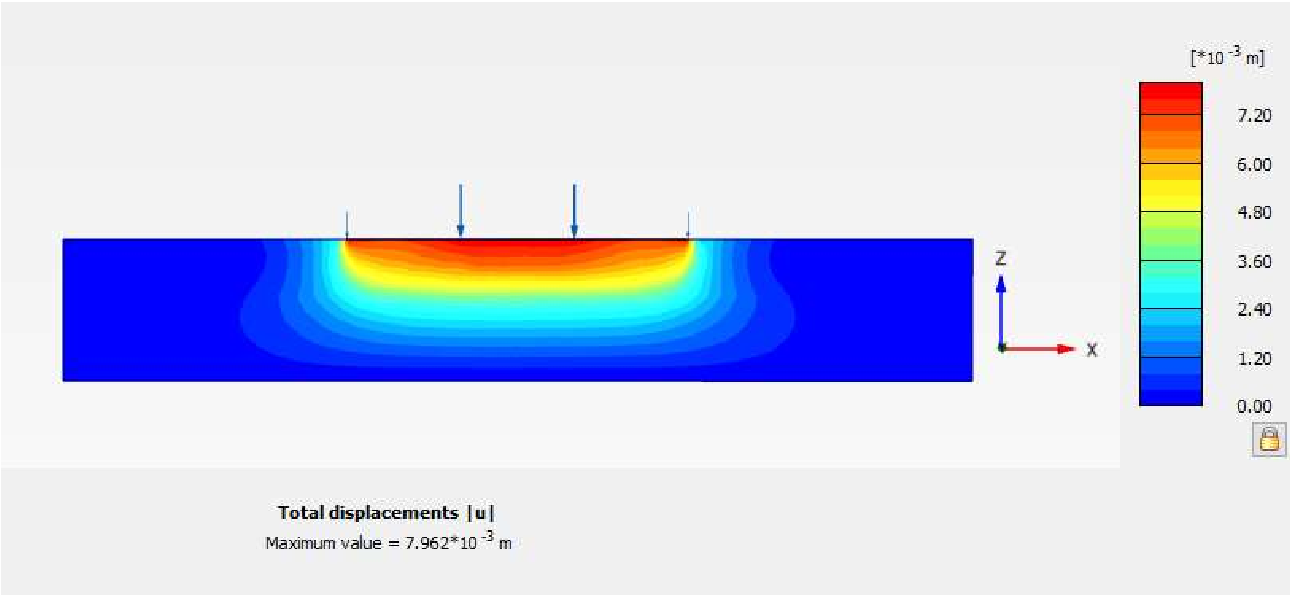


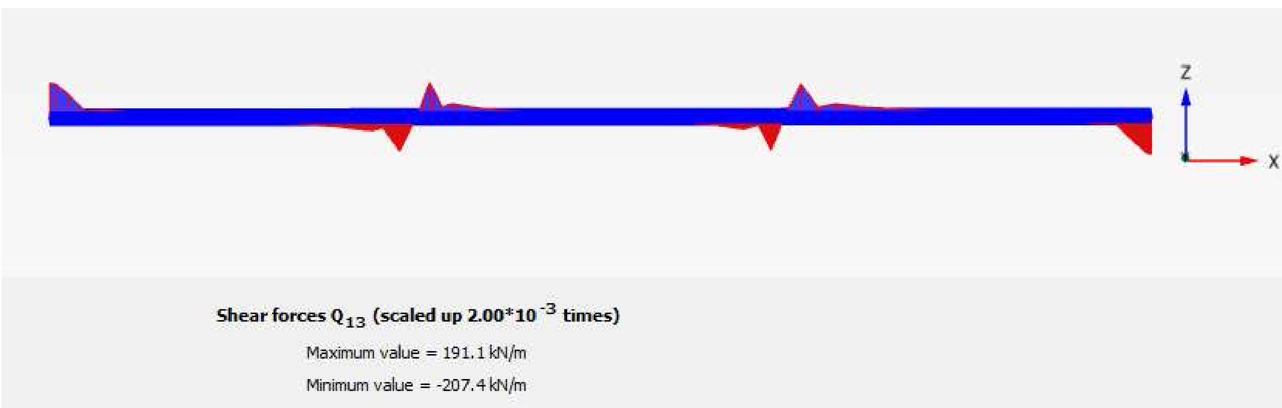
Bending moments M_{11} (scaled up $5.00*10^{-3}$ times)

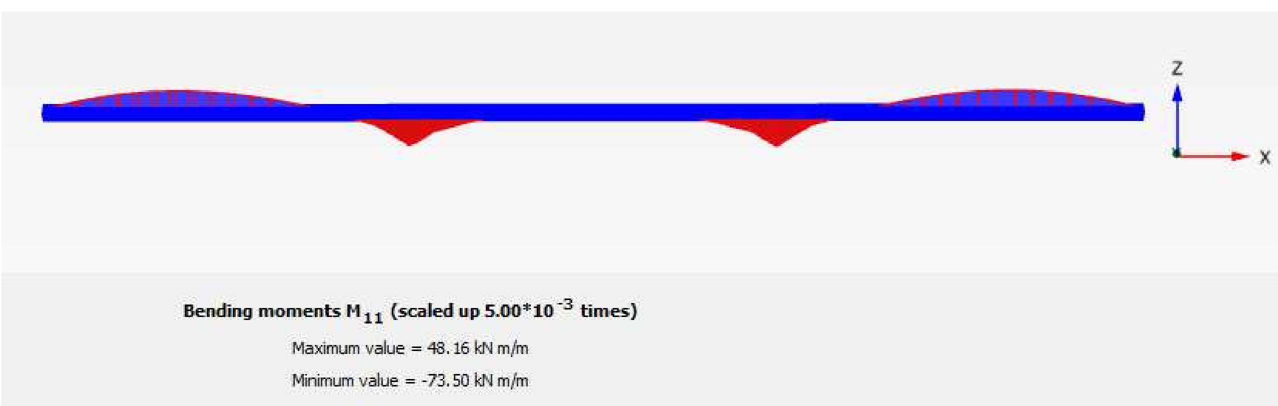
Maximum value = 51.56 kN m/m

Minimum value = -83.14 kN m/m

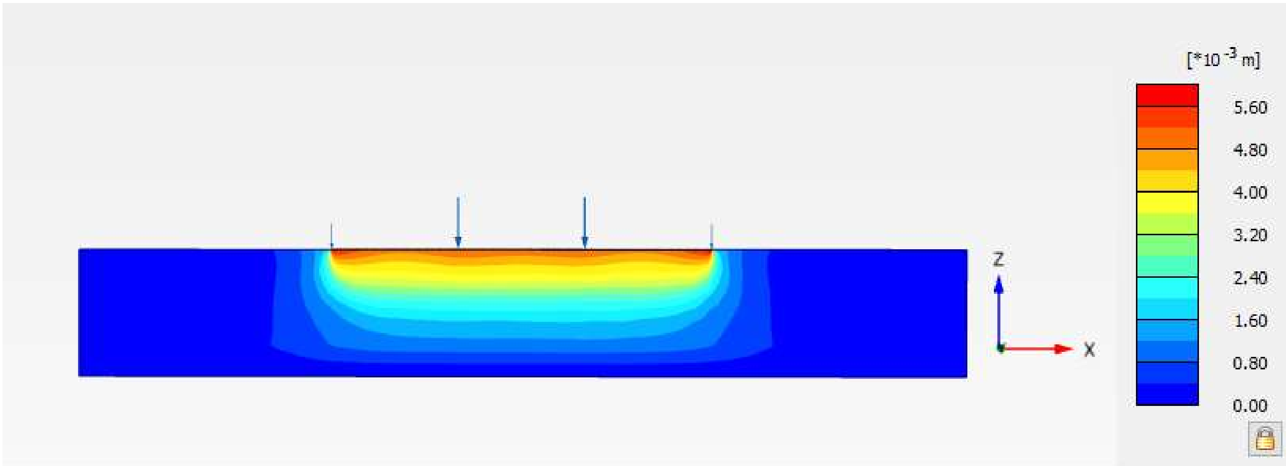
B=1.0m, t=0.4m, L=12.0m, H_1=2.0m, H_2=3.0m, F=80kN, 2F=160kN strip footing





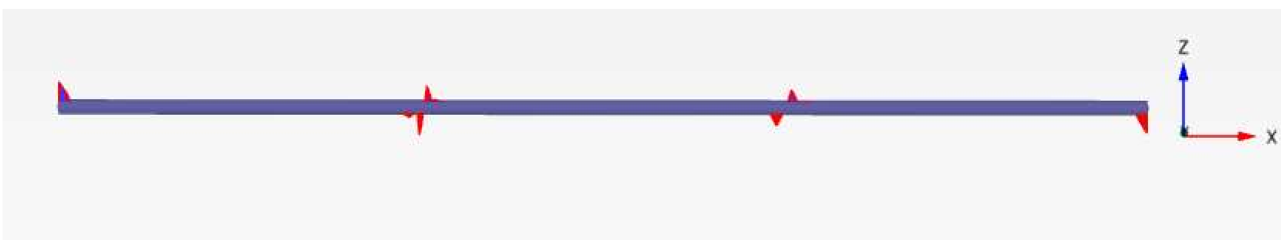


B=1.0m, t=0.4m, L=12.0m, H_1=3.0m, H_2=1.0m, F=80kN, 2F=160kN strip footing



Total displacements |u|

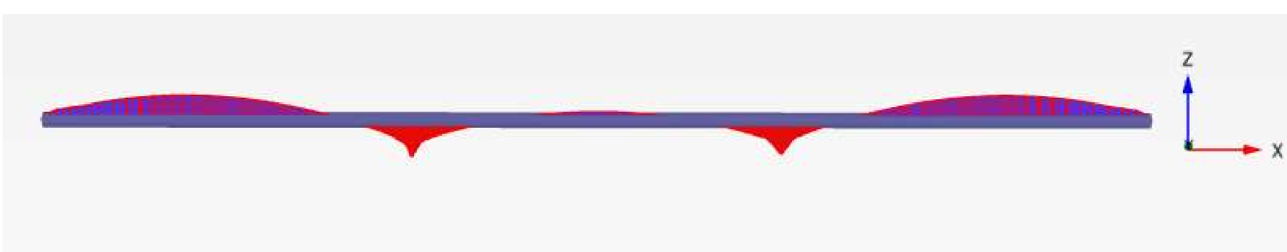
Maximum value = $5.898*10^{-3}$ m



Shear forces Q_{13} (scaled up $0.500*10^{-3}$ times)

Maximum value = 546.0 kN/m

Minimum value = -607.6 kN/m

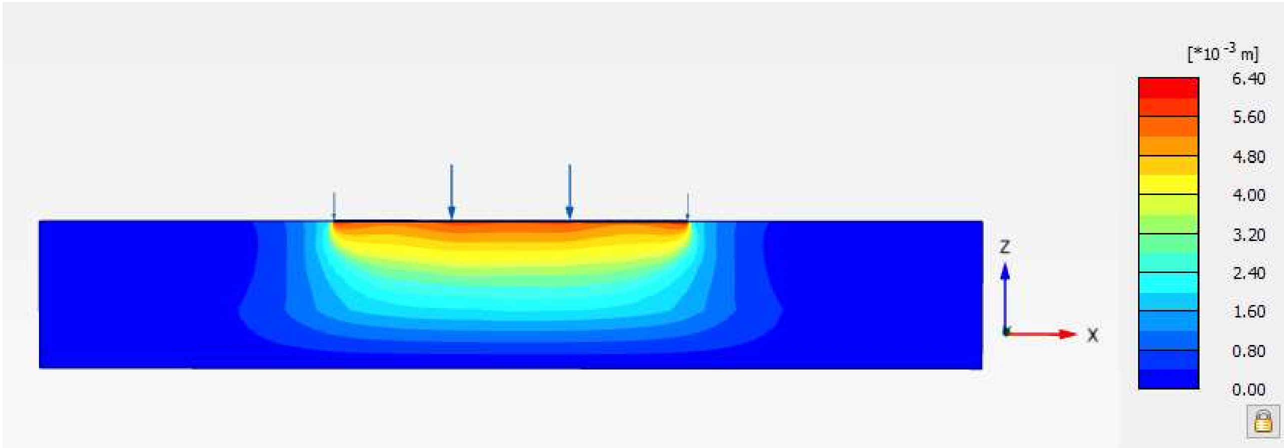


Bending moments M_{11} (scaled up $5.00*10^{-3}$ times)

Maximum value = 54.70 kN m/m

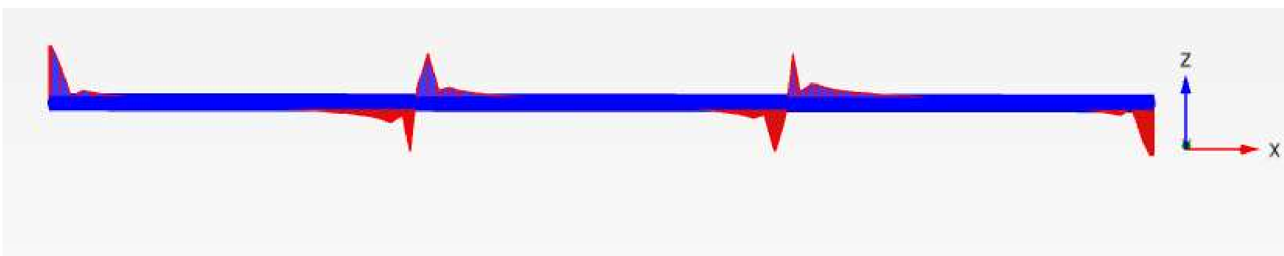
Minimum value = -79.37 kN m/m

B=1.0m, t=0.4m, L=12.0m, H_1=3.0m, H_2=2.0m, F=80kN, 2F=160kN strip footing



Total displacements |u|

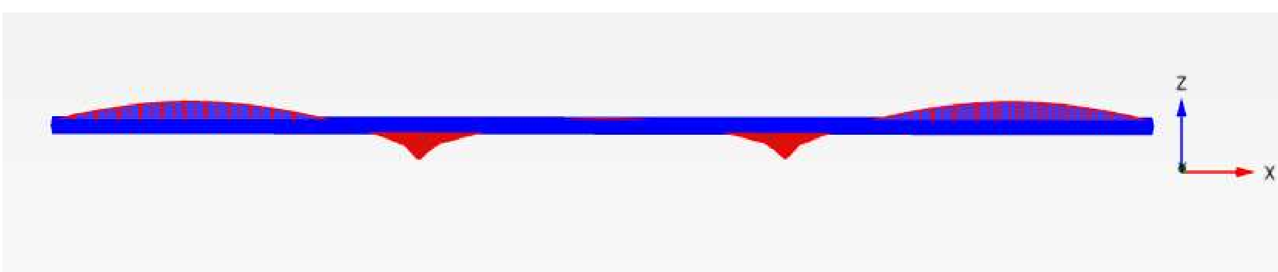
Maximum value = 6.107*10 $^{-3}$ m



Shear forces Q_{13} (scaled up 2.00*10 $^{-3}$ times)

Maximum value = 309.0 kN/m

Minimum value = -282.1 kN/m

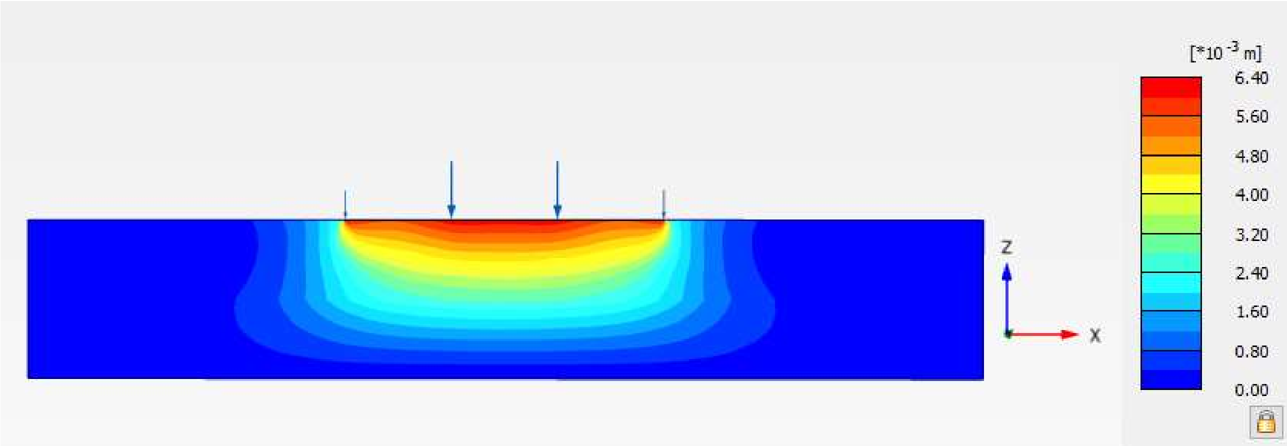


Bending moments M_{11} (scaled up 5.00*10 $^{-3}$ times)

Maximum value = 52.89 kN m/m

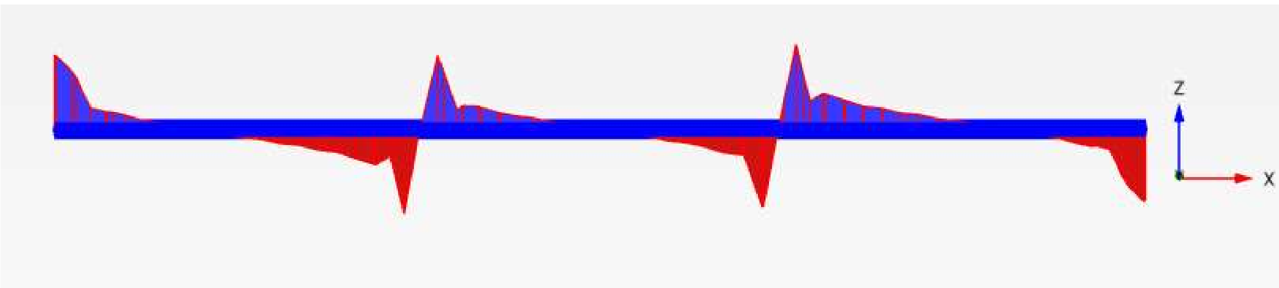
Minimum value = -73.93 kN m/m

B=1.0m, t=0.4m, L=12.0m, H_1=3.0m, H_2=3.0m, F=80kN, 2F=160kN strip footing



Total displacements |u|

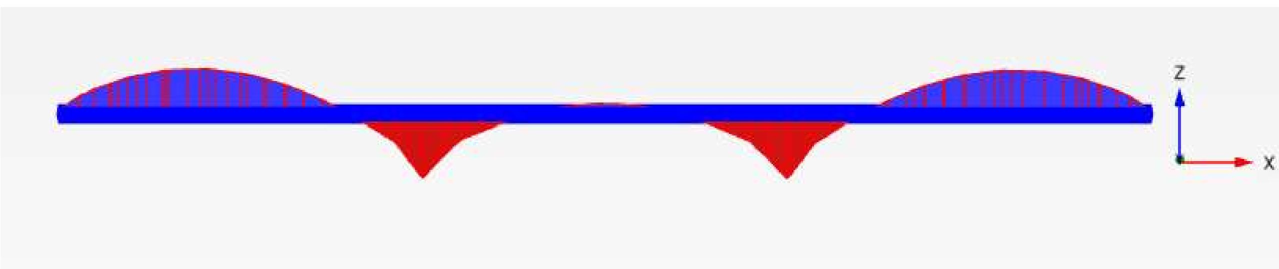
Maximum value = 6.258*10^{-3} m



Shear forces Q_{13} (scaled up 5.00*10^{-3} times)

Maximum value = 186.3 kN/m

Minimum value = -187.0 kN/m

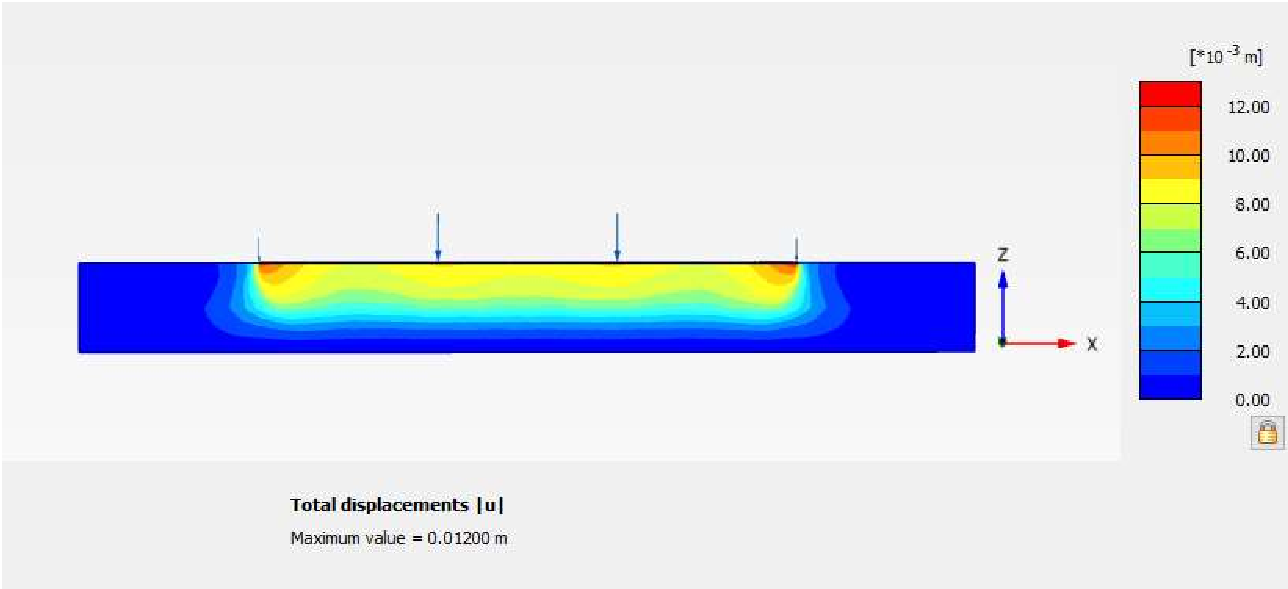


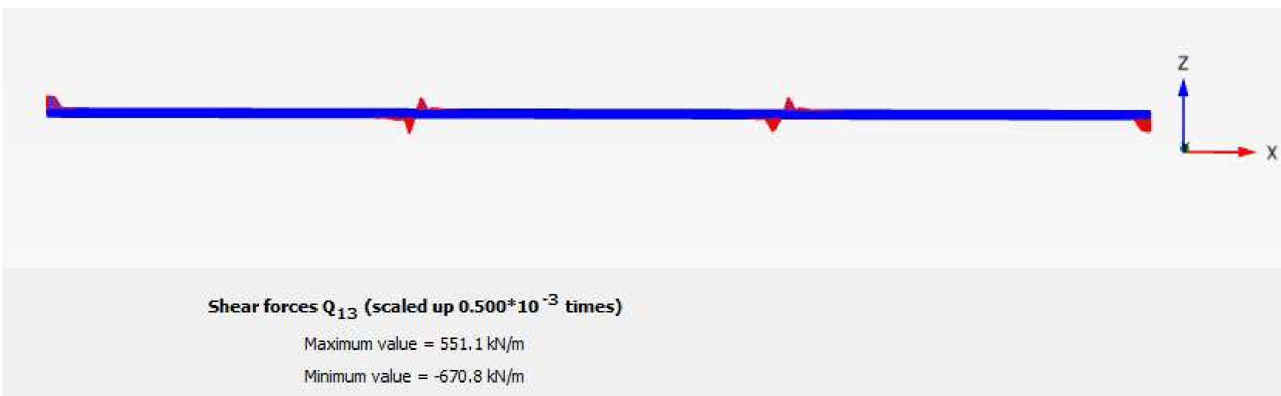
Bending moments M_{11} (scaled up 0.0100 times)

Maximum value = 49.33 kN m/m

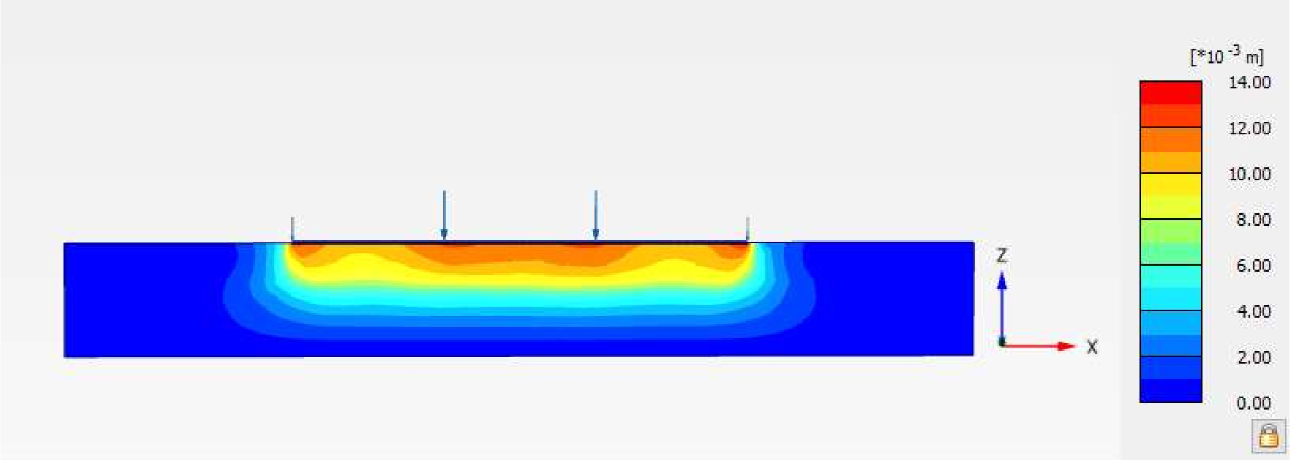
Minimum value = -71.47 kN m/m

B=1.5m, t=0.4m, L=18.0m, H_1=1.5m, H_2=1.5m, F=180kN, 2F=360kN strip footing



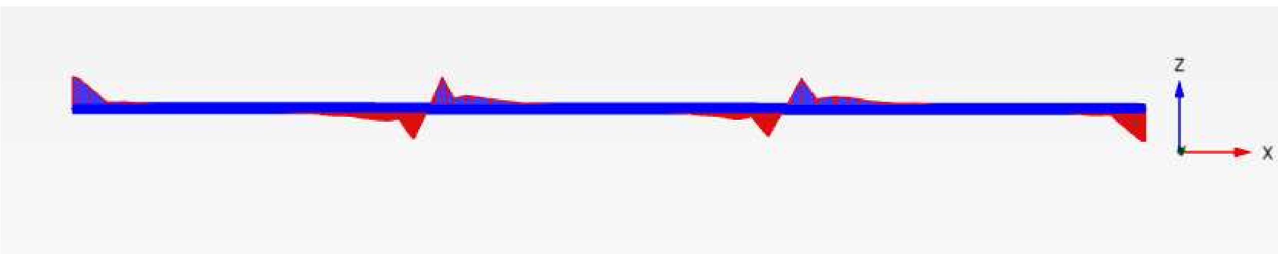


B=1.5m, t=0.4m, L=18.0m, H_1=1.5m, H_2=3.0m, F=180kN, 2F=360kN strip footing



Total displacements |u|

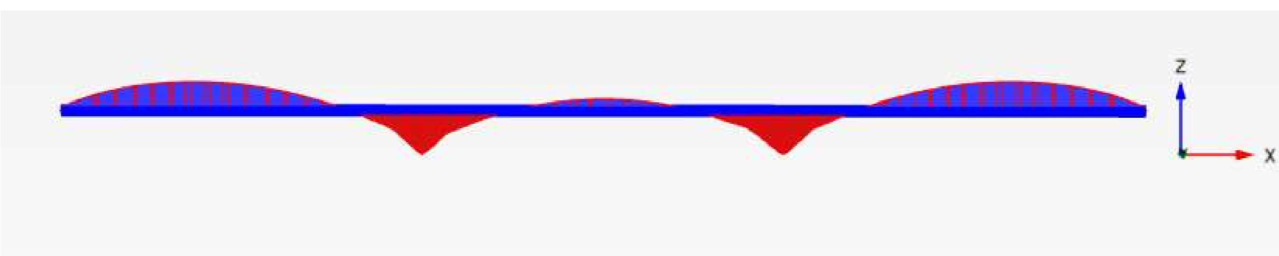
Maximum value = 0.01326 m



Shear forces Q_{13} (scaled up $2.00*10^{-3}$ times)

Maximum value = 261.0 kN/m

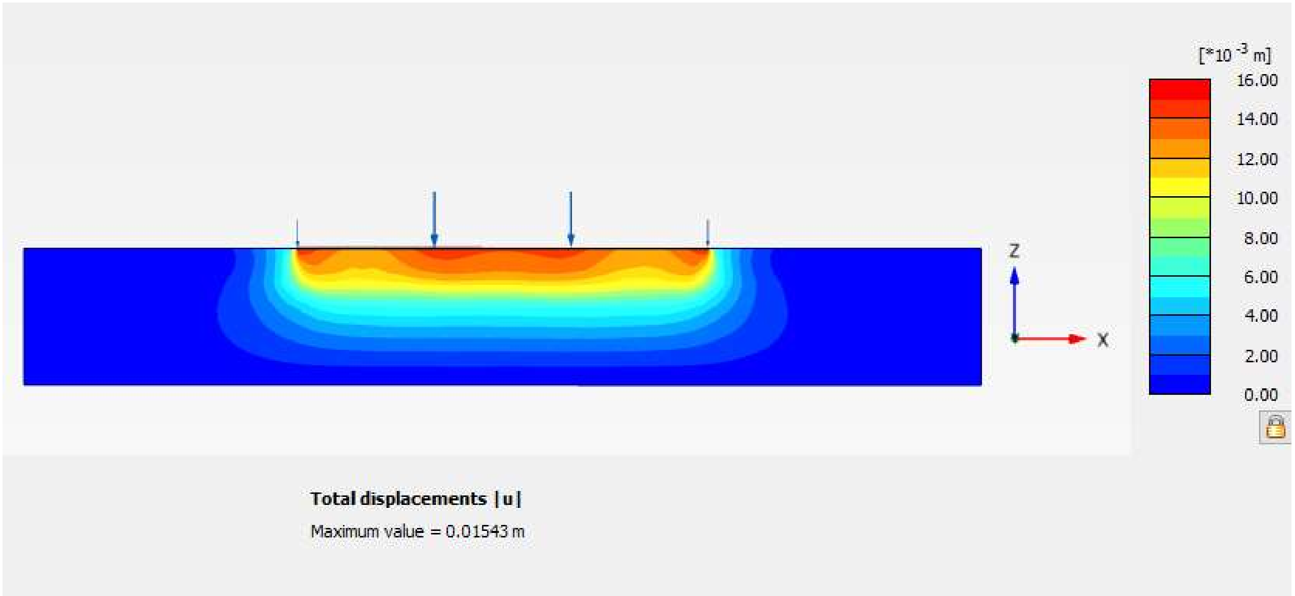
Minimum value = -269.8 kN/m



Bending moments M_{11} (scaled up $5.00*10^{-3}$ times)

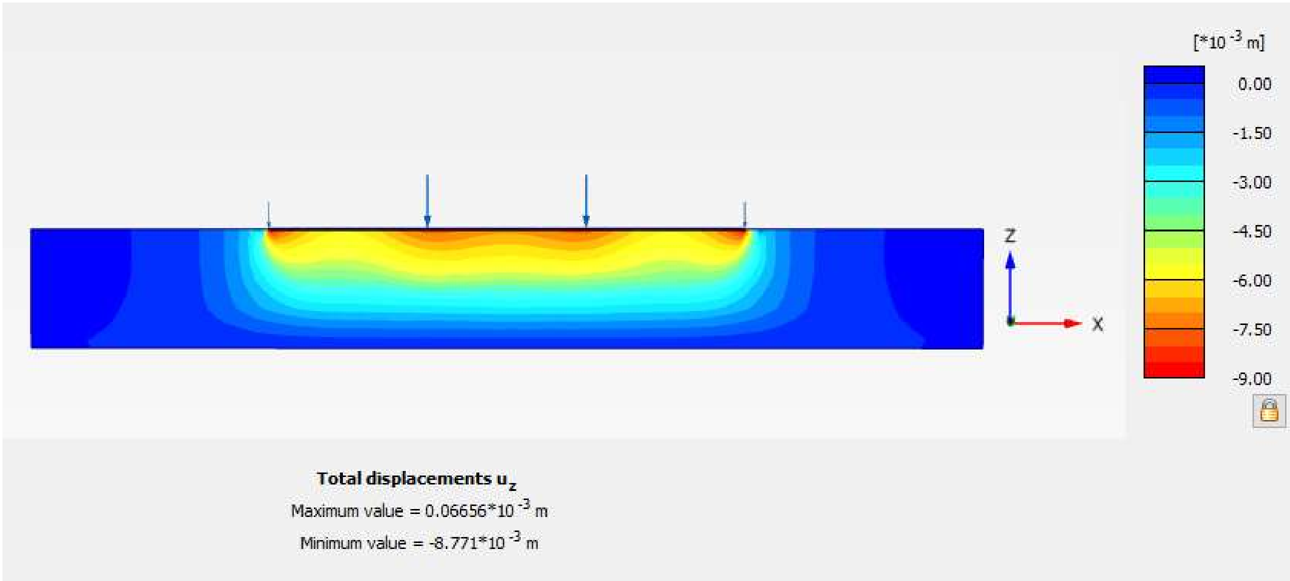
Maximum value = 96.95 kN m/m

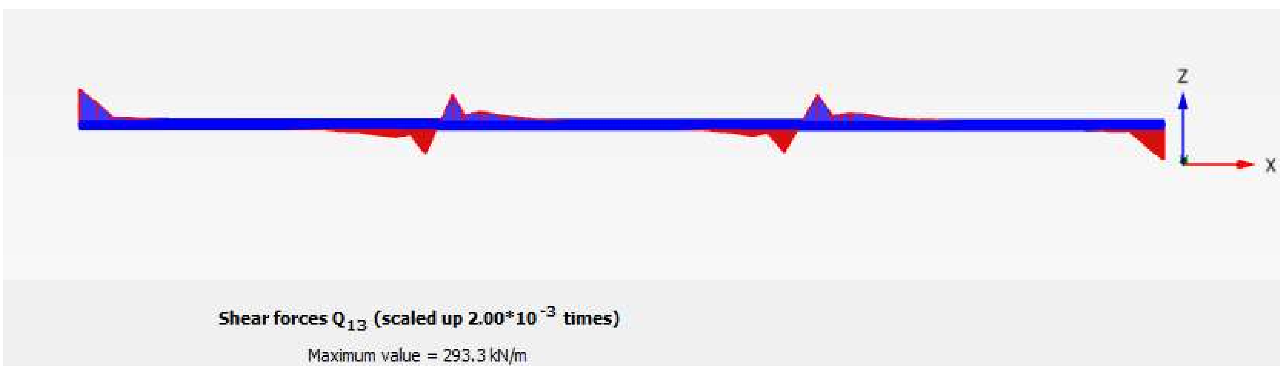
Minimum value = -146.5 kN m/m



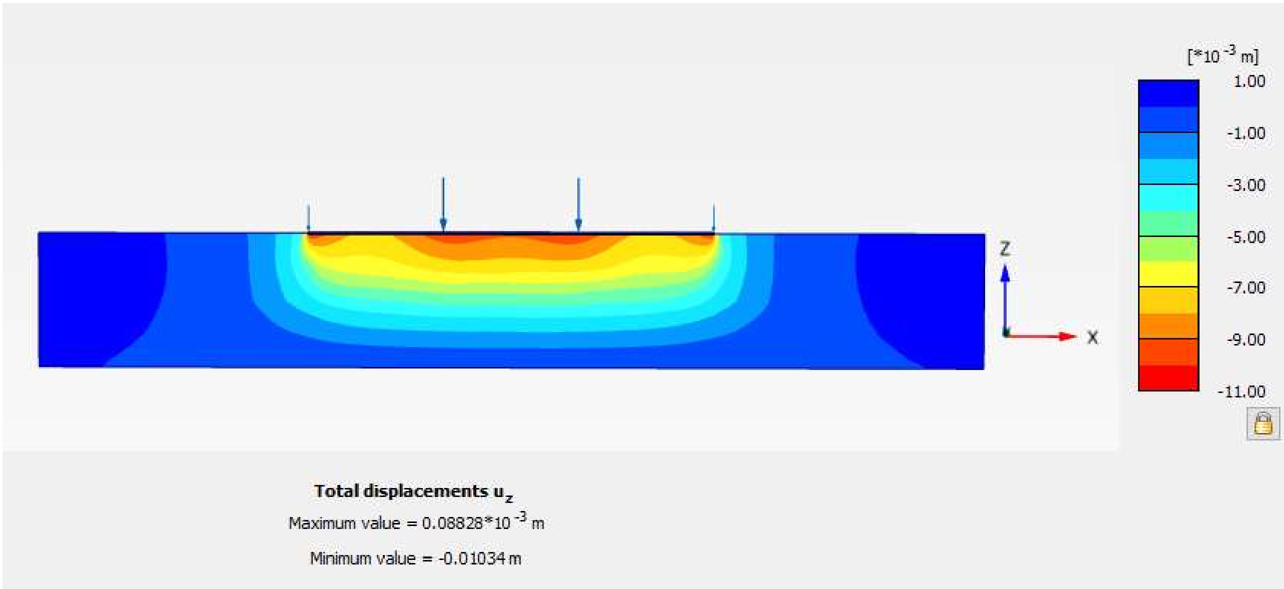
B=1.5m, t=0.4m, L=18.0m, H_1=1.5m, H_2=4.5m, F=180kN, 2F=360kN strip footing

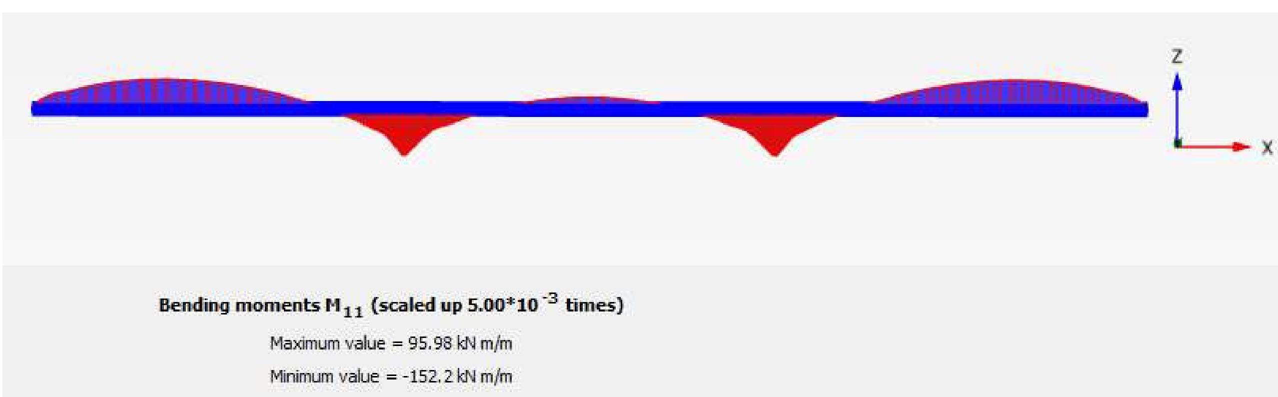
B=1.5m, t=0.4m, L=18.0m, H_1=3.0m, H_2=1.5m, F=180kN, 2F=360kN strip footing



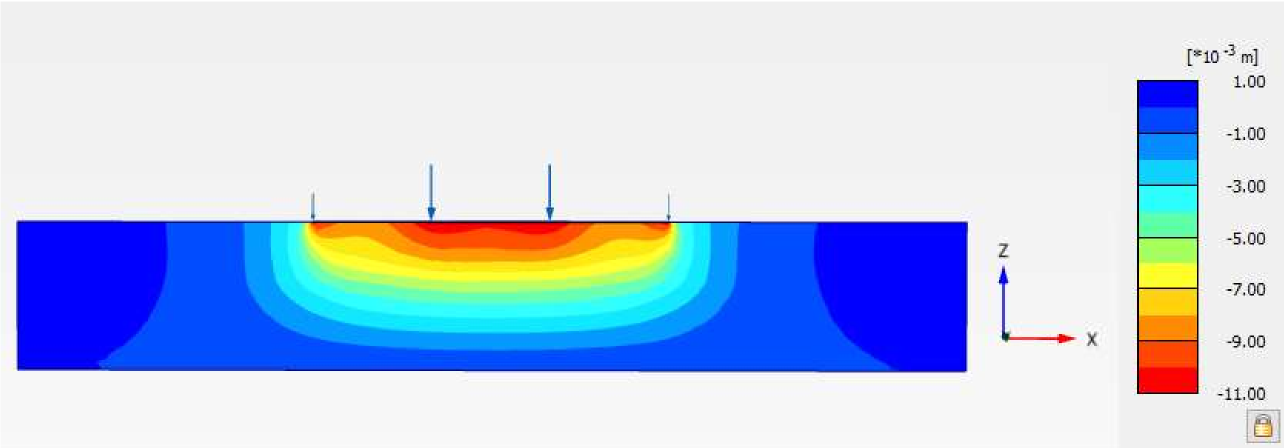


B=1.5m, t=0.4m, L=18.0m, H_1=3.0m, H_2=3.0m, F=180kN, 2F=360kN strip footing





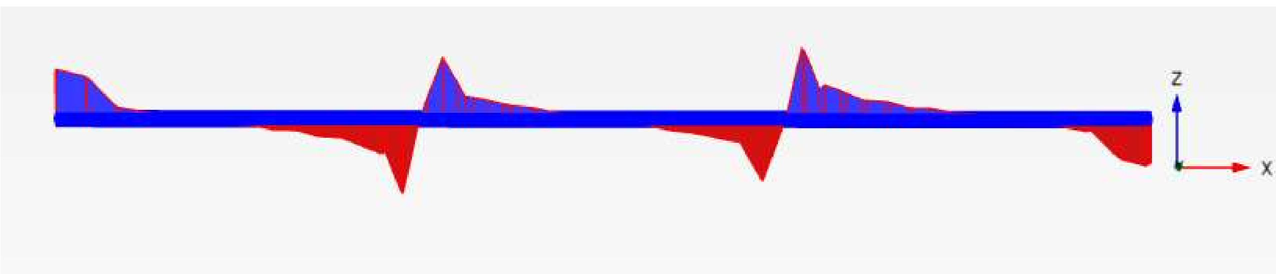
B=1.5m, t=0.4m, L=18.0m, H_1=3.0m, H_2=4.5m, F=180kN, 2F=360kN strip footing



Total displacements u_z

Maximum value = $0.1016*10^{-3}$ m

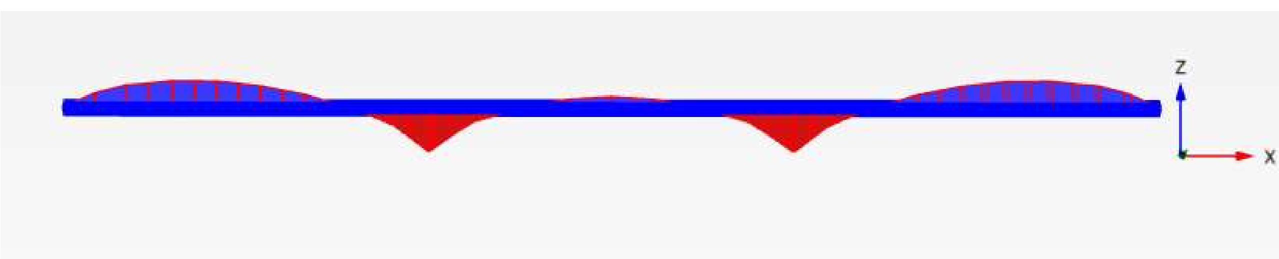
Minimum value = -0.01082 m



Shear forces Q_{13} (scaled up $5.00*10^{-3}$ times)

Maximum value = 235.2 kN/m

Minimum value = -245.2 kN/m

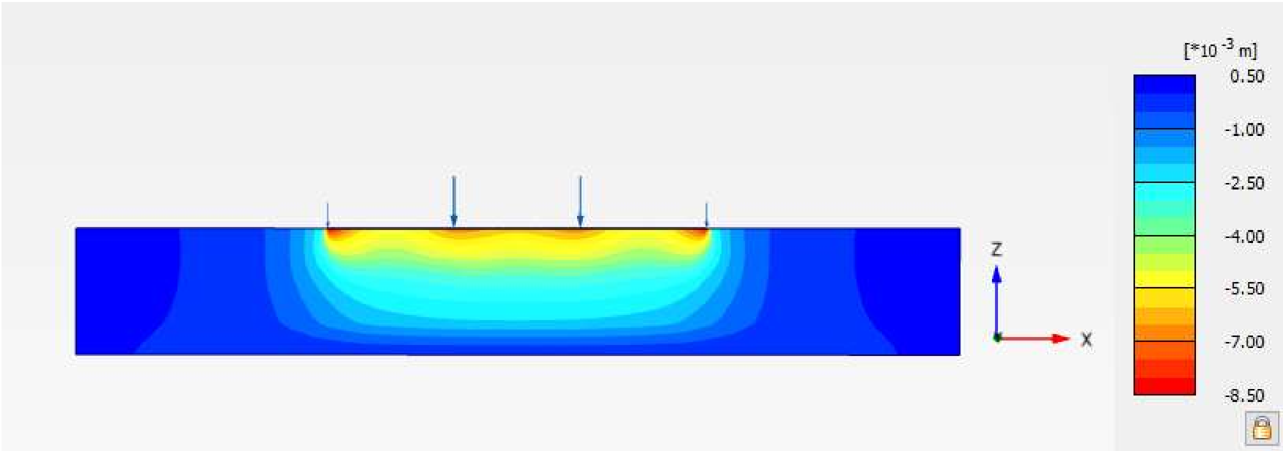


Bending moments M_{11} (scaled up $5.00*10^{-3}$ times)

Maximum value = 89.38 kN m/m

Minimum value = -142.8 kN m/m

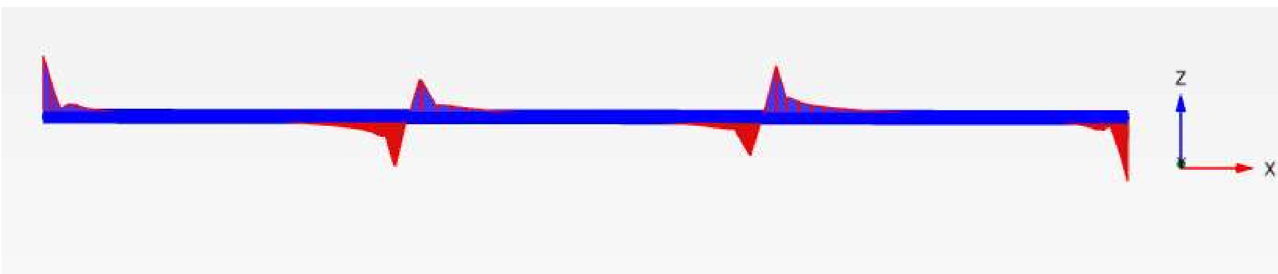
B=1.5m, t=0.4m, L=18.0m, H_1=4.5m, H_2=1.5m, F=180kN, 2F=360kN strip footing



Total displacements u_z

Maximum value = $0.04736 \cdot 10^{-3}$ m

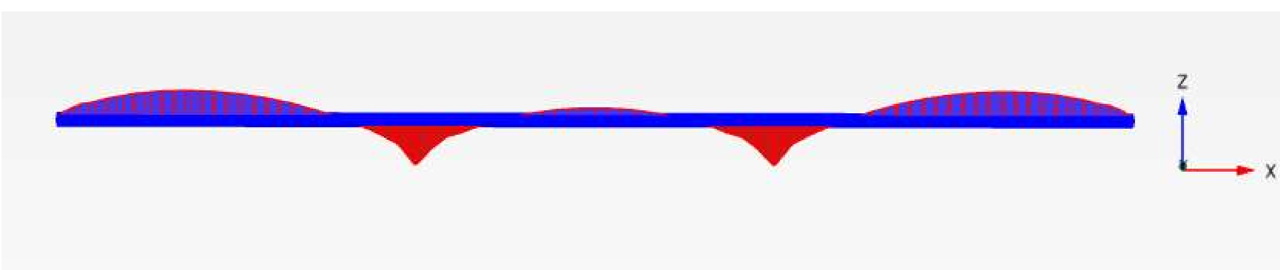
Minimum value = $-8.444 \cdot 10^{-3}$ m



Shear forces Q_{13} (scaled up $2.00 \cdot 10^{-3}$ times)

Maximum value = 505.4 kN/m

Minimum value = -527.2 kN/m

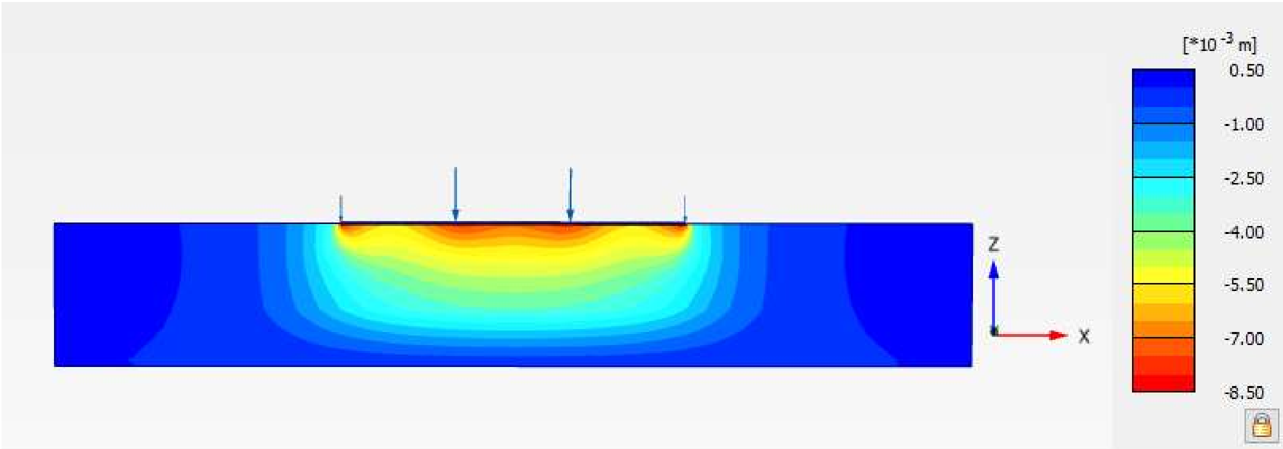


Bending moments M_{11} (scaled up $5.00 \cdot 10^{-3}$ times)

Maximum value = 98.43 kN m/m

Minimum value = -152.9 kN m/m

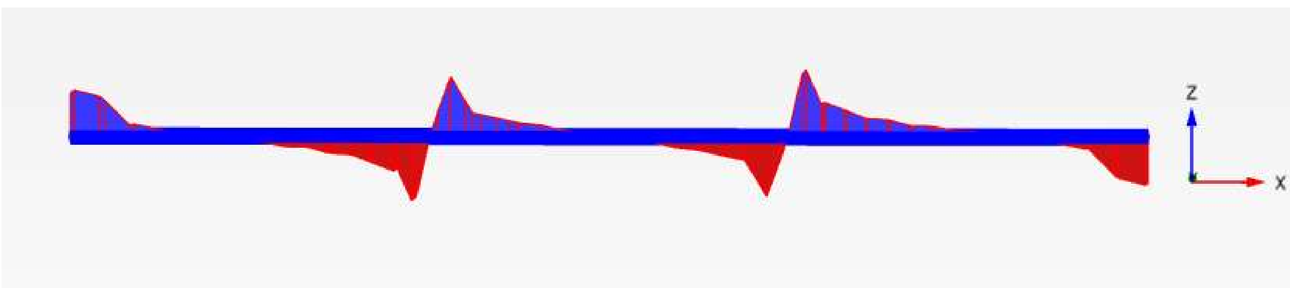
B=1.5m, t=0.4m, L=18.0m, H_1=4.5m, H_2=3.0m, F=180kN, 2F=360kN strip footing



Total displacements u_z

Maximum value = $0.06894*10^{-3}$ m

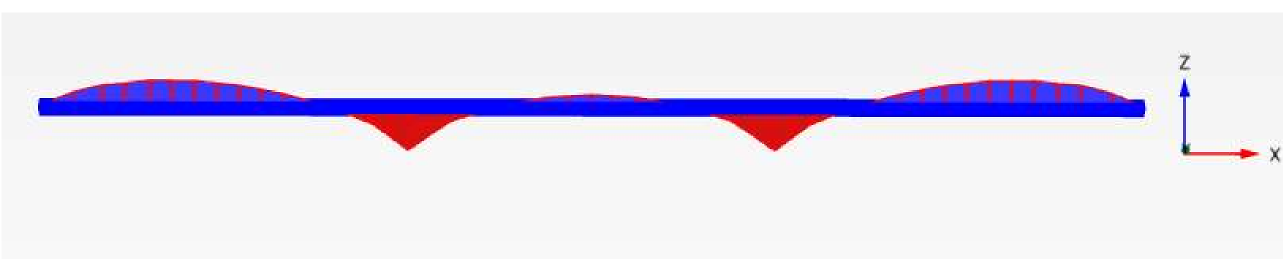
Minimum value = $-8.370*10^{-3}$ m



Shear forces Q_{13} (scaled up $5.00*10^{-3}$ times)

Maximum value = 222.2 kN/m

Minimum value = -213.5 kN/m

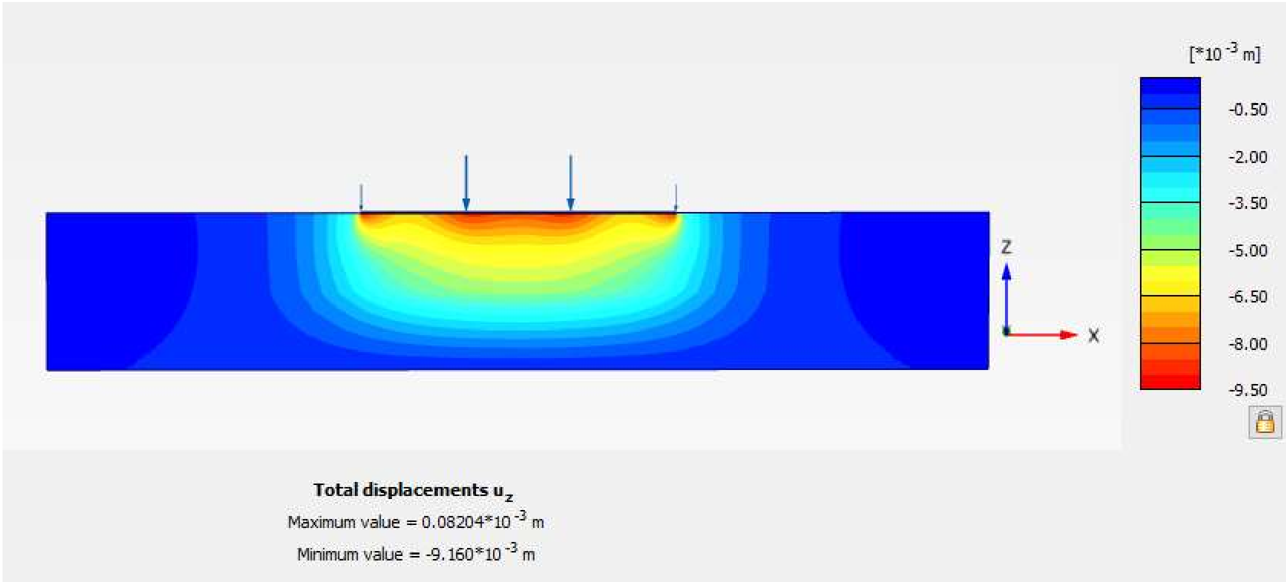


Bending moments M_{11} (scaled up $5.00*10^{-3}$ times)

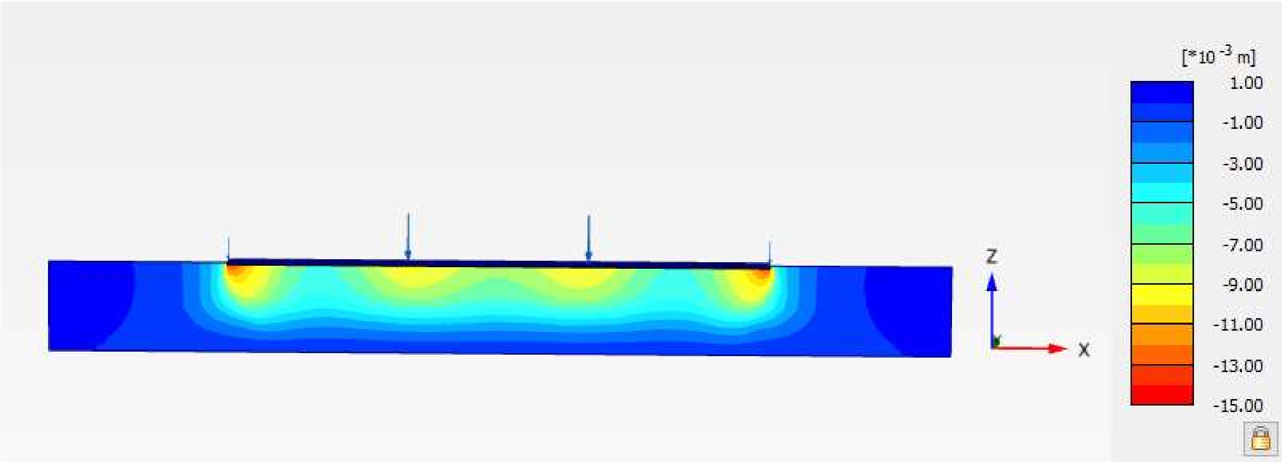
Maximum value = 89.34 kN m/m

Minimum value = -140.0 kN m/m

B=1.5m, t=0.4m, L=18.0m, H_1=4.5m, H_2=4.5m, F=180kN, 2F=360kN strip footing



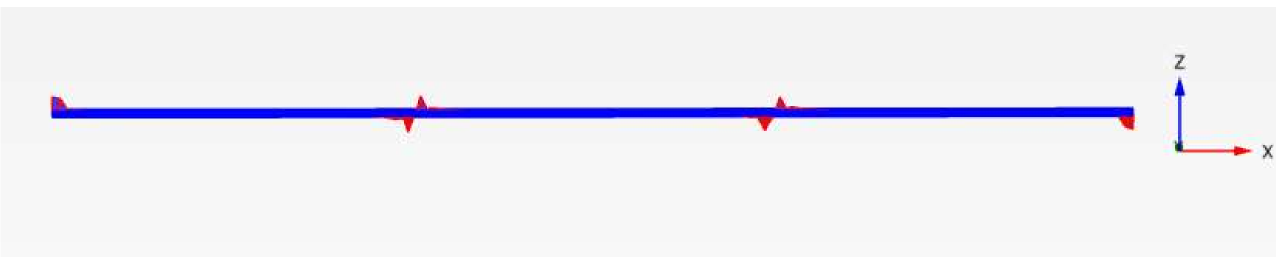
B=2.0m, t=0.4m, L=24.0m, H_1=2.0m, H_2=2.0m, F=320kN, 2F=640kN strip footing



Total displacements u_z

Maximum value = 0.1180*10^{-3} m

Minimum value = -0.01458 m



Shear forces Q_{13} (scaled up 0.500*10^{-3} times)

Maximum value = 756.5 kN/m

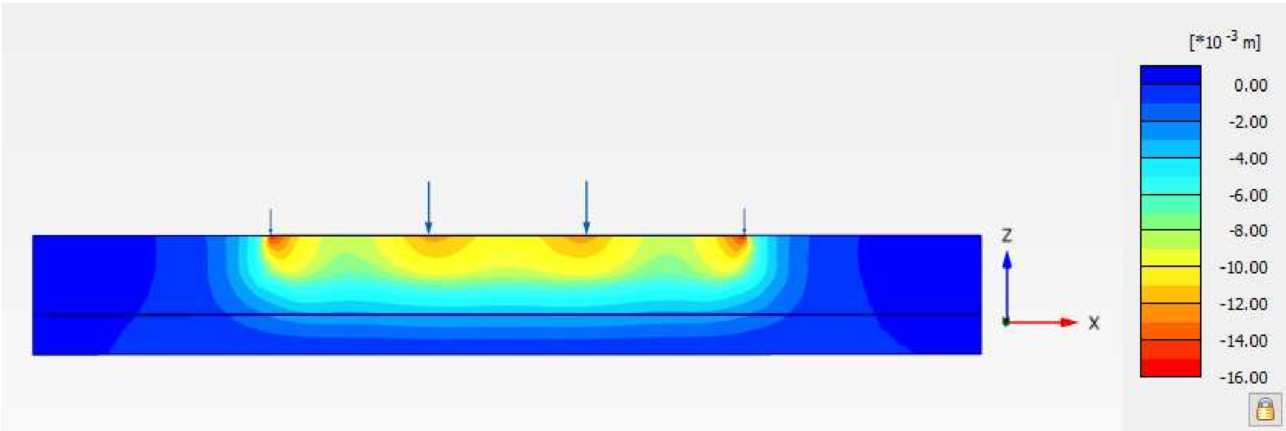
Minimum value = -854.3 kN/m

Bending moments M_{11} (scaled up 2.00*10^{-3} times)

Maximum value = 153.9 kN m/m

Minimum value = -295.2 kN m/m

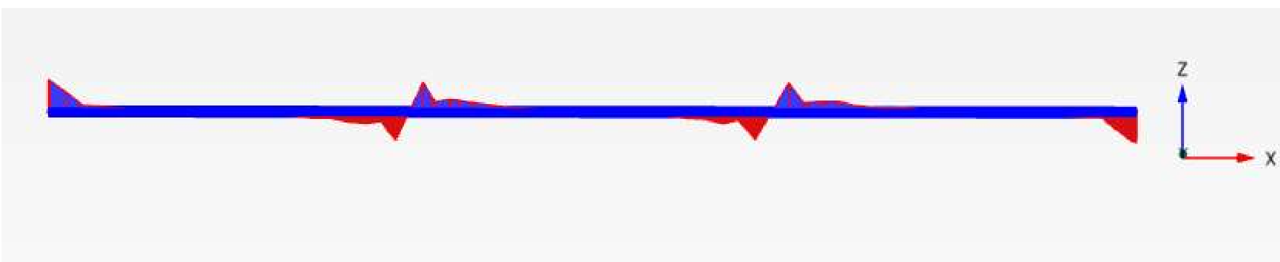
B=2.0m, t=0.4m, L=24.0m, H_1=2.0m, H_2=4.0m, F=320kN, 2F=640kN strip footing



Total displacements u_z

Maximum value = $0.1429*10^{-3}$ m

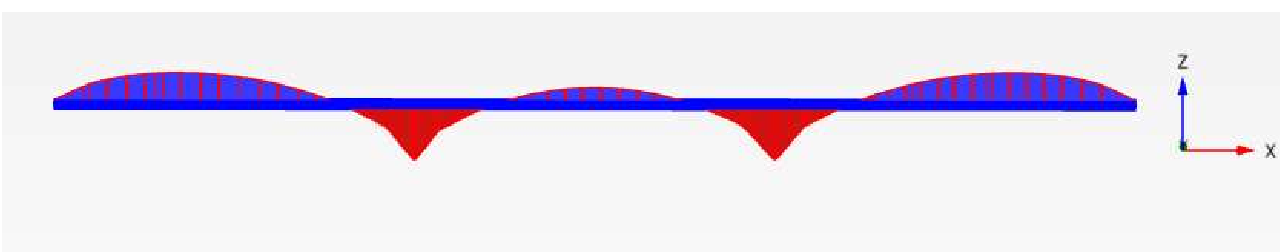
Minimum value = -0.01540 m



Shear forces Q_{13} (scaled up $2.00*10^{-3}$ times)

Maximum value = 349.7 kN/m

Minimum value = -332.1 kN/m

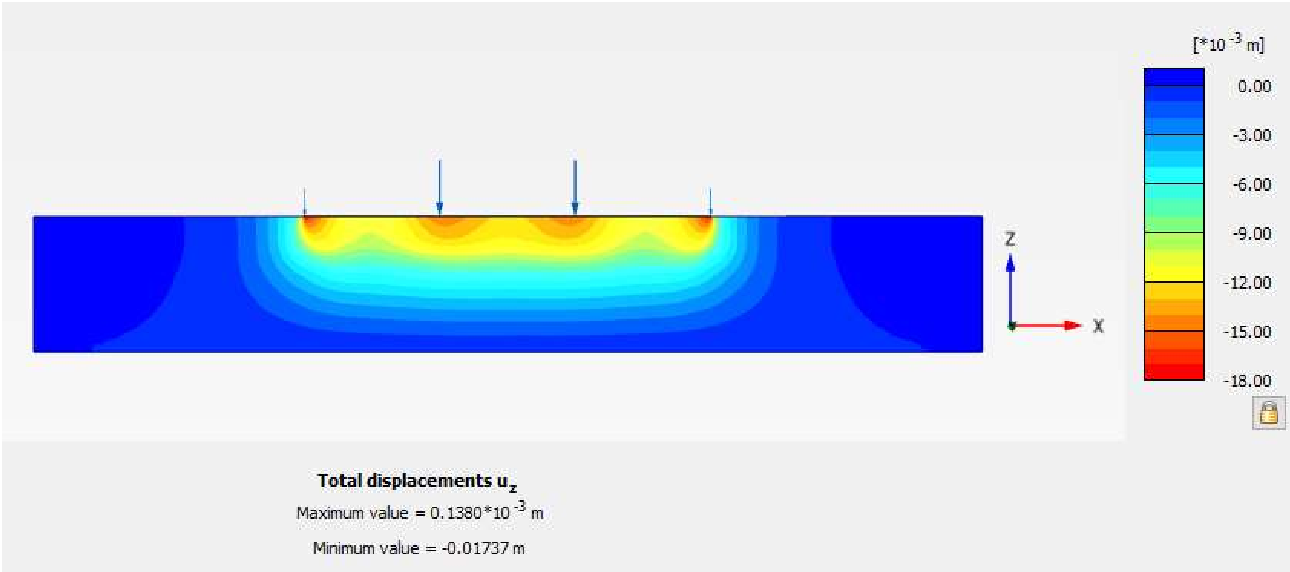


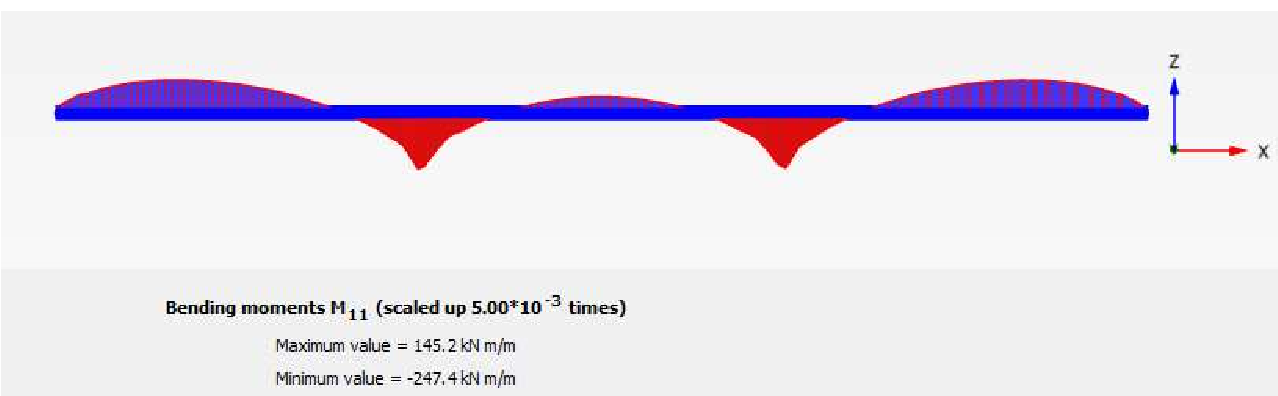
Bending moments M_{11} (scaled up $5.00*10^{-3}$ times)

Maximum value = 141.8 kN m/m

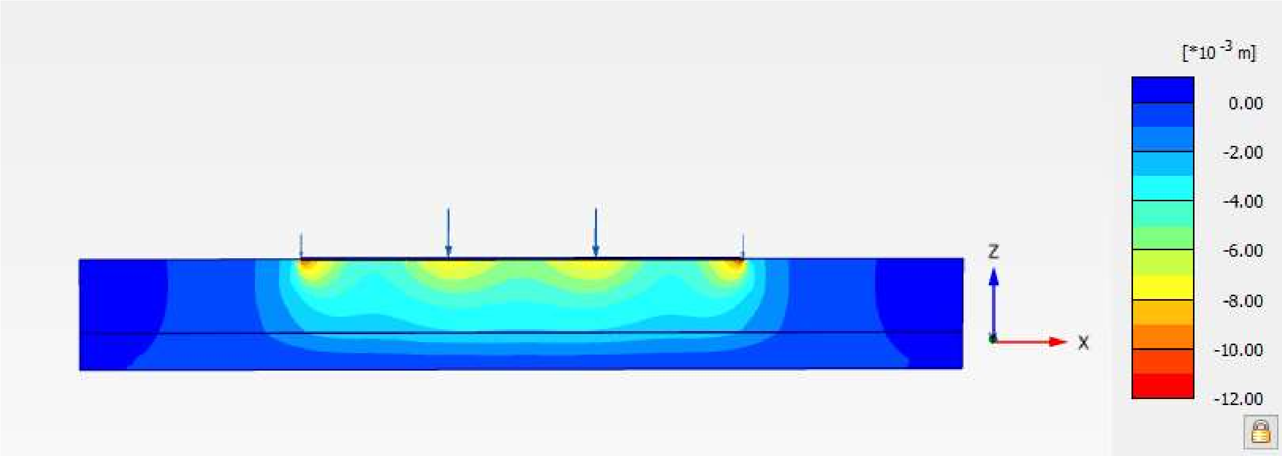
Minimum value = -251.0 kN m/m

B=2.0m, t=0.4m, L=24.0m, H_1=2.0m, H_2=6.0m, F=320kN, 2F=640kN strip footing





B=2.0m, t=0.4m, L=24.0m, H_1=4.0m, H_2=2.0m, F=320kN, 2F=640kN strip footing



Total displacements u_z

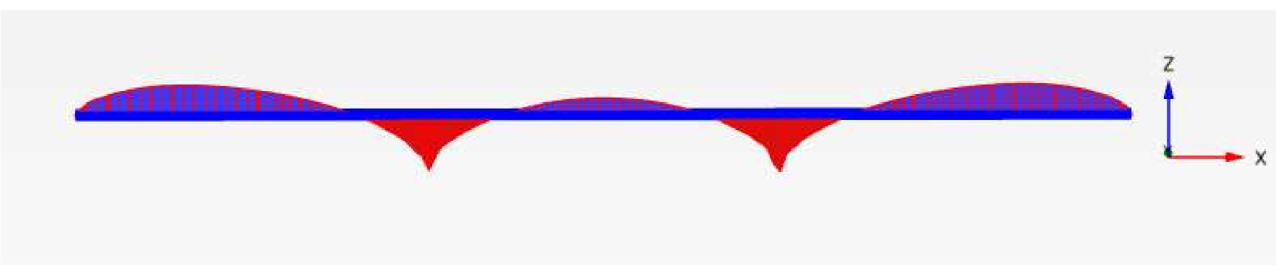
Maximum value = 0.06782*10^{-3} m

Minimum value = -0.01132 m

Shear forces Q_{13} (scaled up 2.00*10^{-3} times)

Maximum value = 482.4 kN/m

Minimum value = -568.8 kN/m

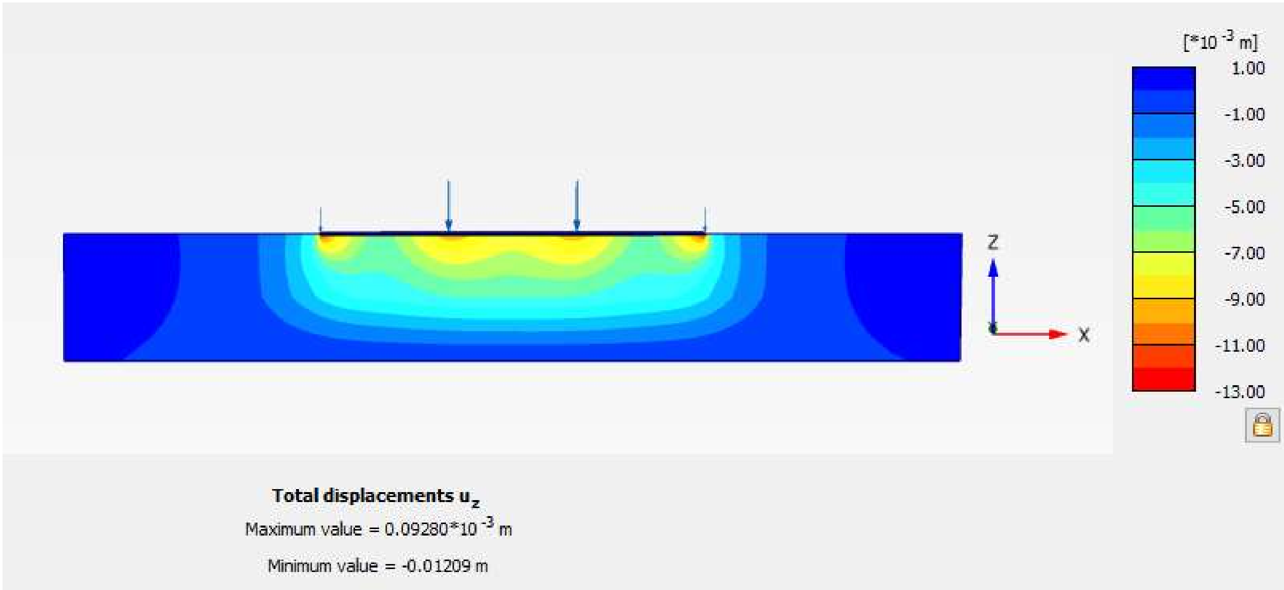


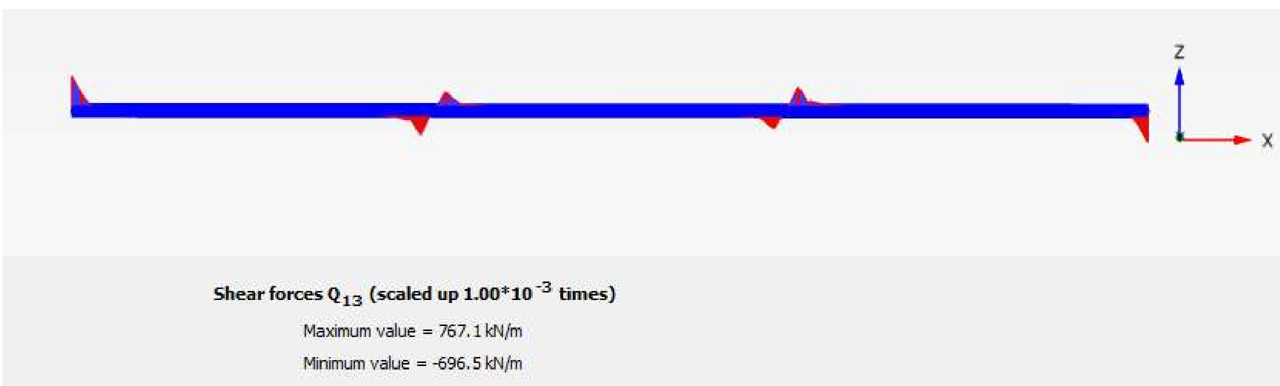
Bending moments M_{11} (scaled up 5.00*10^{-3} times)

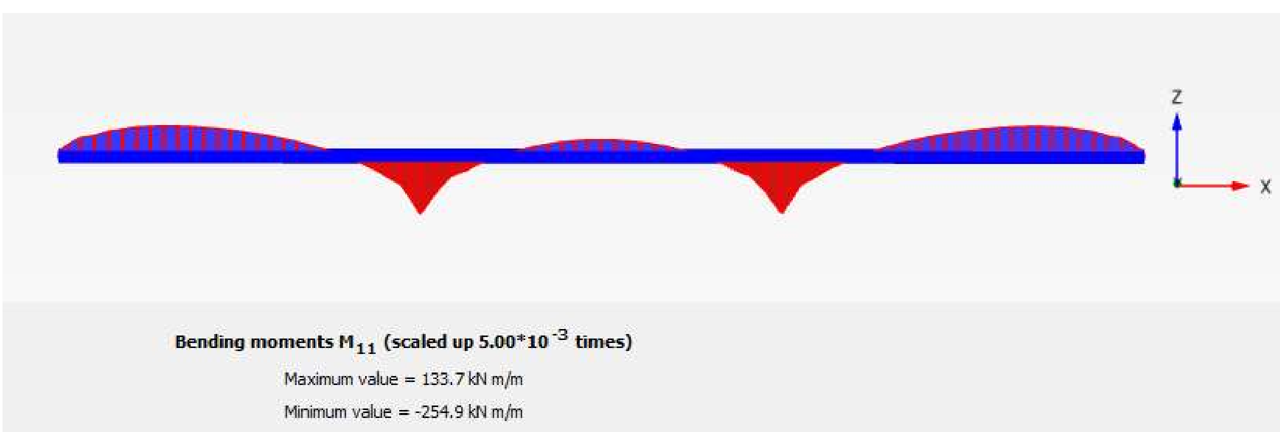
Maximum value = 139.6 kN m/m

Minimum value = -262.5 kN m/m

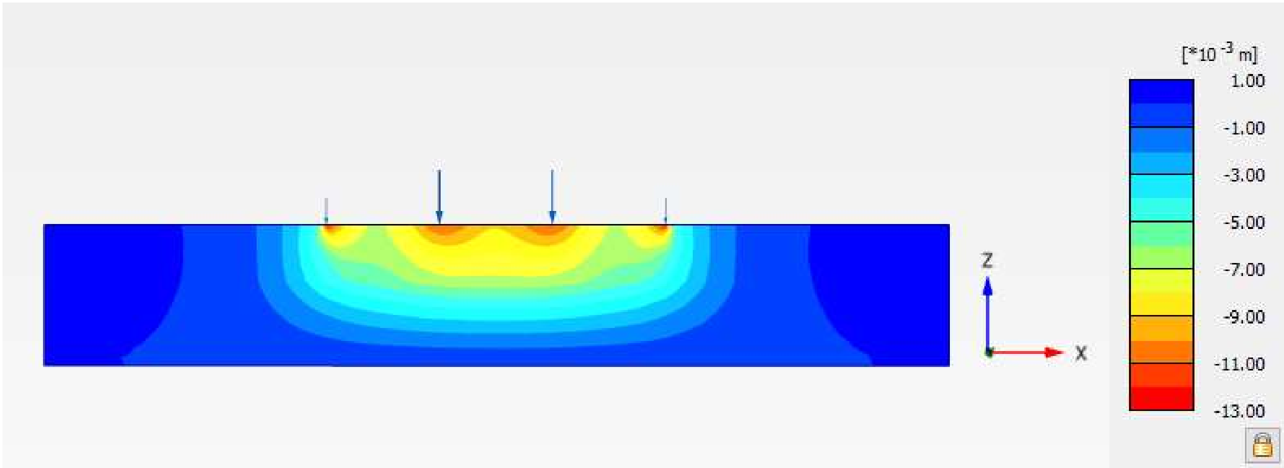
B=2.0m, t=0.4m, L=24.0m, H_1=4.0m, H_2=4.0m, F=320kN, 2F=640kN strip footing







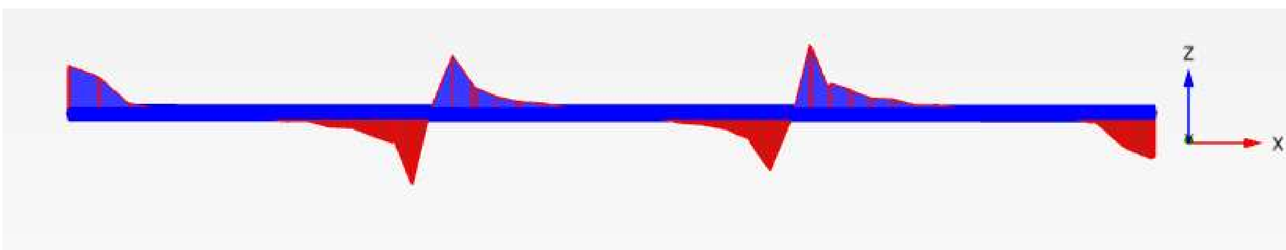
B=2.0m, t=0.4m, L=24.0m, H_1=4.0m, H_2=6.0m, F=320kN, 2F=640kN strip footing



Total displacements u_z

Maximum value = $0.1075*10^{-3}$ m

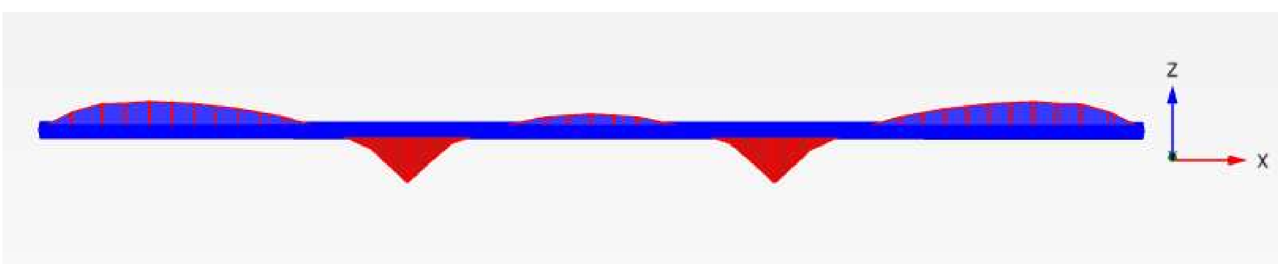
Minimum value = -0.01236 m



Shear forces Q_{13} (scaled up $5.00*10^{-3}$ times)

Maximum value = 303.0 kN/m

Minimum value = -319.2 kN/m

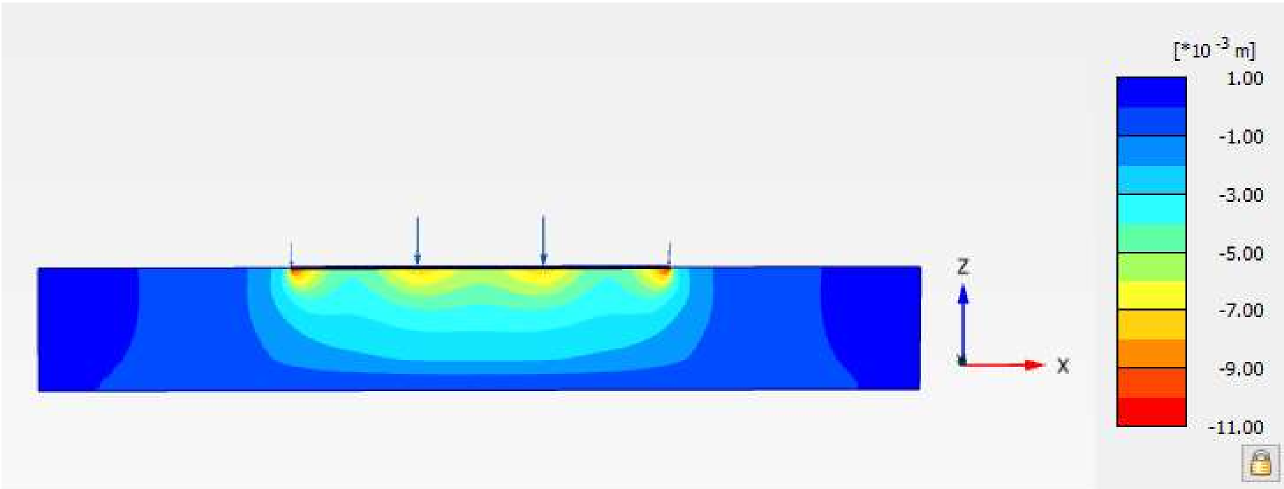


Bending moments M_{11} (scaled up $5.00*10^{-3}$ times)

Maximum value = 126.0 kN m/m

Minimum value = -229.6 kN m/m

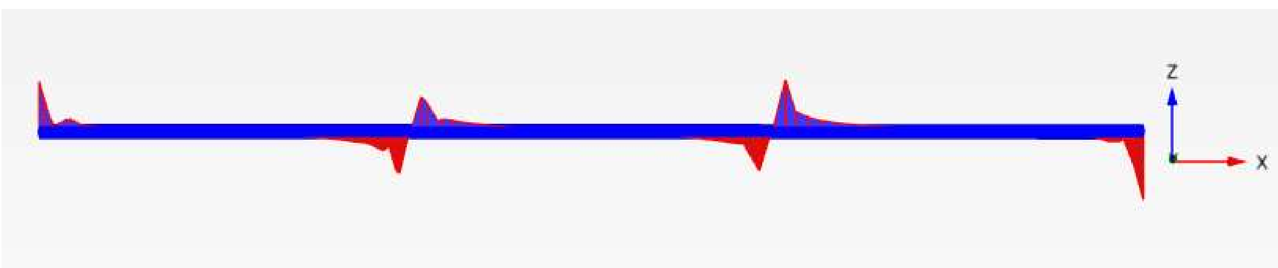
B=2.0m, t=0.4m, L=24.0m, H_1=6.0m, H_2=2.0m, F=320kN, 2F=640kN strip footing



Total displacements u_z

Maximum value = 0.05095×10^{-3} m

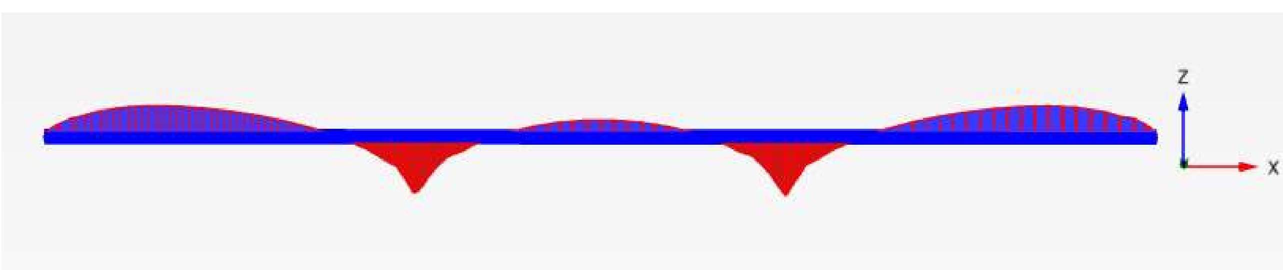
Minimum value = -0.01093 m



Shear forces Q_{13} (scaled up 2.00×10^{-3} times)

Maximum value = 565.1 kN/m

Minimum value = -736.8 kN/m

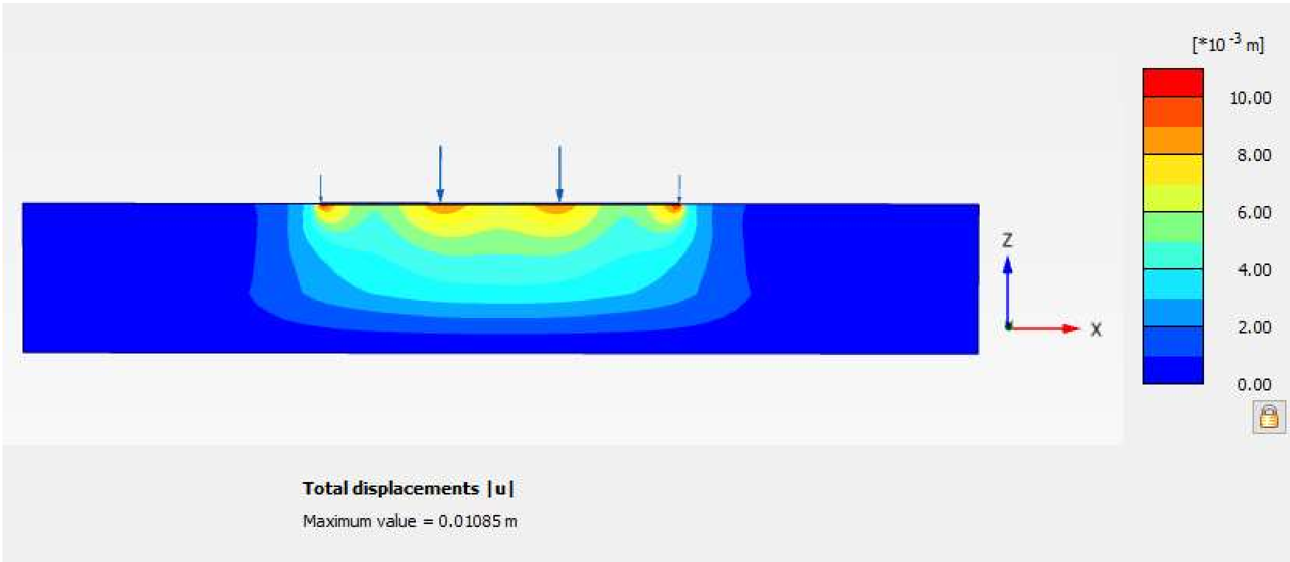


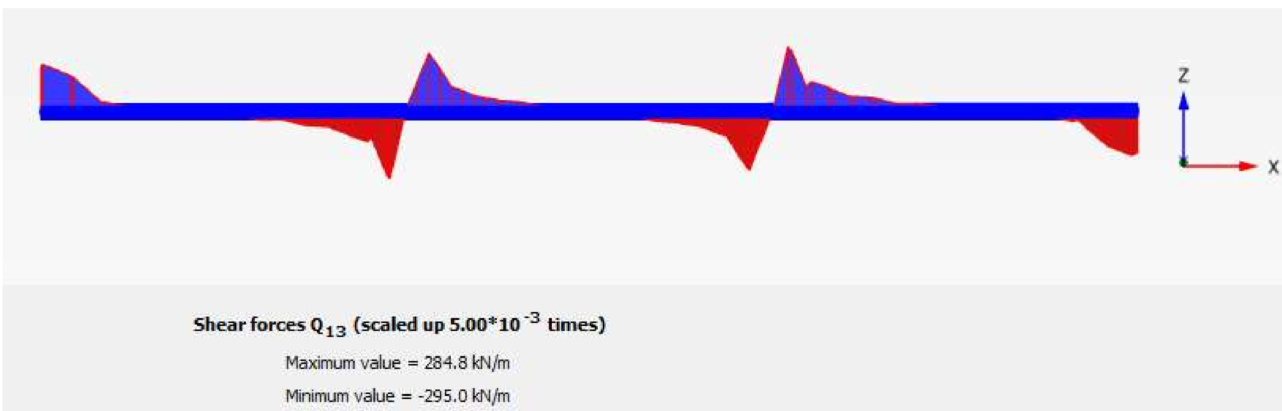
Bending moments M_{11} (scaled up 5.00×10^{-3} times)

Maximum value = 135.3 kN m/m

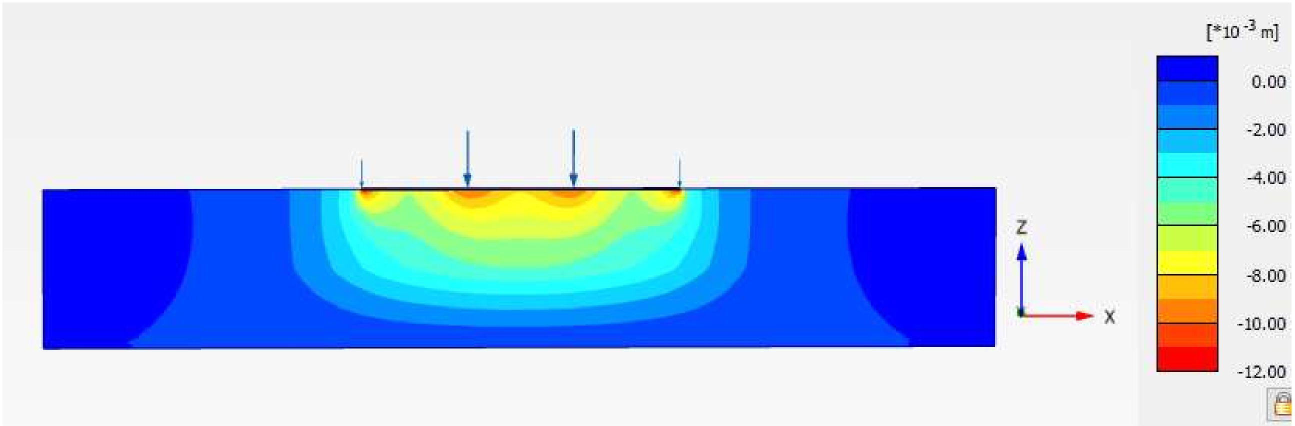
Minimum value = -255.6 kN m/m

B=2.0m, t=0.4m, L=24.0m, H_1=6.0m, H_2=4.0m, F=320kN, 2F=640kN strip footing





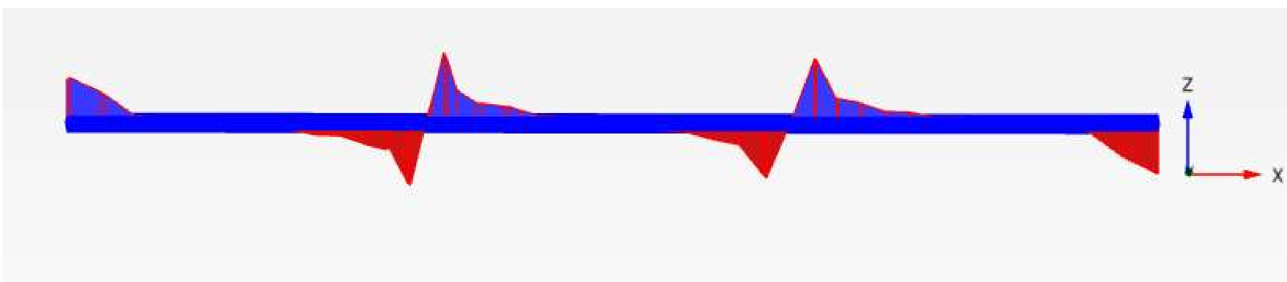
B=2.0m, t=0.4m, L=24.0m, H_1=6.0m, H_2=6.0m, F=320kN, 2F=640kN strip footing



Total displacements u_z

Maximum value = $0.08852*10^{-3}$ m

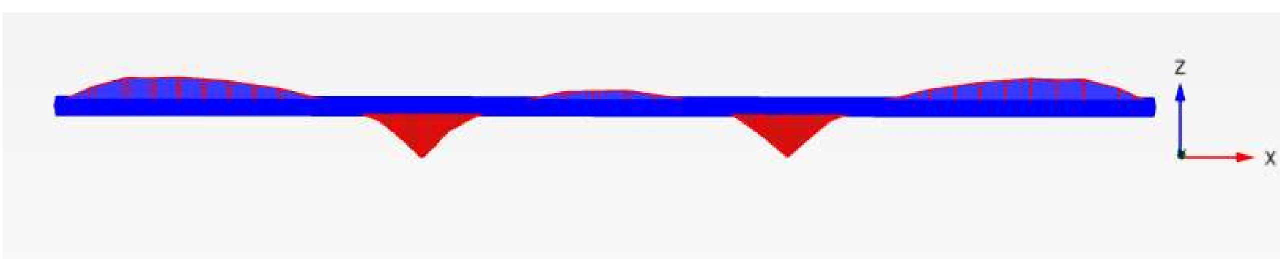
Minimum value = -0.01160 m



Shear forces Q_{13} (scaled up $5.00*10^{-3}$ times)

Maximum value = 311.5 kN/m

Minimum value = -274.5 kN/m

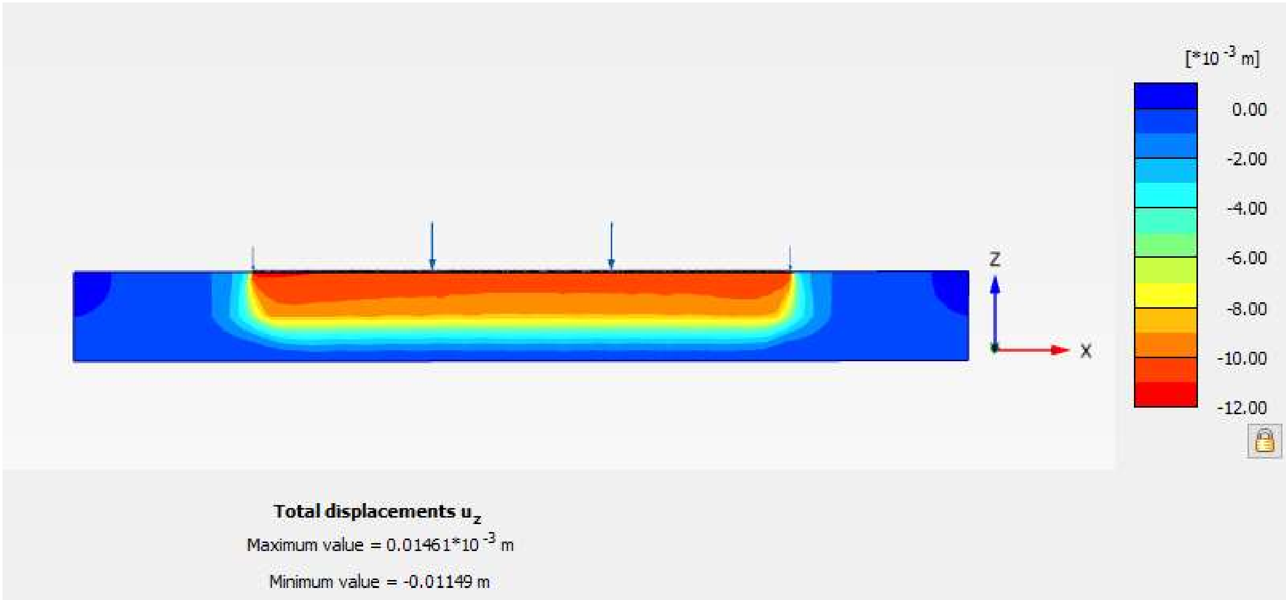


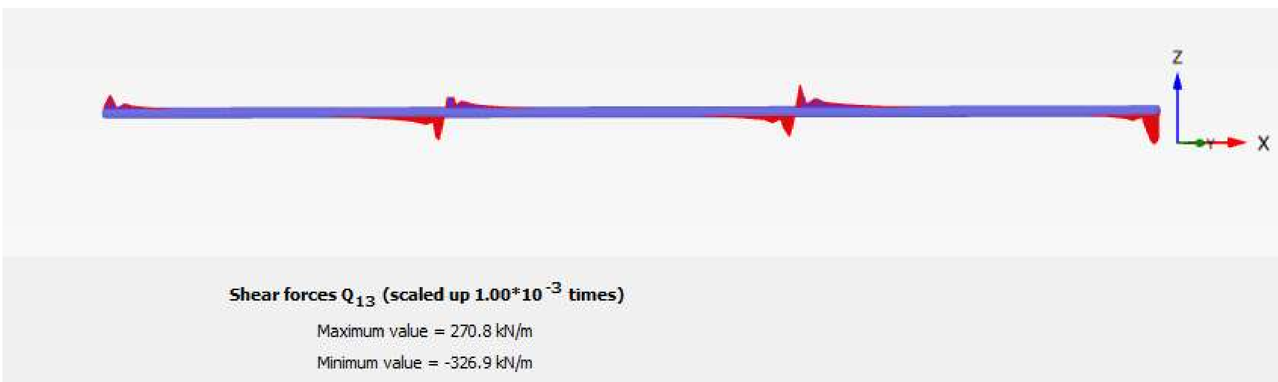
Bending moments M_{11} (scaled up $5.00*10^{-3}$ times)

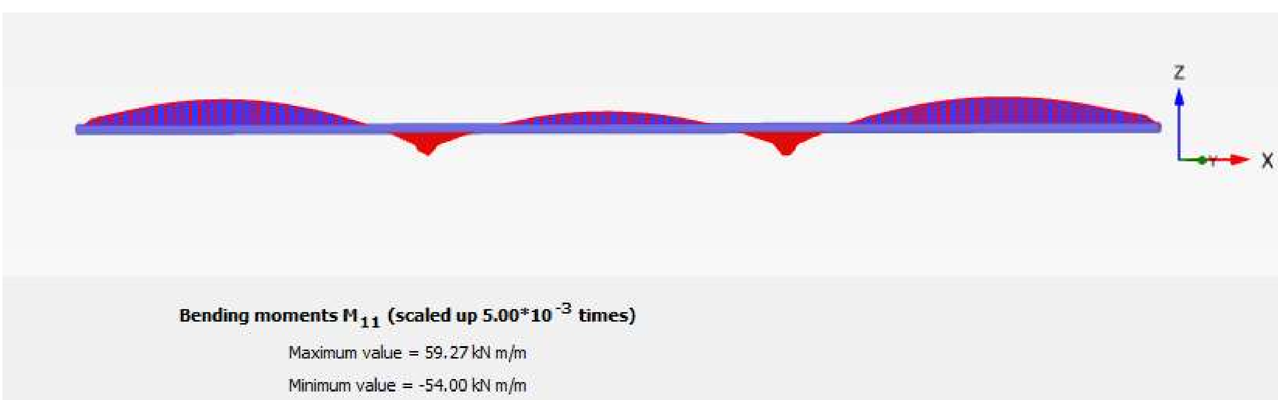
Maximum value = 125.3 kN m/m

Minimum value = -223.4 kN m/m

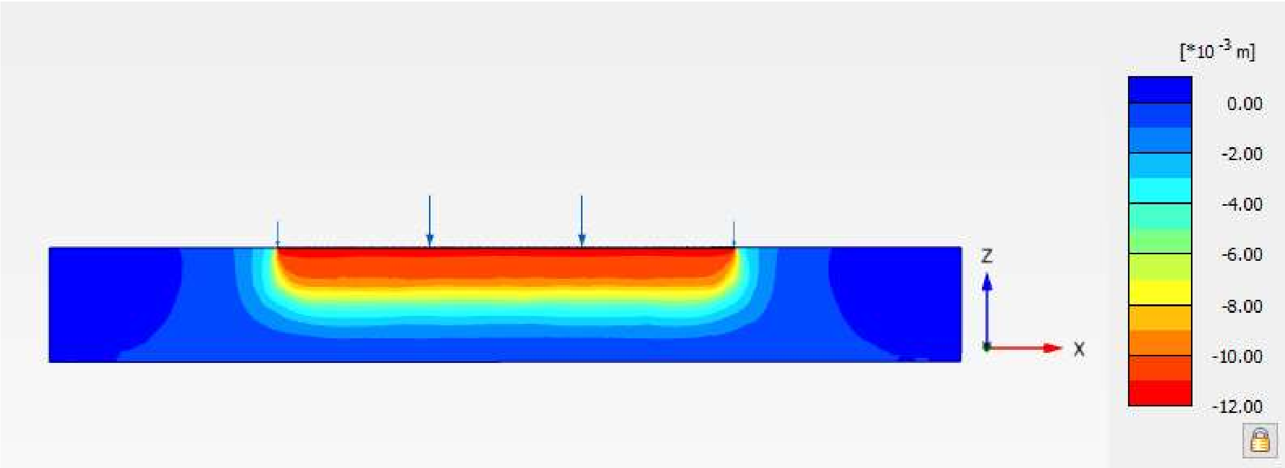
B=1.0m, t=0.6m, L=12.0m, H_1=1.0m, H_2=1.0m, F=80kN, 2F=160kN strip footing







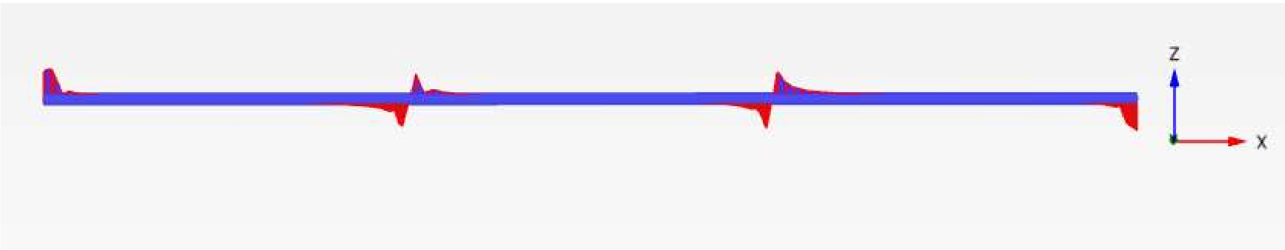
B=1.0m, t=0.6m, L=12.0m, H_1=1.0m, H_2=2.0m, F=80kN, 2F=160kN strip footing



Total displacements u_z

Maximum value = $0.09815*10^{-3}$ m

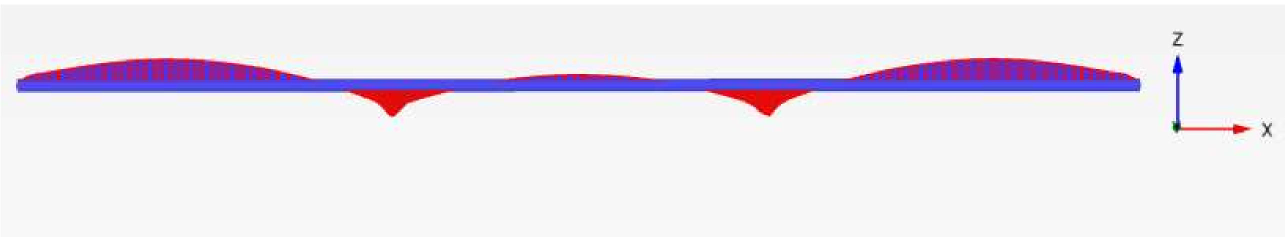
Minimum value = -0.01176 m



Shear forces Q_{13} (scaled up $1.00*10^{-3}$ times)

Maximum value = 331.1 kN/m

Minimum value = -351.6 kN/m

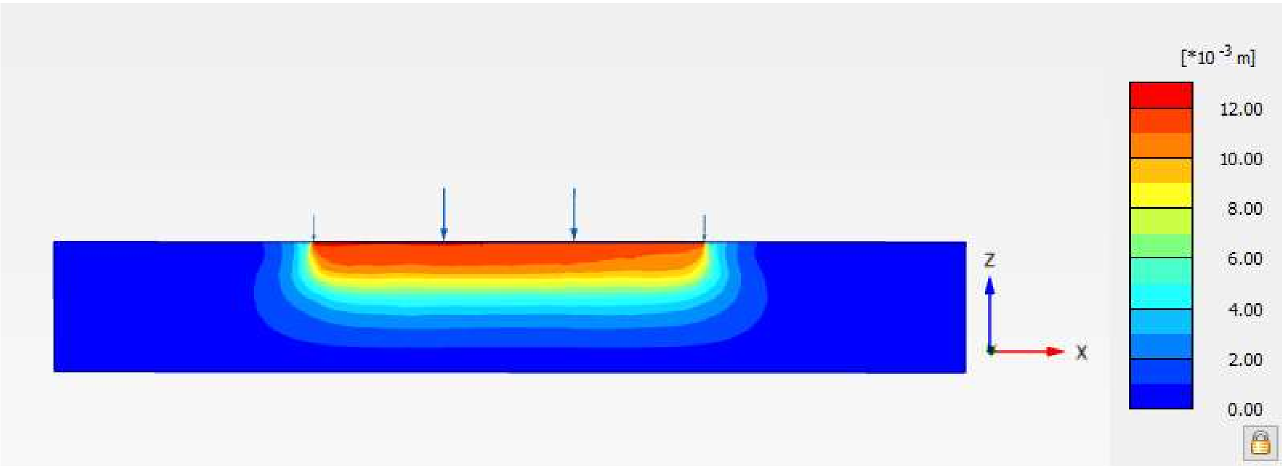


Bending moments M_{11} (scaled up $5.00*10^{-3}$ times)

Maximum value = 55.81 kN m/m

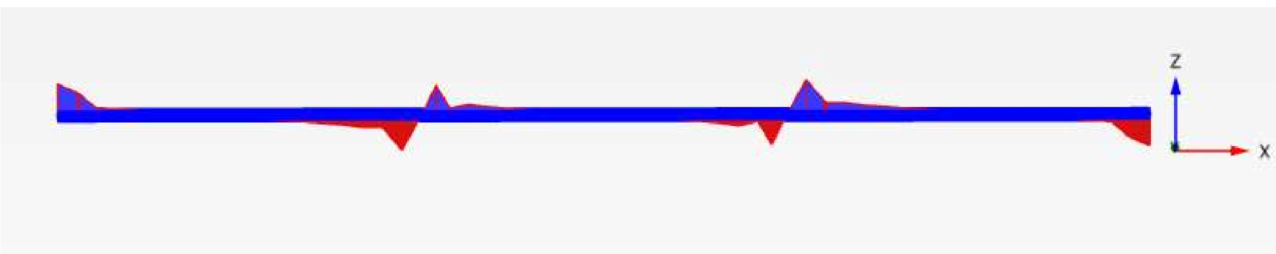
Minimum value = -65.98 kN m/m

B=1.0m, t=0.6m, L=12.0m, H_1=1.0m, H_2=3.0m, F=80kN, 2F=160kN strip footing



Total displacements |u|

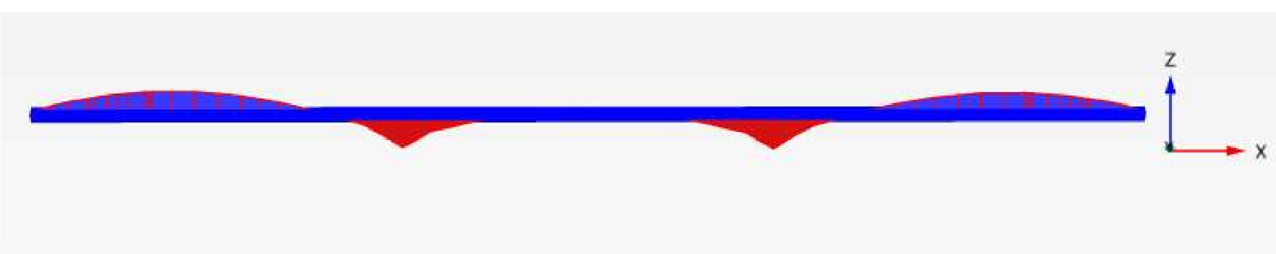
Maximum value = 0.01237 m



Shear forces Q_{13} (scaled up $2.00*10^{-3}$ times)

Maximum value = 193.7 kN/m

Minimum value = -194.4 kN/m

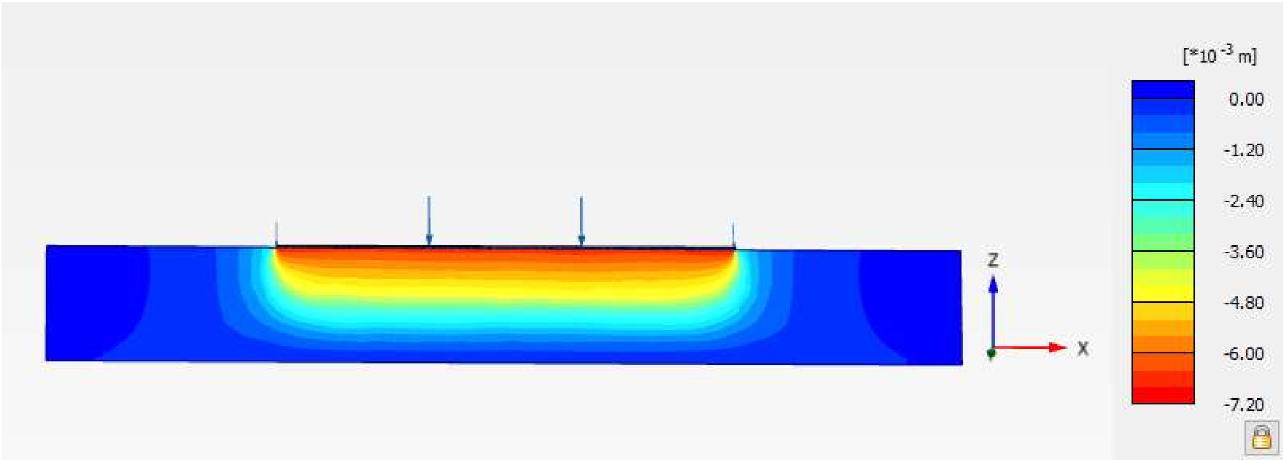


Bending moments M_{11} (scaled up $5.00*10^{-3}$ times)

Maximum value = 50.38 kN m/m

Minimum value = -74.58 kN m/m

B=1.0m, t=0.6m, L=12.0m, H_1=2.0m, H_2=1.0m, F=80kN, 2F=160kN strip footing



Total displacements u_z

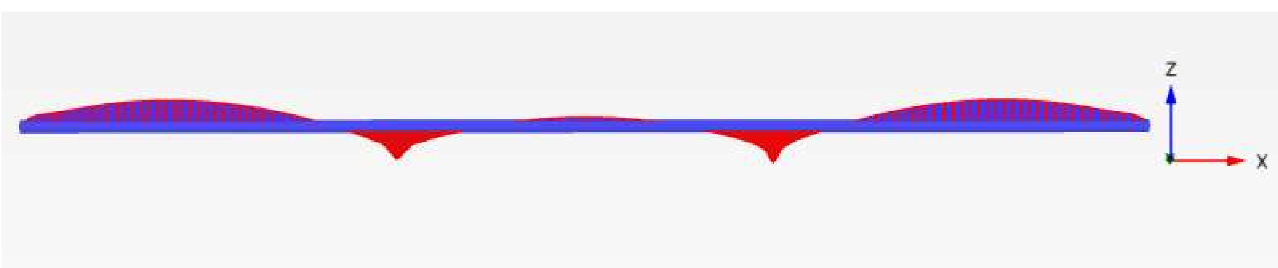
Maximum value = 0.04955×10^{-3} m

Minimum value = -6.922×10^{-3} m

Shear forces Q_{13} (scaled up 0.500×10^{-3} times)

Maximum value = 322.8 kN/m

Minimum value = -567.5 kN/m

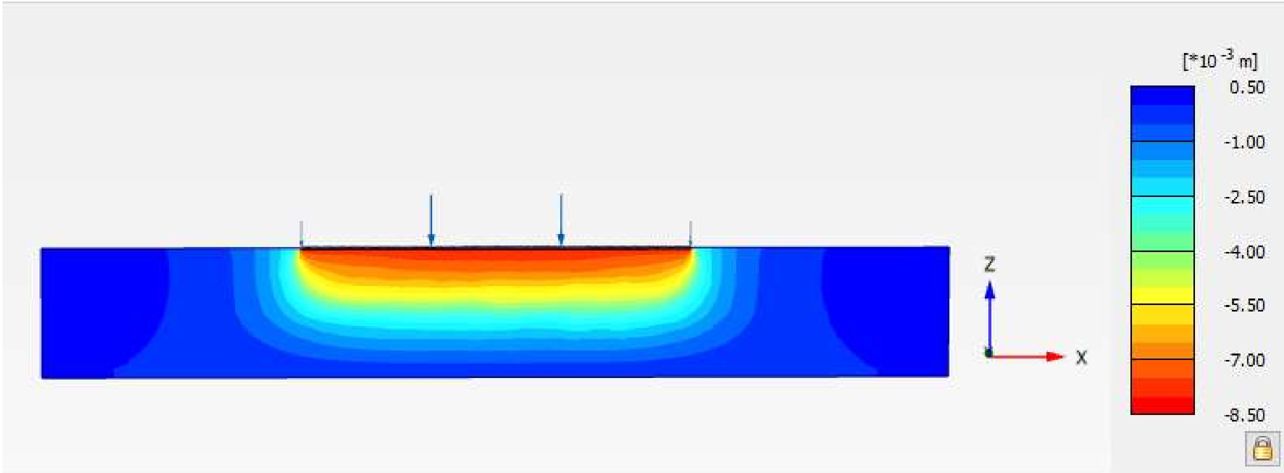


Bending moments M_{11} (scaled up 5.00×10^{-3} times)

Maximum value = 55.81 kN m/m

Minimum value = -79.13 kN m/m

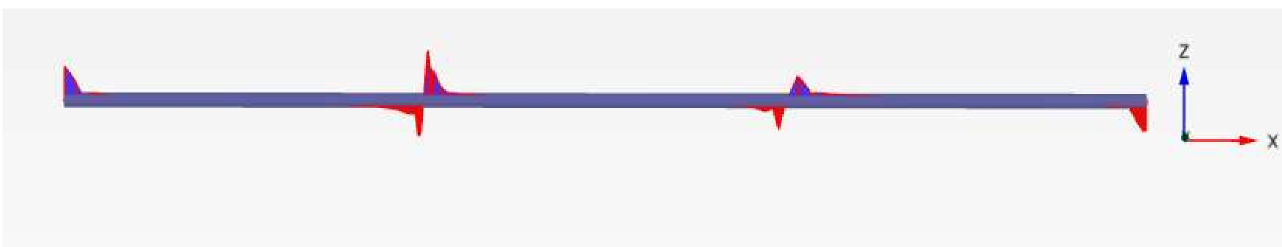
B=1.0m, t=0.6m, L=12.0m, H_1=2.0m, H_2=2.0m, F=80kN, 2F=160kN strip footing



Total displacements u_z

Maximum value = 0.06599*10 $^{-3}$ m

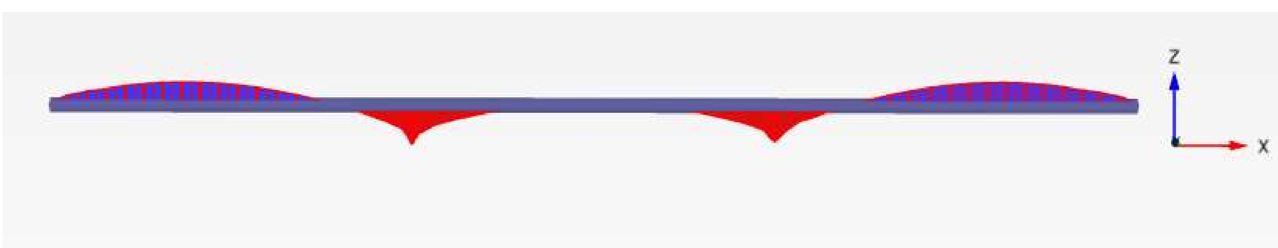
Minimum value = -8.137*10 $^{-3}$ m



Shear forces Q_{13} (scaled up 1.00*10 $^{-3}$ times)

Maximum value = 550.4 kN/m

Minimum value = -399.3 kN/m

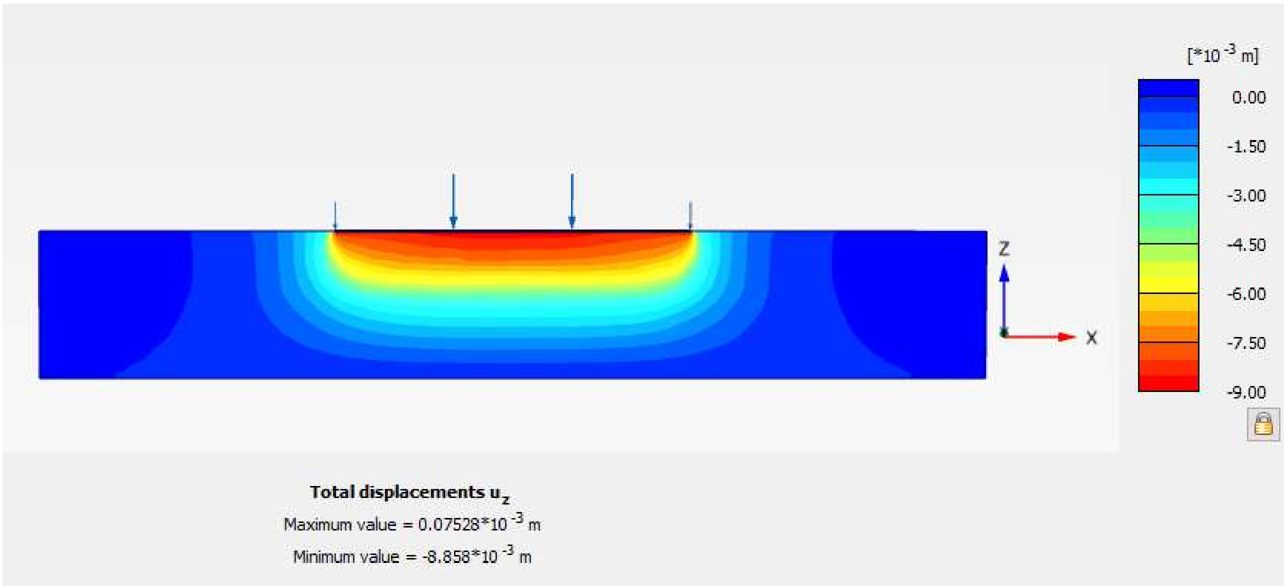


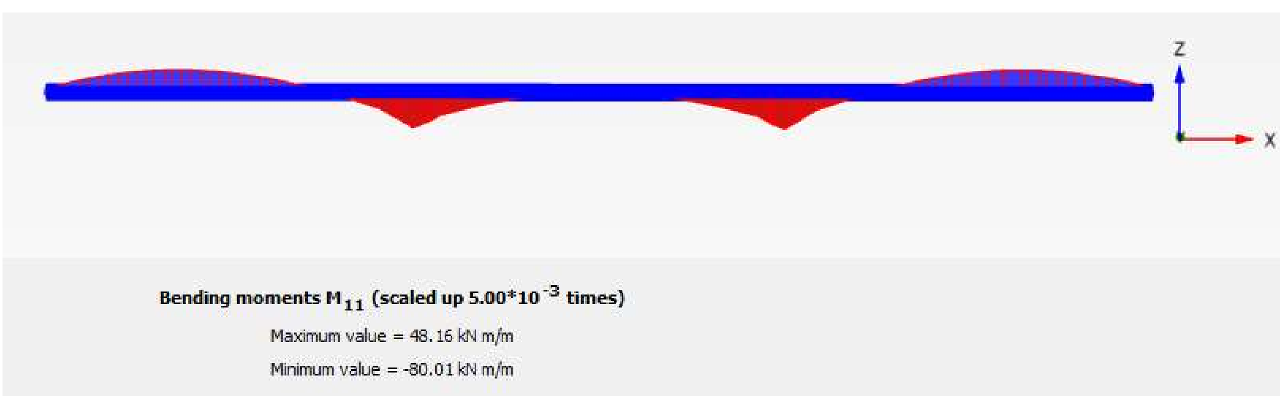
Bending moments M_{11} (scaled up 5.00*10 $^{-3}$ times)

Maximum value = 52.75 kN m/m

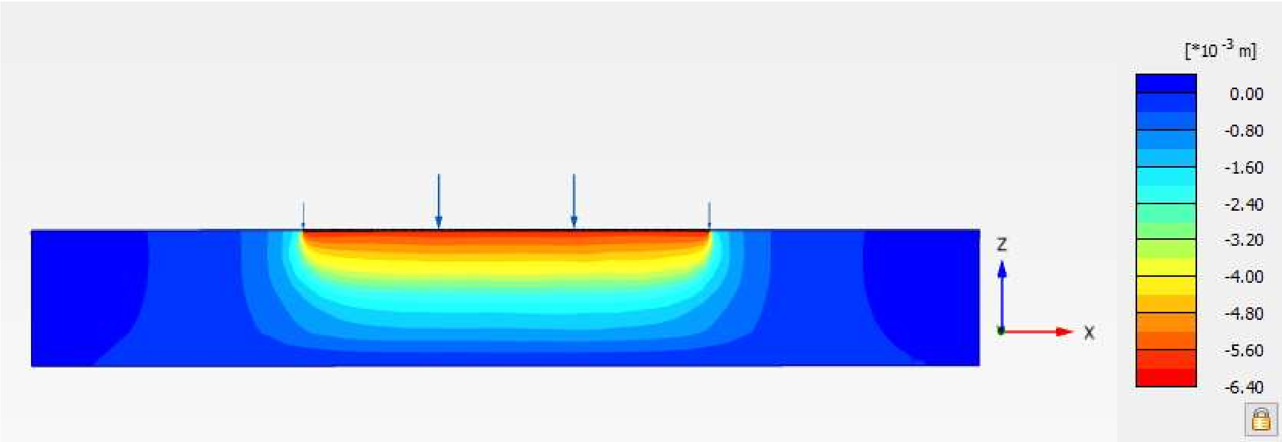
Minimum value = -87.60 kN m/m

B=1.0m, t=0.6m, L=12.0m, H_1=2.0m, H_2=3.0m, F=80kN, 2F=160kN strip footing





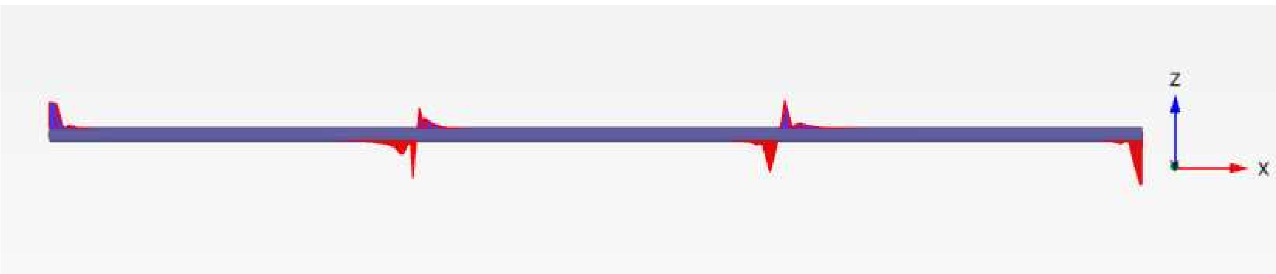
B=1.0m, t=0.6m, L=12.0m, H_1=3.0m, H_2=1.0m, F=80kN, 2F=160kN strip footing



Total displacements u_z

Maximum value = 0.03585×10^{-3} m

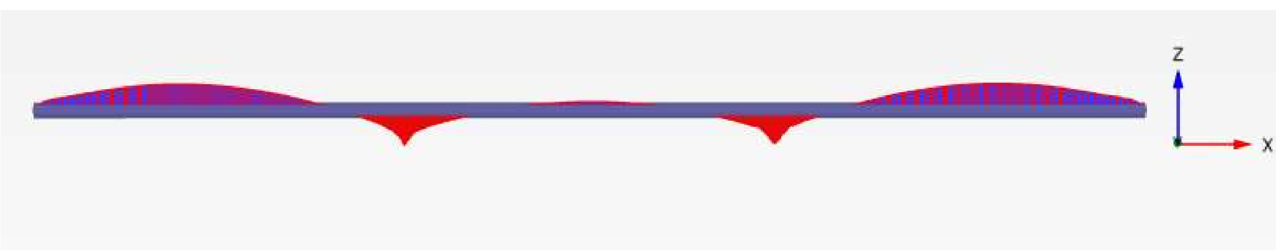
Minimum value = -6.223×10^{-3} m



Shear forces Q_{13} (scaled up 1.00×10^{-3} times)

Maximum value = 374.6 kN/m

Minimum value = -553.1 kN/m

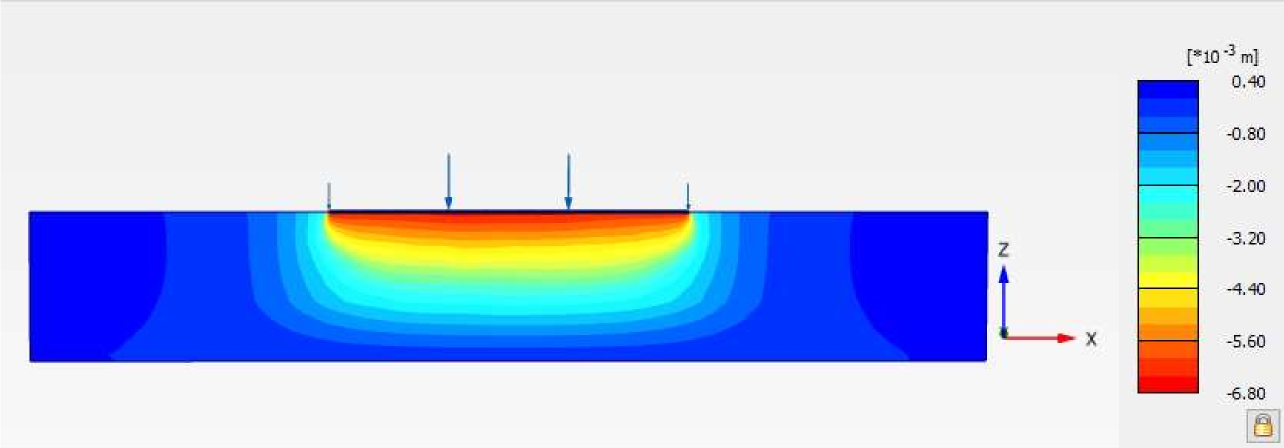


Bending moments M_{11} (scaled up 5.00×10^{-3} times)

Maximum value = 57.63 kN m/m

Minimum value = -76.38 kN m/m

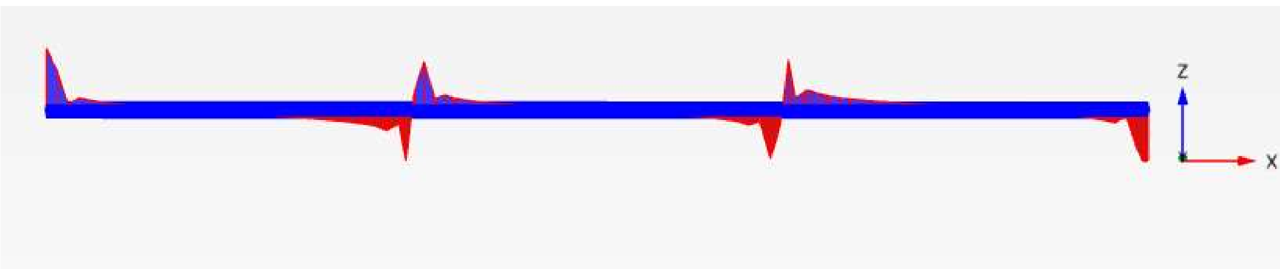
B=1.0m, t=0.6m, L=12.0m, H_1=3.0m, H_2=2.0m, F=80kN, 2F=160kN strip footing



Total displacements u_z

Maximum value = 0.05084*10^{-3} m

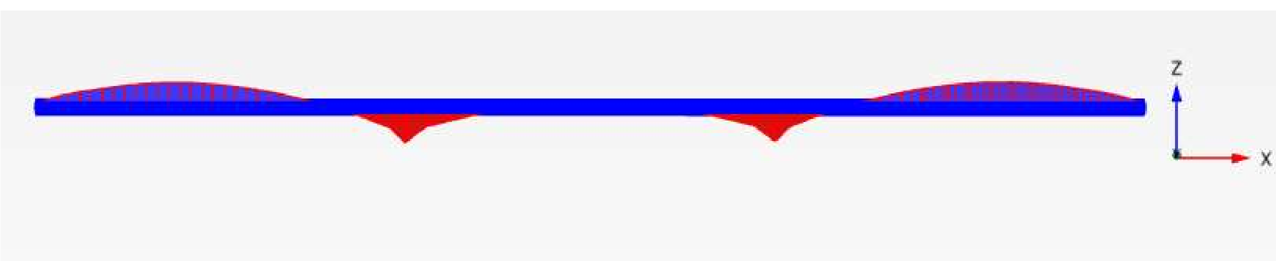
Minimum value = -6.623*10^{-3} m



Shear forces Q_{13} (scaled up 2.00*10^{-3} times)

Maximum value = 336.0 kN/m

Minimum value = -278.0 kN/m

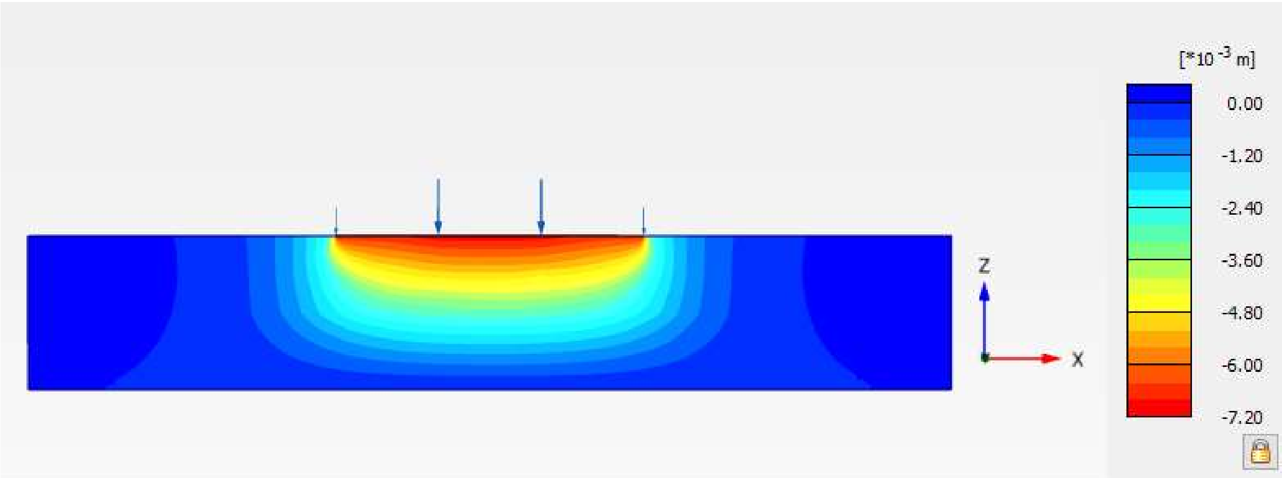


Bending moments M_{11} (scaled up 5.00*10^{-3} times)

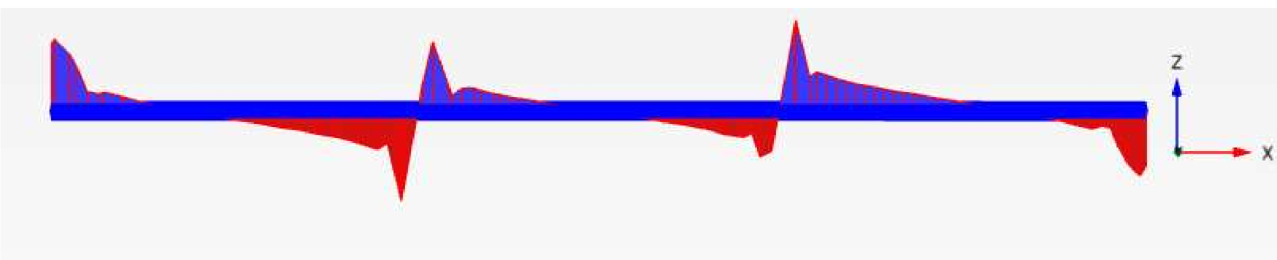
Maximum value = 54.83 kN m/m

Minimum value = -77.14 kN m/m

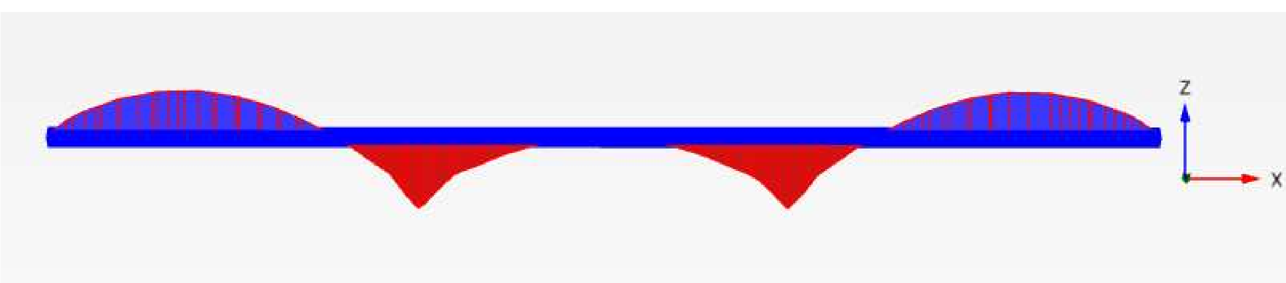
B=1.0m, t=0.6m, L=12.0m, H_1=3.0m, H_2=3.0m, F=80kN, 2F=160kN strip footing



Total displacements u_z
Maximum value = 0.06048*10^{-3} m
Minimum value = -7.072*10^{-3} m

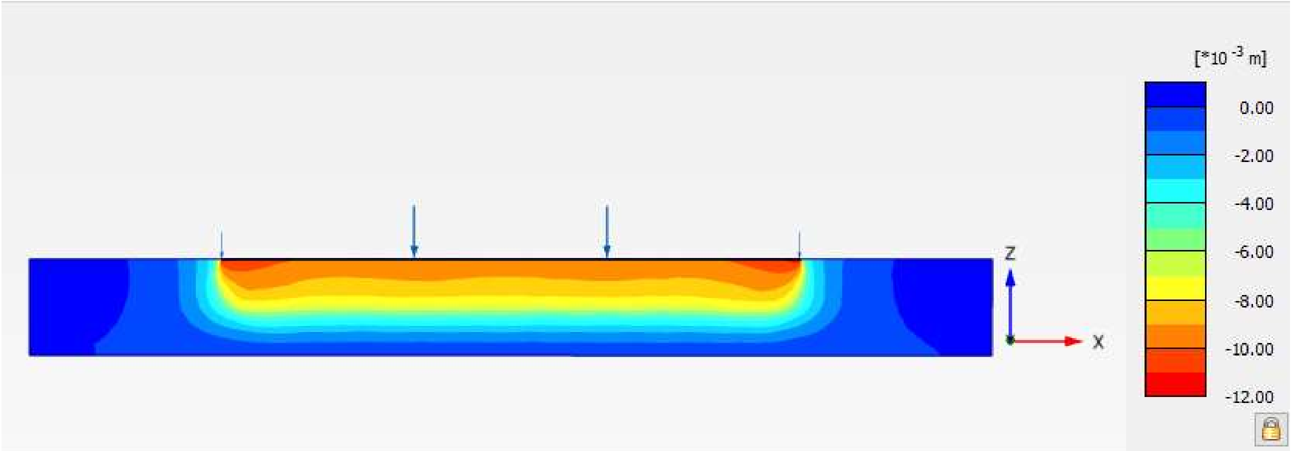


Shear forces Q_{13} (scaled up 5.00*10^{-3} times)
Maximum value = 198.3 kN/m
Minimum value = -197.9 kN/m



Bending moments M_{11} (scaled up 0.0100 times)
Maximum value = 49.72 kN m/m
Minimum value = -77.11 kN m/m

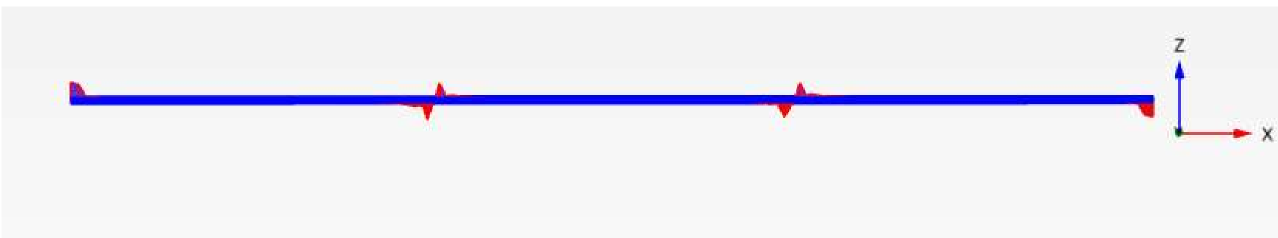
B=1.5m, t=0.6m, L=18.0m, H_1=1.5m, H_2=1.5m, F=160kN, 2F=320kN strip footing



Total displacements u_z

Maximum value = 0.1154×10^{-3} m

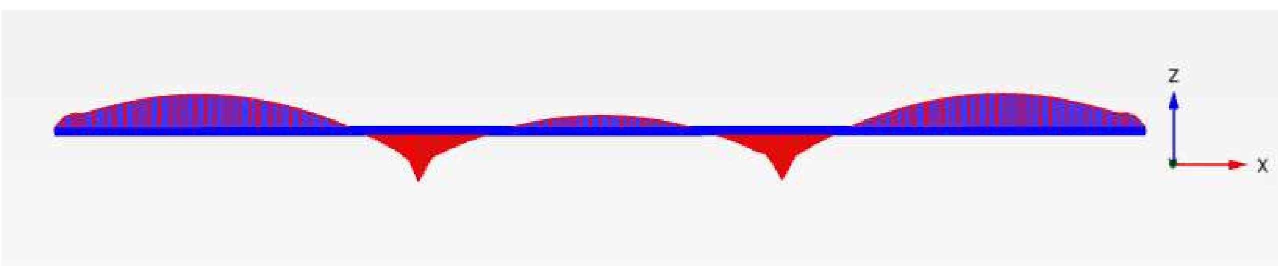
Minimum value = -0.01150 m



Shear forces Q_{13} (scaled up 0.500×10^{-3} times)

Maximum value = 602.2 kN/m

Minimum value = -661.6 kN/m

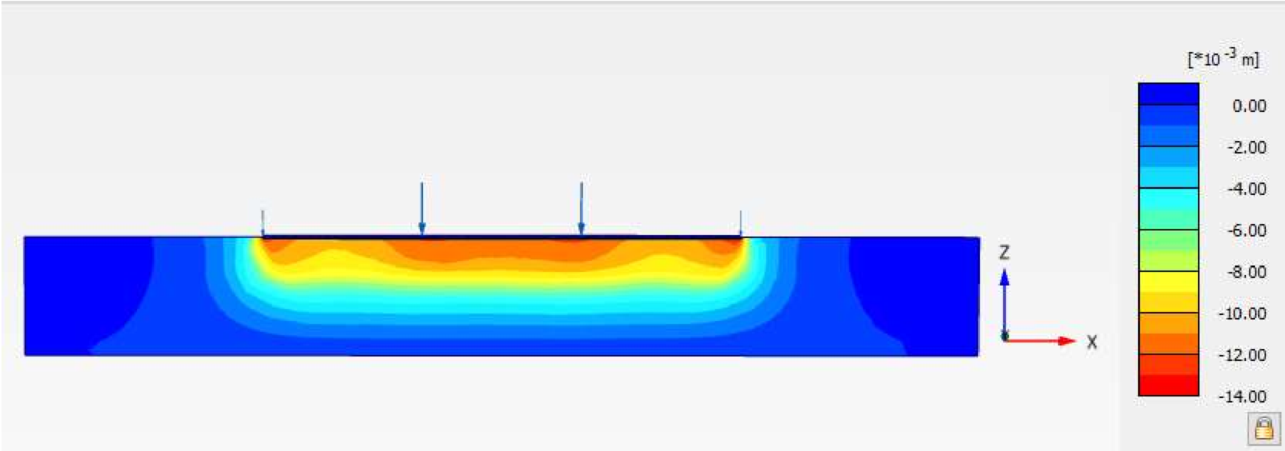


Bending moments M_{11} (scaled up 5.00×10^{-3} times)

Maximum value = 121.1 kN m/m

Minimum value = -166.5 kN m/m

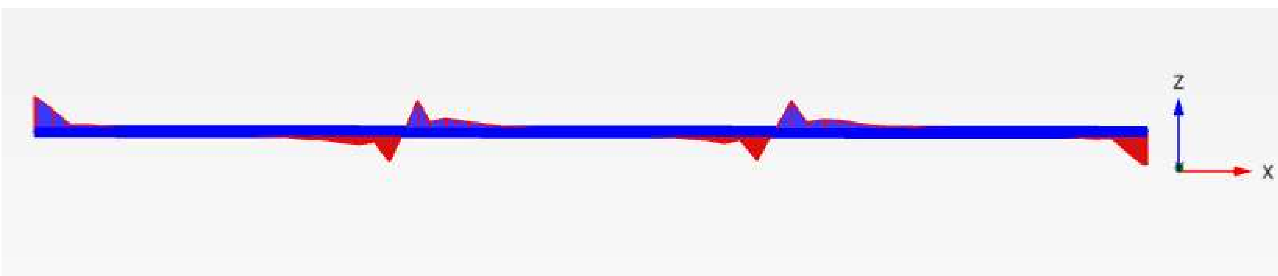
B=1.5m, t=0.6m, L=18.0m, H_1=1.5m, H_2=3.0m, F=180kN, 2F=360kN strip footing



Total displacements u_z

Maximum value = $0.1271*10^{-3}$ m

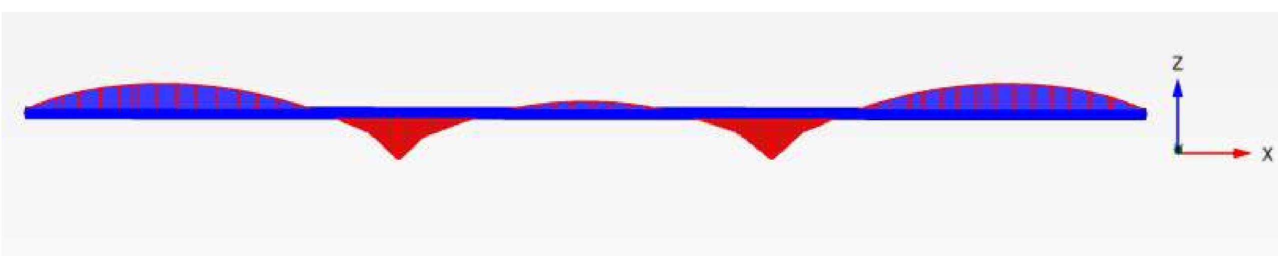
Minimum value = -0.01327 m



Shear forces Q_{13} (scaled up $2.00*10^{-3}$ times)

Maximum value = 283.1 kN/m

Minimum value = -267.1 kN/m

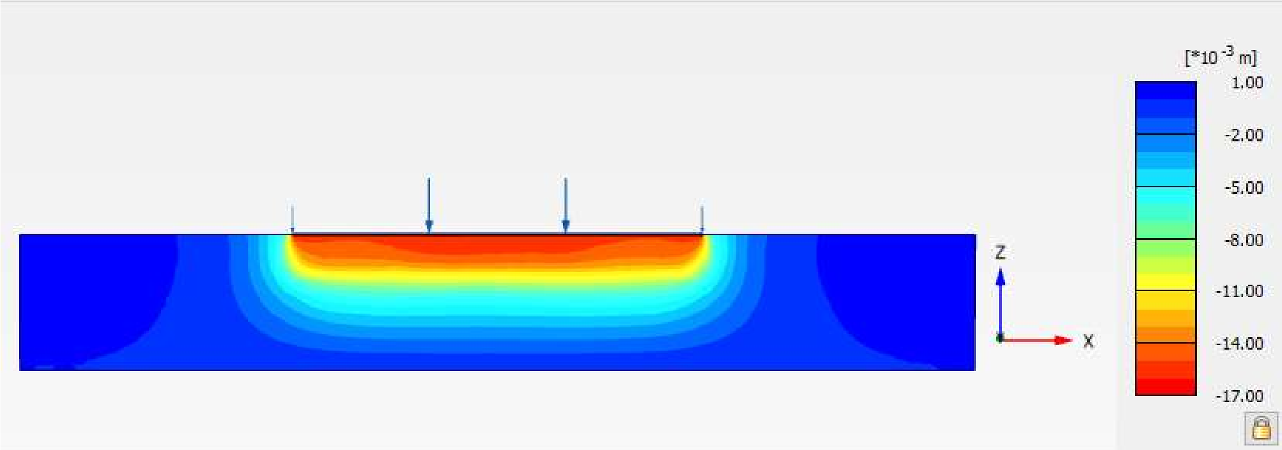


Bending moments M_{11} (scaled up $5.00*10^{-3}$ times)

Maximum value = 96.94 kN m/m

Minimum value = -149.9 kN m/m

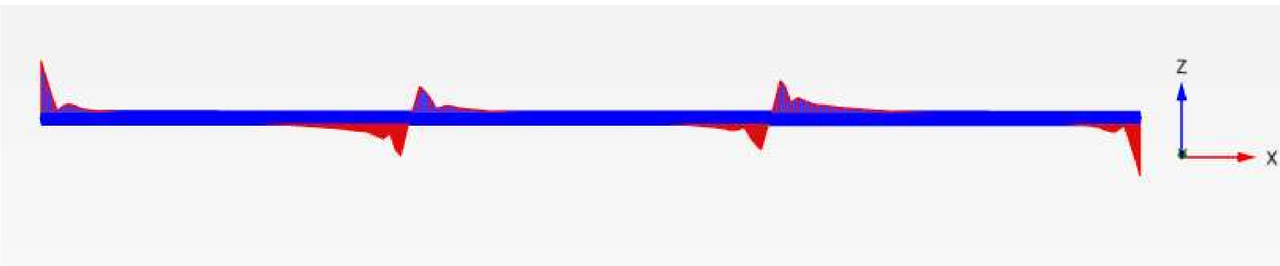
B=1.5m, t=0.6m, L=18.0m, H_1=1.5m, H_2=4.5m, F=180kN, 2F=360kN strip footing



Total displacements u_z

Maximum value = $0.1455*10^{-3}$ m

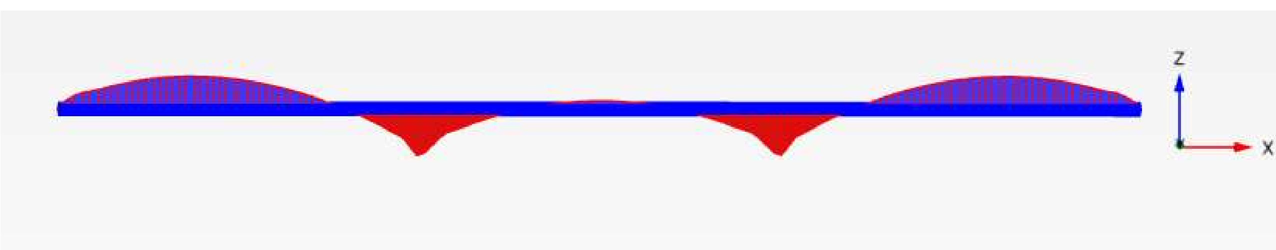
Minimum value = -0.01614 m



Shear forces Q_{13} (scaled up $2.00*10^{-3}$ times)

Maximum value = 470.1 kN/m

Minimum value = -470.5 kN/m

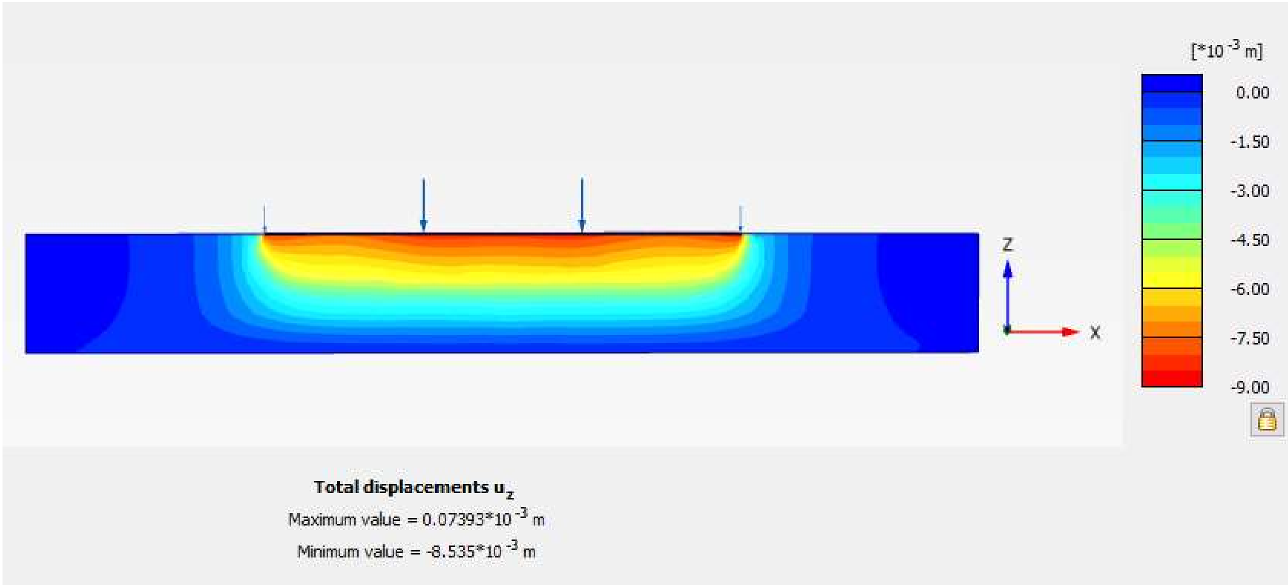


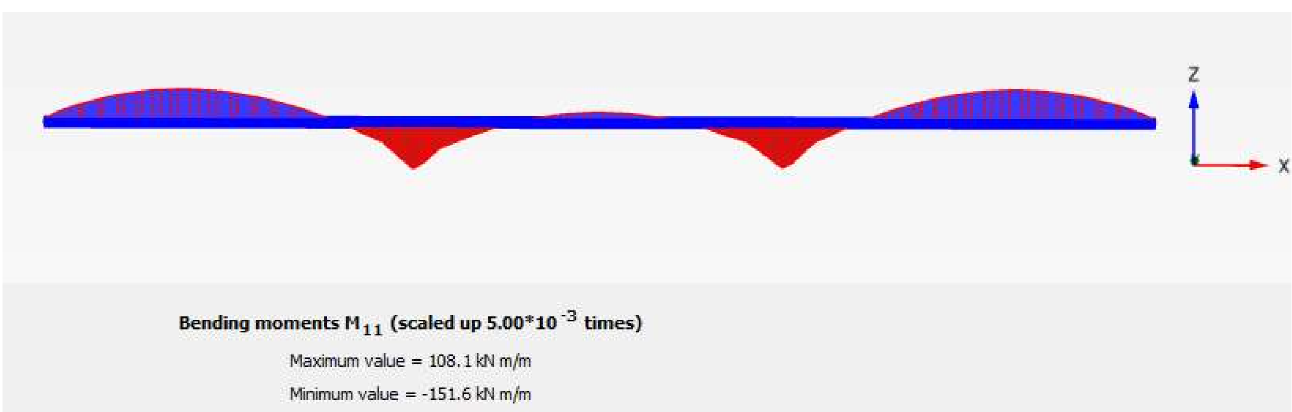
Bending moments M_{11} (scaled up $5.00*10^{-3}$ times)

Maximum value = 108.4 kN m/m

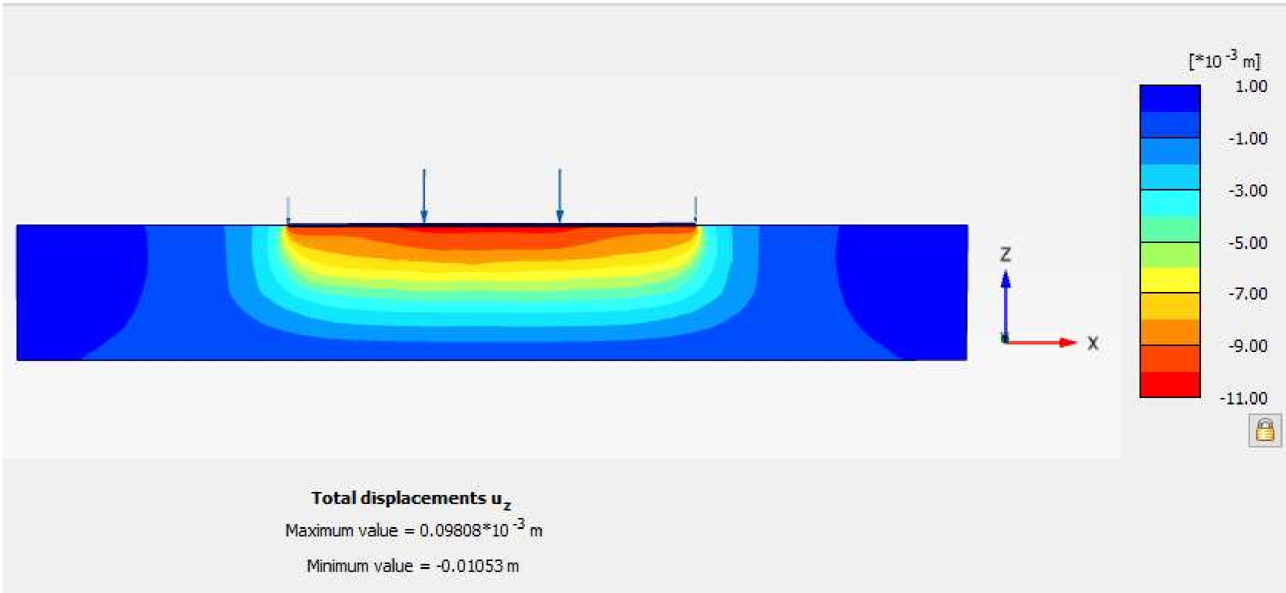
Minimum value = -155.7 kN m/m

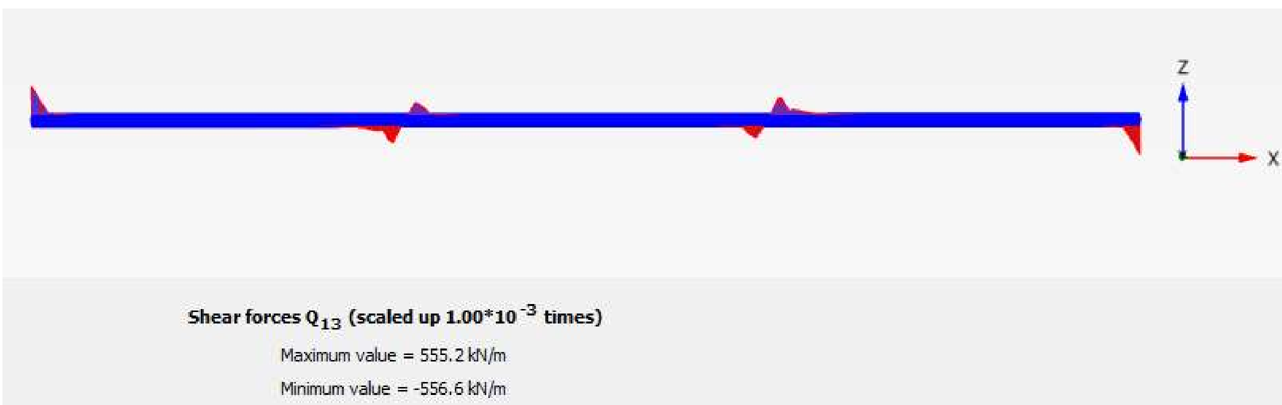
B=1.5m, t=0.6m, L=18.0m, H_1=3.0m, H_2=1.5m, F=180kN, 2F=360kN strip footing



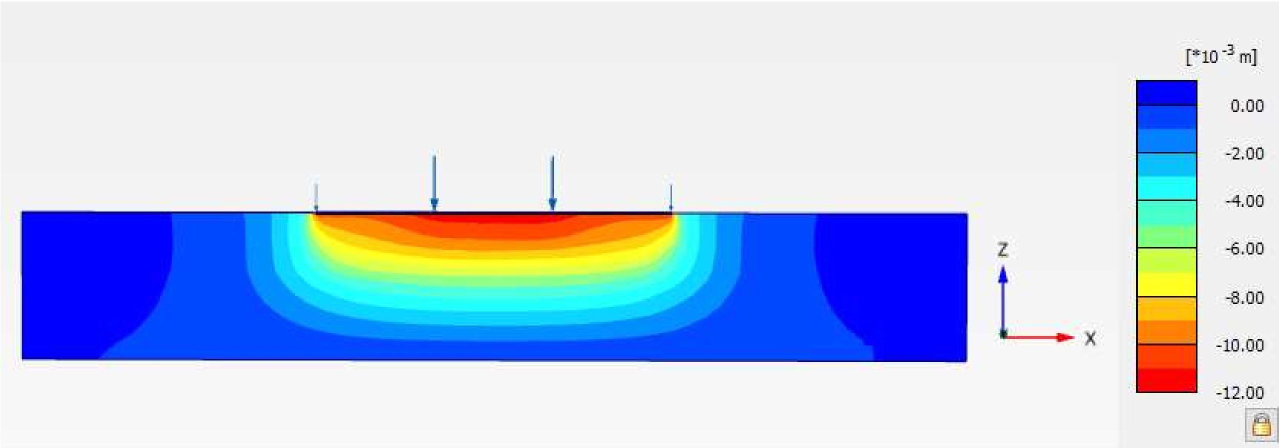


B=1.5m, t=0.6m, L=18.0m, H_1=3.0m, H_2=3.0m, F=180kN, 2F=360kN strip footing





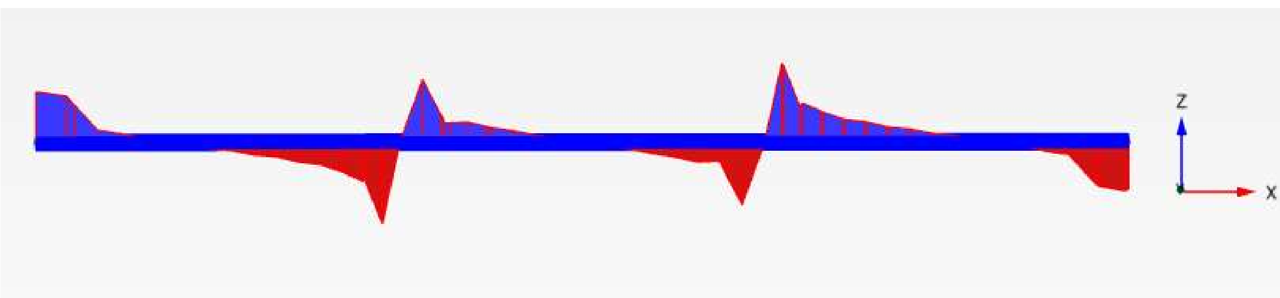
B=1.5m, t=0.6m, L=18.0m, H_1=3.0m, H_2=4.5m, F=180kN, 2F=360kN strip footing



Total displacements u_z

Maximum value = $0.1127*10^{-3}$ m

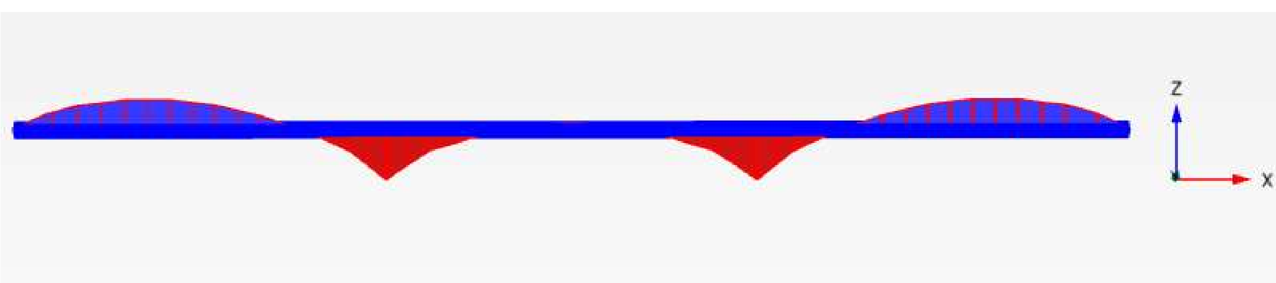
Minimum value = -0.01167 m



Shear forces Q_{13} (scaled up $5.00*10^{-3}$ times)

Maximum value = 258.8 kN/m

Minimum value = -266.1 kN/m

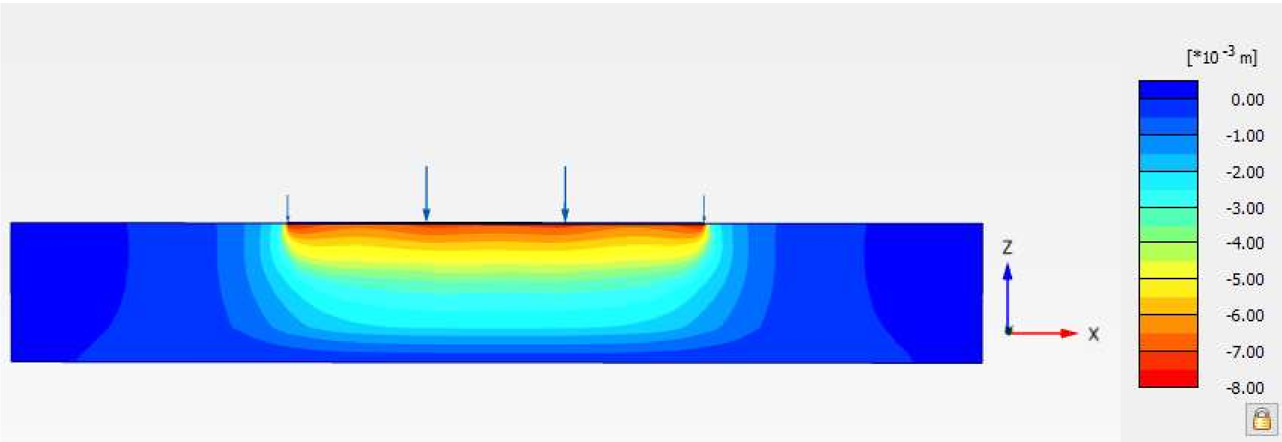


Bending moments M_{11} (scaled up $5.00*10^{-3}$ times)

Maximum value = 99.11 kN m/m

Minimum value = -161.4 kN m/m

B=1.5m, t=0.6m, L=18.0m, H_1=4.5m, H_2=1.5m, F=180kN, 2F=360kN strip footing



Total displacements u_z

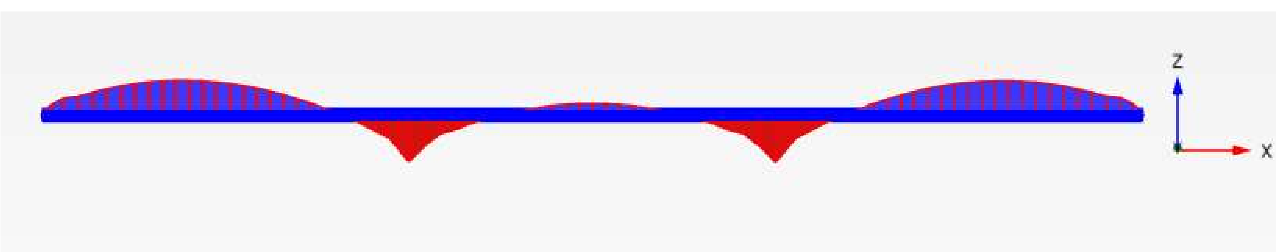
Maximum value = 0.05256×10^{-3} m

Minimum value = -7.945×10^{-3} m

Shear forces Q_{13} (scaled up 1.00×10^{-3} times)

Maximum value = 606.4 kN/m

Minimum value = -678.9 kN/m

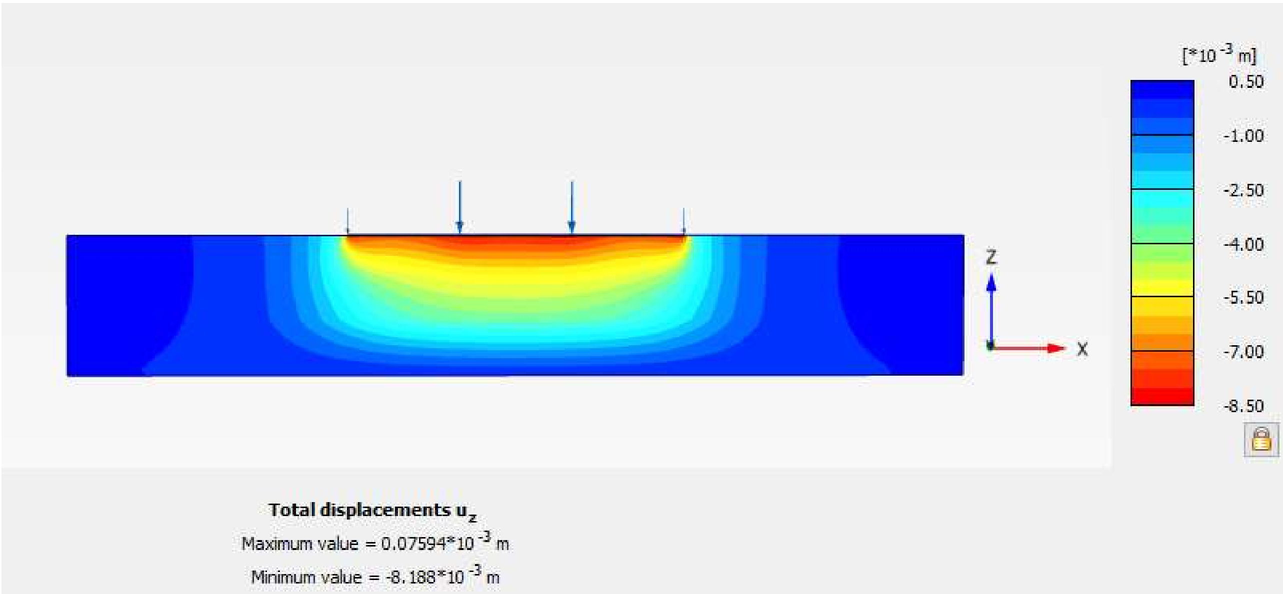


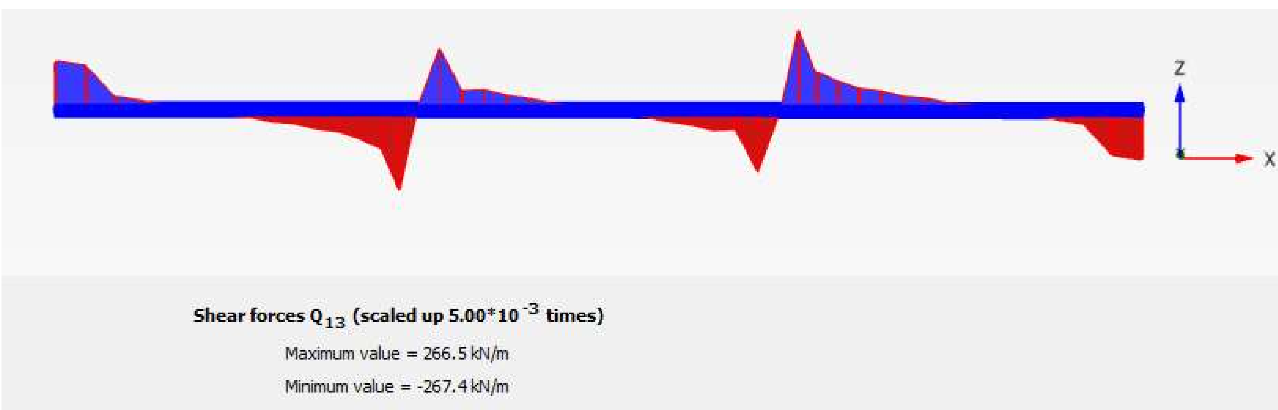
Bending moments M_{11} (scaled up 5.00×10^{-3} times)

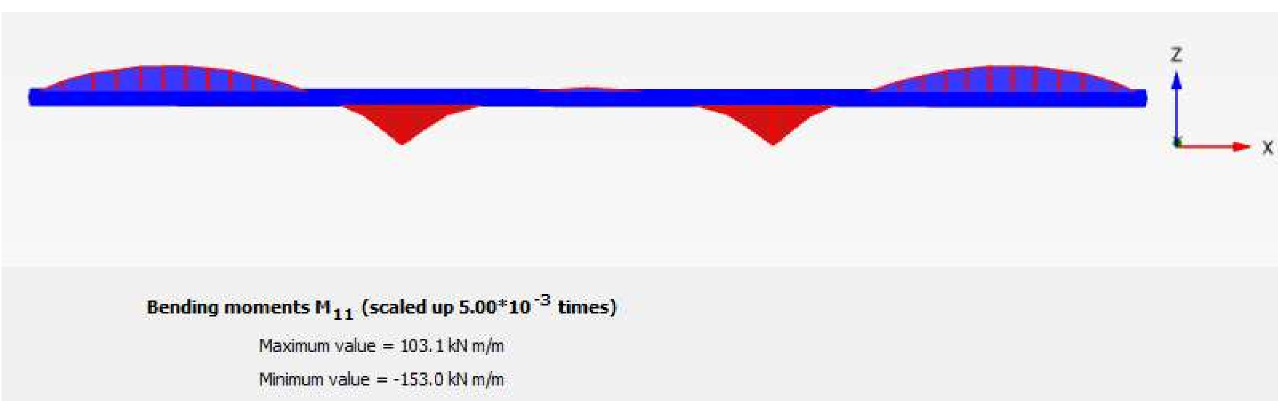
Maximum value = 114.1 kN m/m

Minimum value = -156.6 kN m/m

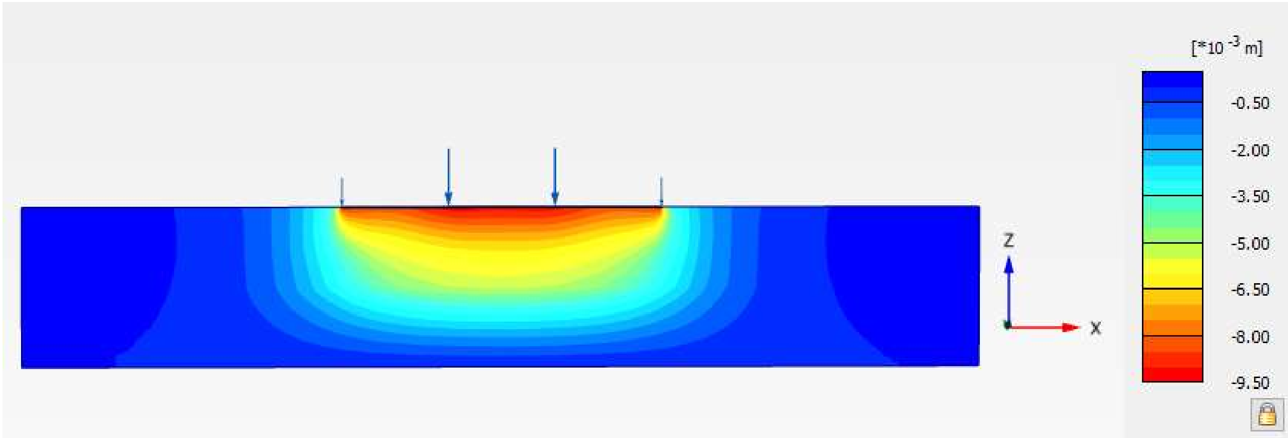
B=1.5m, t=0.6m, L=18.0m, H_1=4.5m, H_2=3.0m, F=180kN, 2F=360kN strip footing







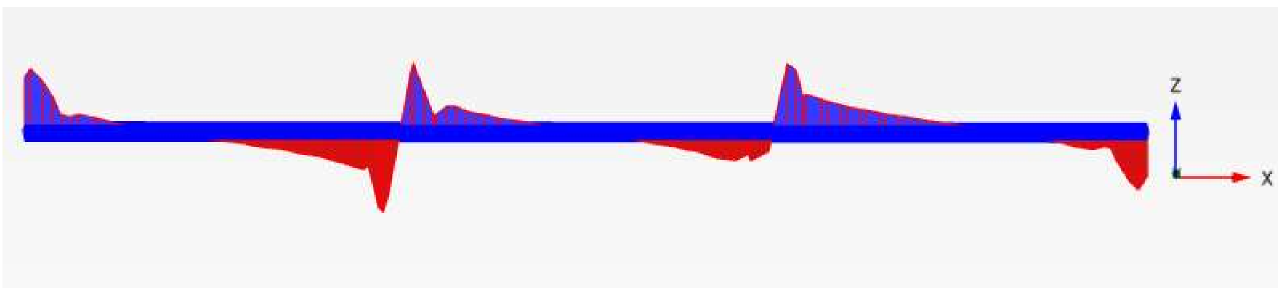
B=1.5m, t=0.6m, L=18.0m, H_1=4.5m, H_2=4.5m, F=180kN, 2F=360kN strip footing



Total displacements u_z

Maximum value = $0.09049*10^{-3}$ m

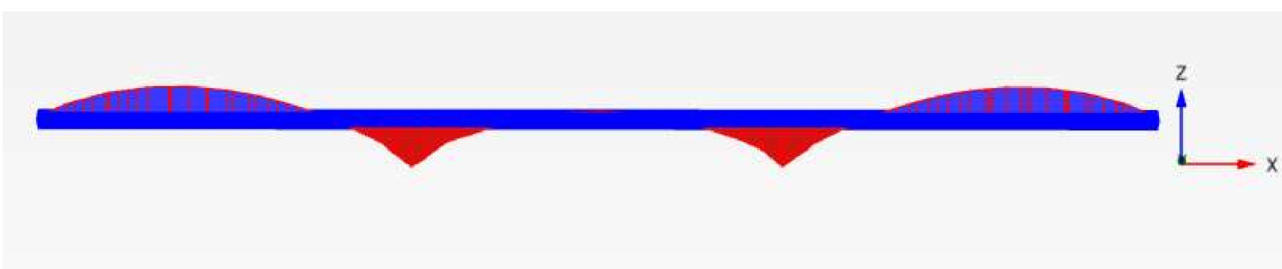
Minimum value = $-9.315*10^{-3}$ m



Shear forces Q_{13} (scaled up $5.00*10^{-3}$ times)

Maximum value = 222.6 kN/m

Minimum value = -257.2 kN/m

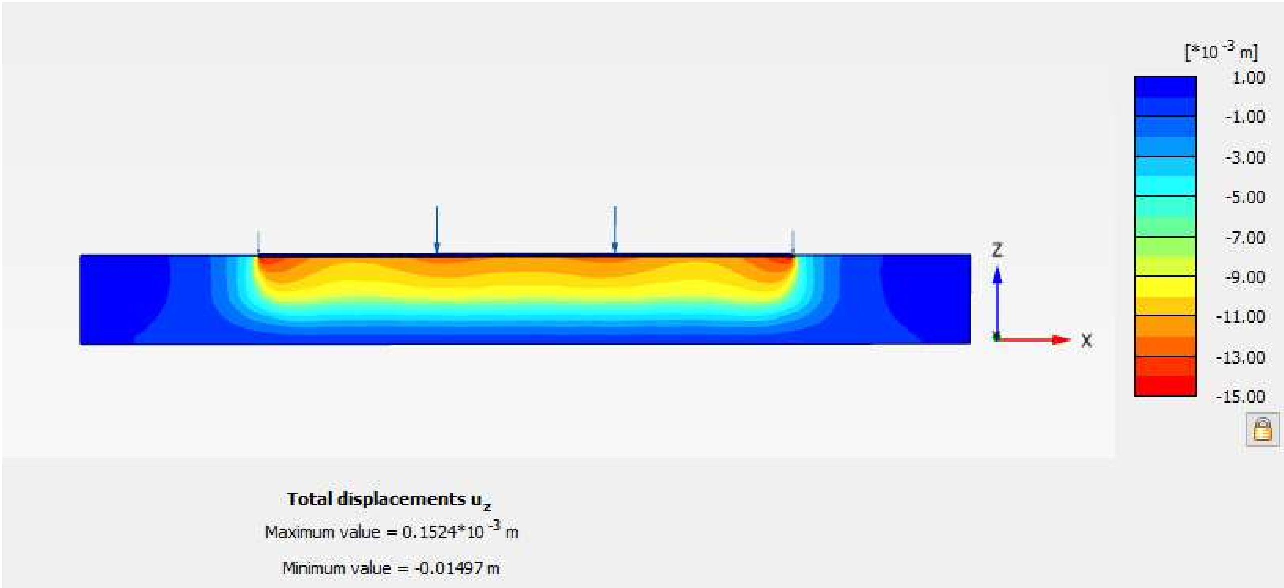


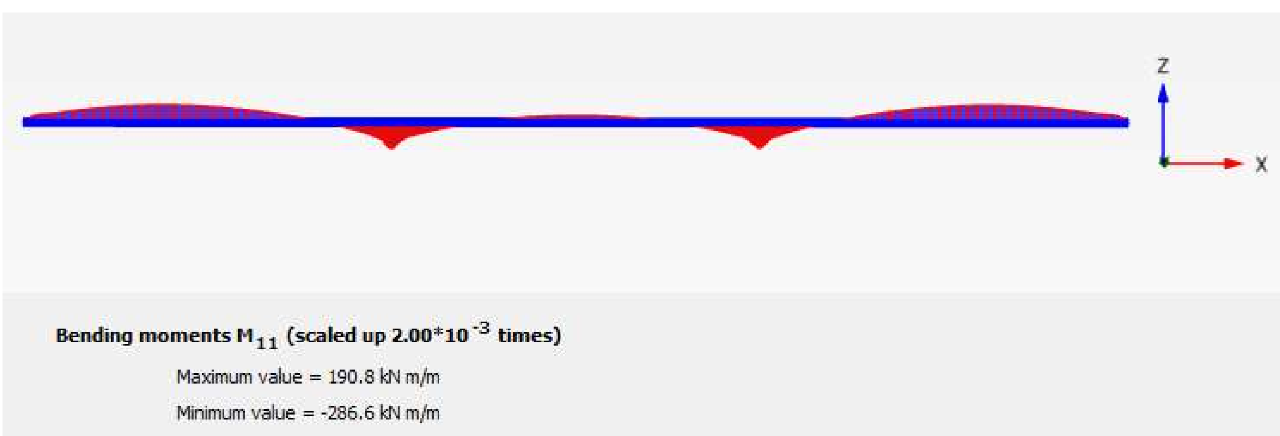
Bending moments M_{11} (scaled up $5.00*10^{-3}$ times)

Maximum value = 105.1 kN m/m

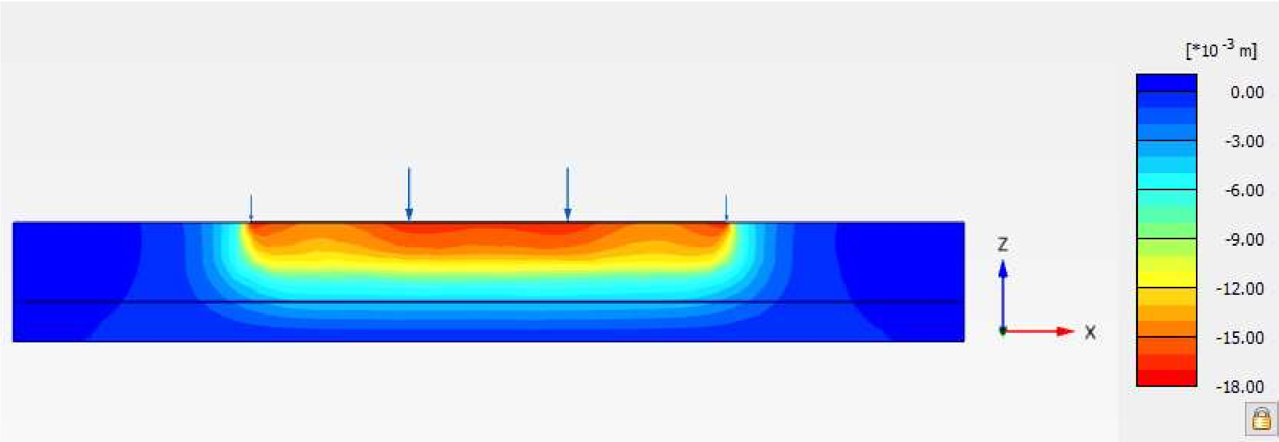
Minimum value = -151.5 kN m/m

B=2.0m, t=0.6m, L=24.0m, H_1=2.0m, H_2=2.0m, F=320kN, 2F=640kN strip footing

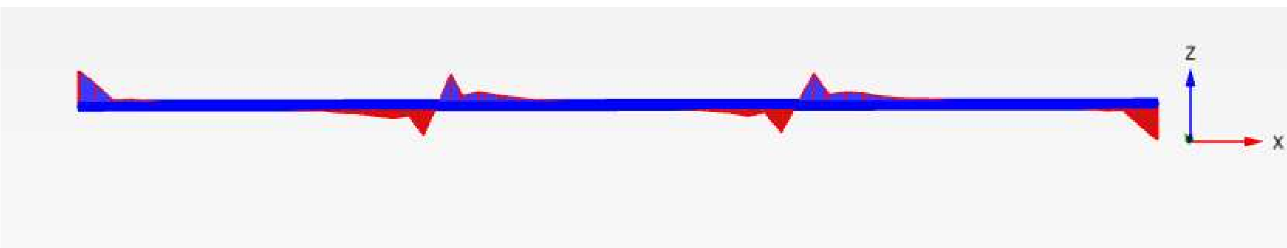




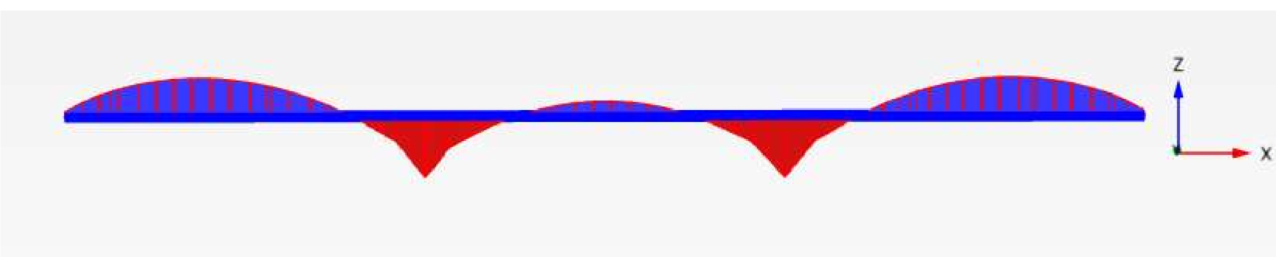
B=2.0m, t=0.6m, L=24.0m, H_1=2.0m, H_2=4.0m, F=320kN, 2F=640kN strip footing



Total displacements u_z
Maximum value = $0.1922*10^{-3}$ m
Minimum value = -0.01721 m

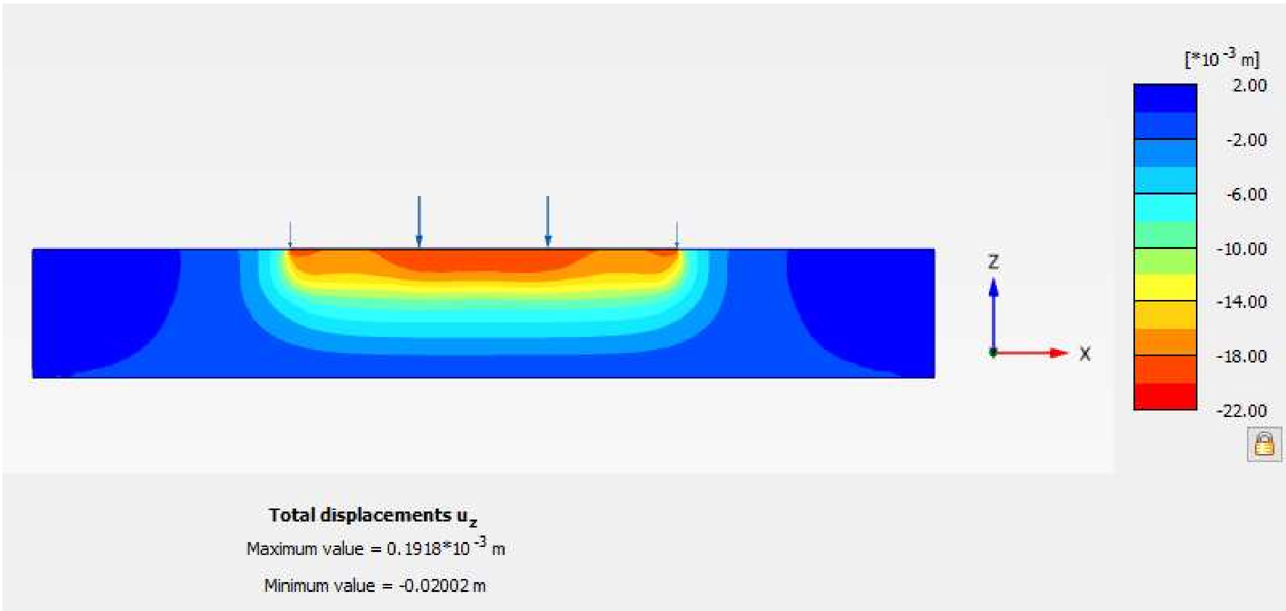


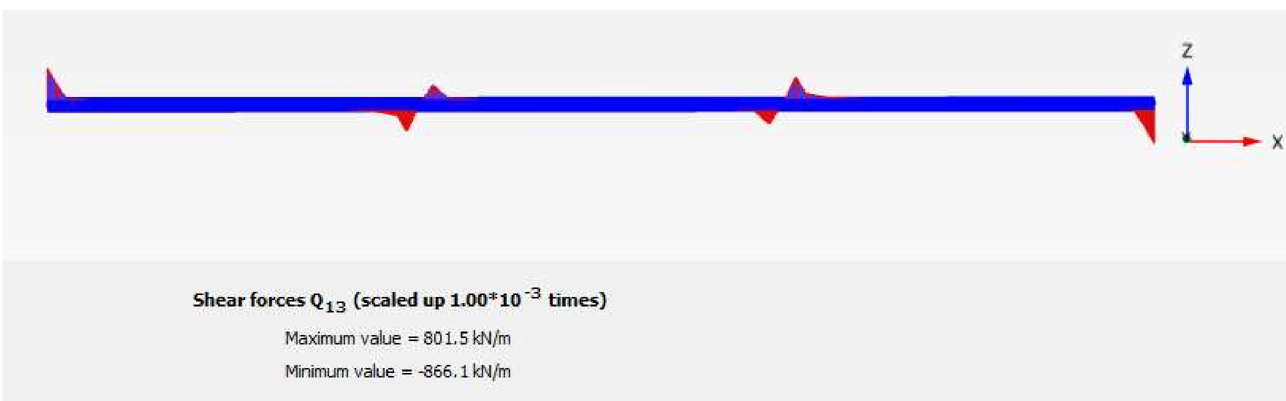
Shear forces Q_{13} (scaled up $2.00*10^{-3}$ times)
Maximum value = 384.3 kN/m
Minimum value = -393.7 kN/m

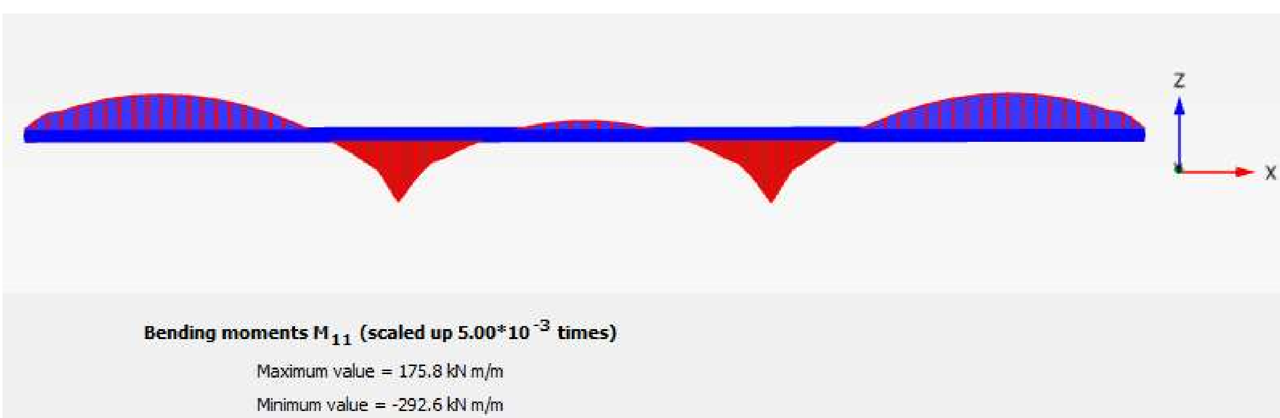


Bending moments M_{11} (scaled up $5.00*10^{-3}$ times)
Maximum value = 171.6 kN m/m
Minimum value = -273.8 kN m/m

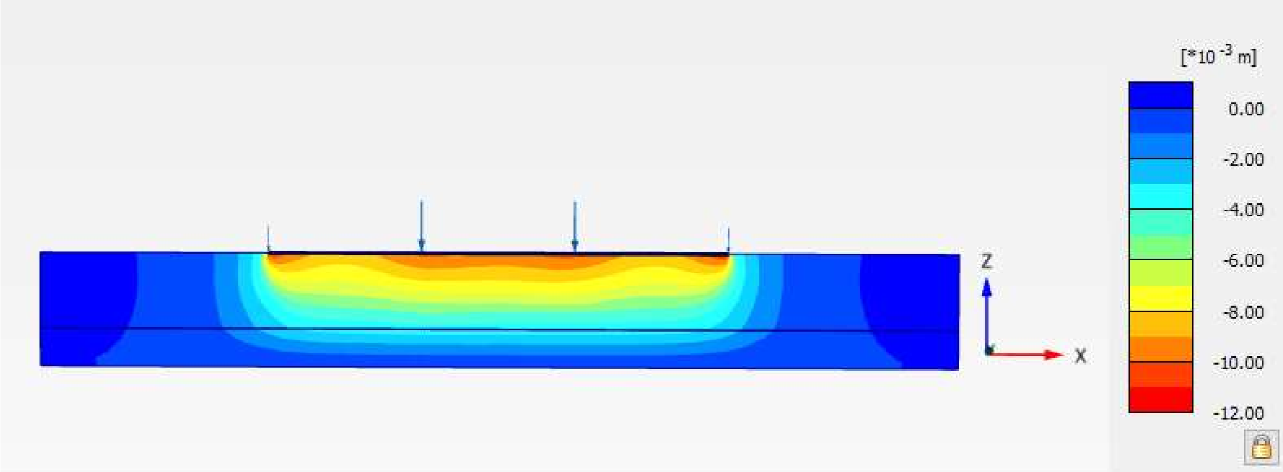
B=2.0m, t=0.6m, L=24.0m, H_1=2.0m, H_2=6.0m, F=320kN, 2F=640kN strip footing







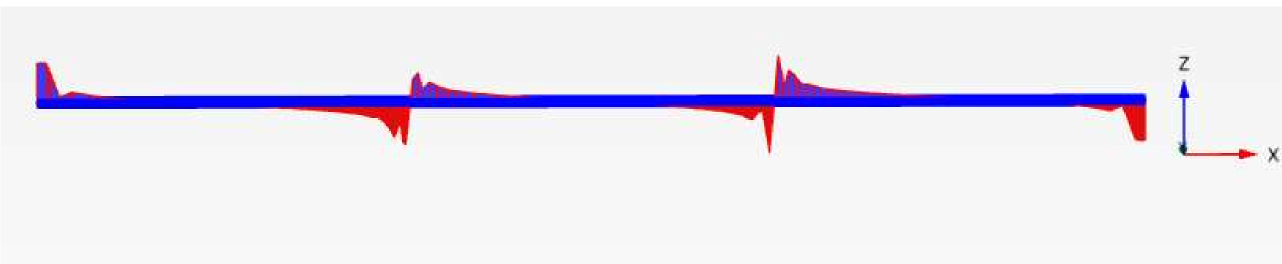
B=2.0m, t=0.6m, L=24.0m, H_1=4.0m, H_2=2.0m, F=320kN, 2F=640kN strip footing



Total displacements u_z

Maximum value = 0.09510×10^{-3} m

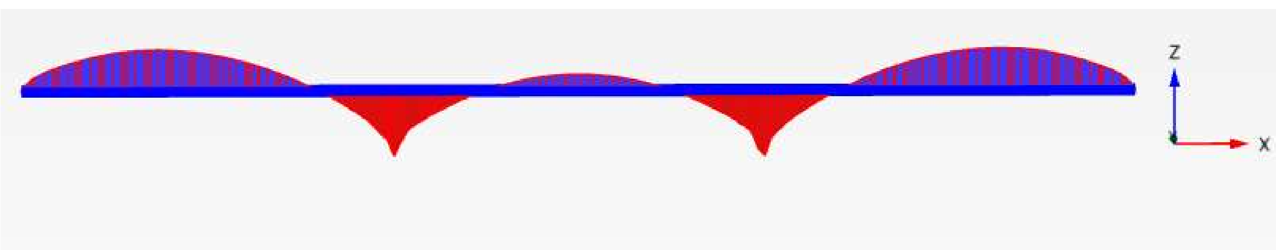
Minimum value = -0.01145 m



Shear forces Q_{13} (scaled up 2.00×10^{-3} times)

Maximum value = 498.2 kN/m

Minimum value = -571.0 kN/m

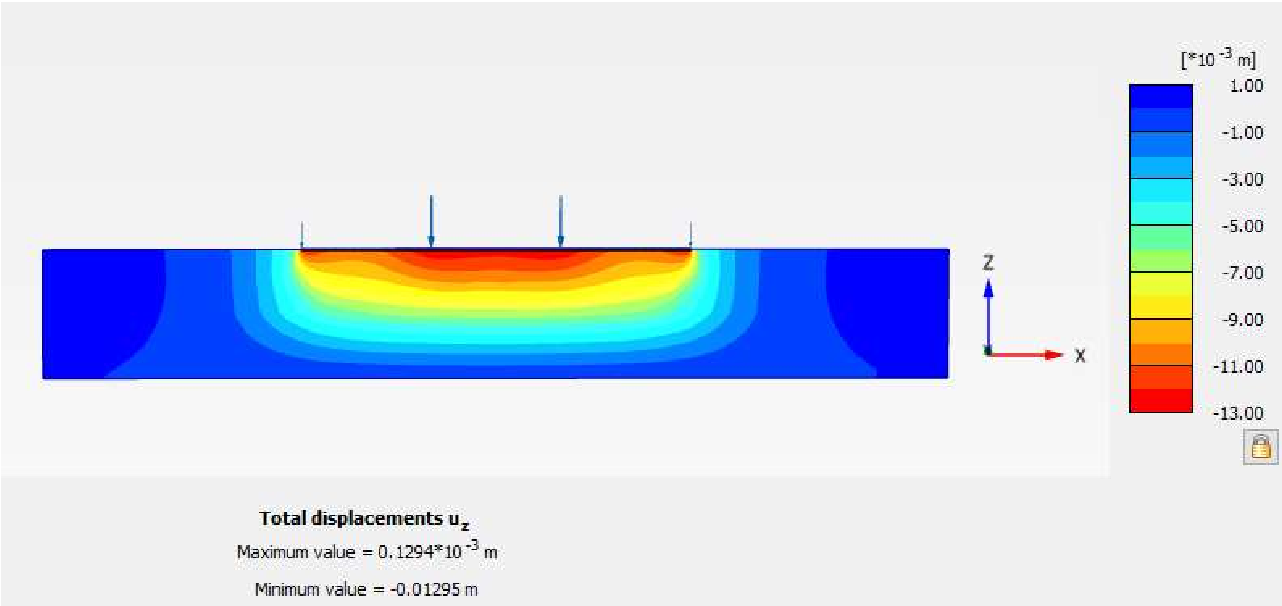


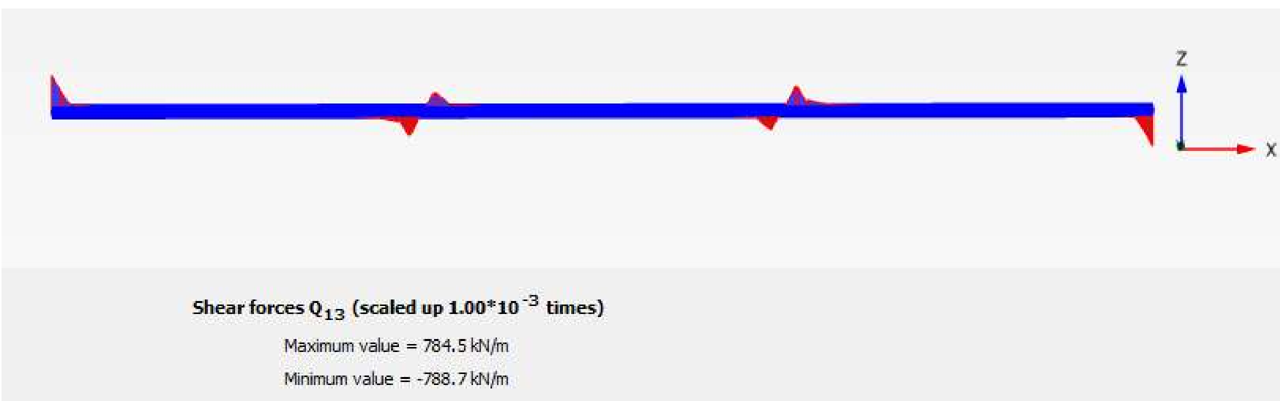
Bending moments M_{11} (scaled up 5.00×10^{-3} times)

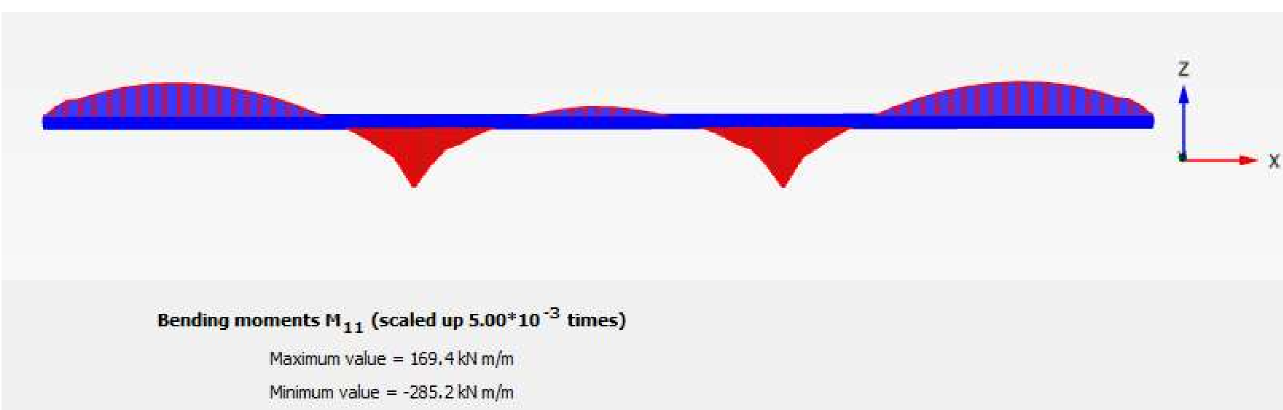
Maximum value = 181.8 kN m/m

Minimum value = -285.3 kN m/m

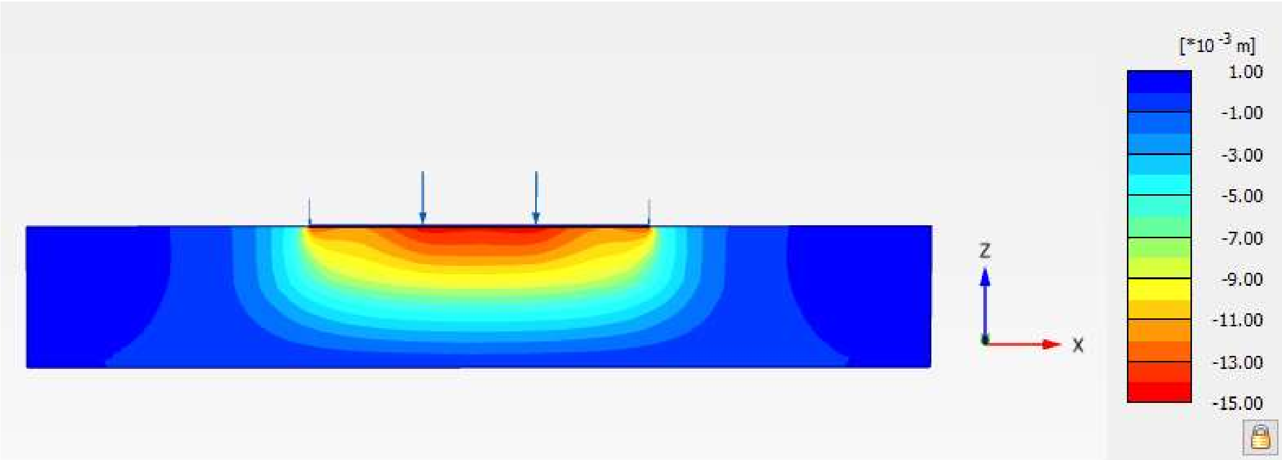
B=2.0m, t=0.6m, L=24.0m, H_1=4.0m, H_2=4.0m, F=320kN, 2F=640kN strip footing







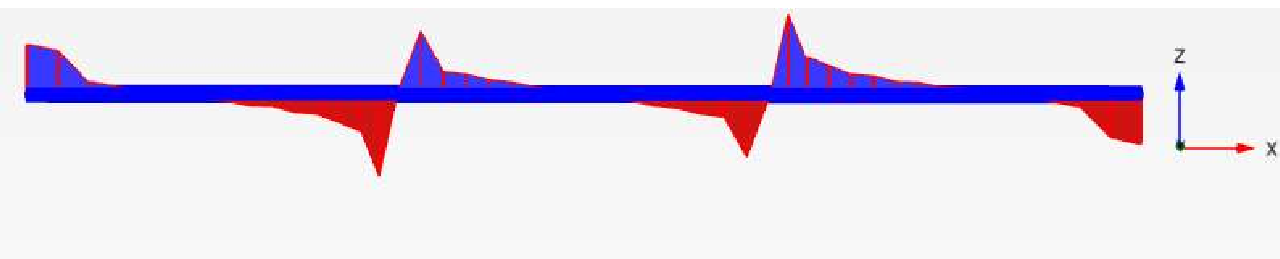
B=2.0m, t=0.6m, L=24.0m, H_1=4.0m, H_2=6.0m, F=320kN, 2F=640kN strip footing



Total displacements u_z

Maximum value = $0.1501*10^{-3}$ m

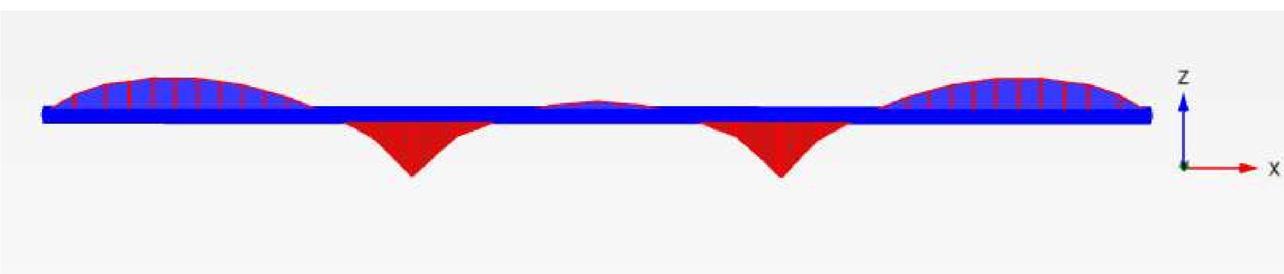
Minimum value = -0.01470 m



Shear forces Q_{13} (scaled up $5.00*10^{-3}$ times)

Maximum value = 342.4 kN/m

Minimum value = -354.3 kN/m

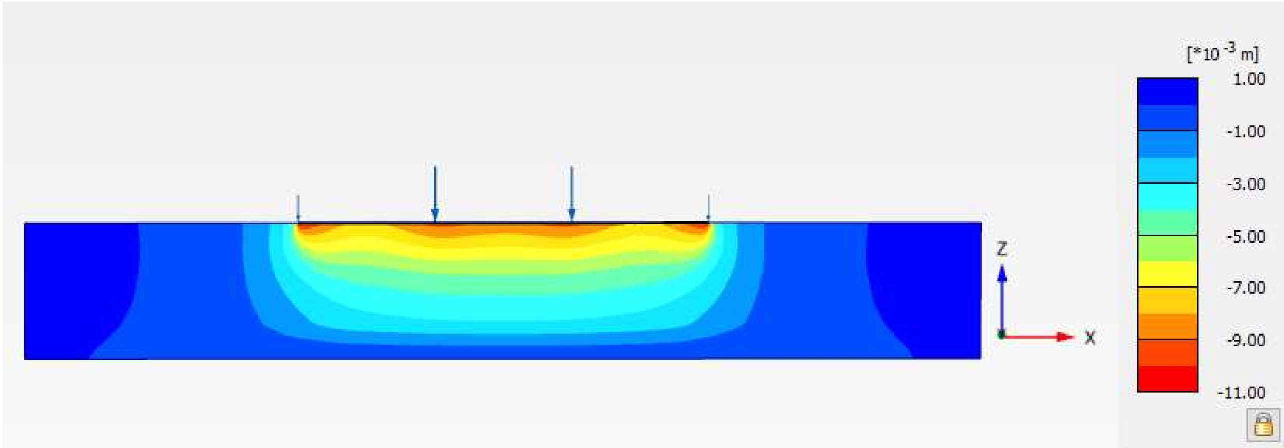


Bending moments M_{11} (scaled up $5.00*10^{-3}$ times)

Maximum value = 160.1 kN m/m

Minimum value = -269.4 kN m/m

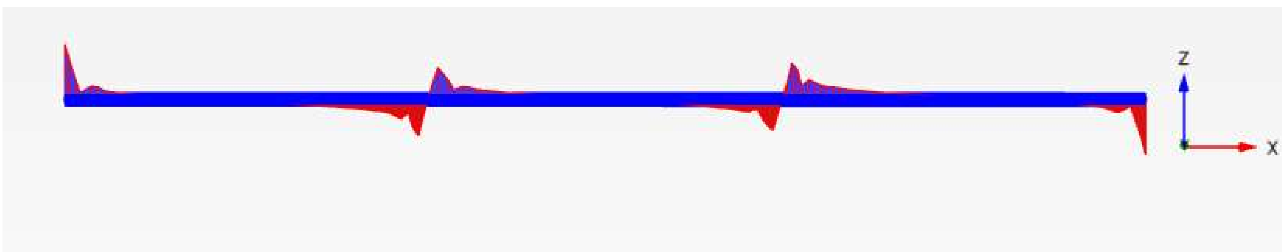
B=2.0m, t=0.6m, L=24.0m, H_1=6.0m, H_2=2.0m, F=320kN, 2F=640kN strip footing



Total displacements u_z

Maximum value = 0.06994*10^{-3} m

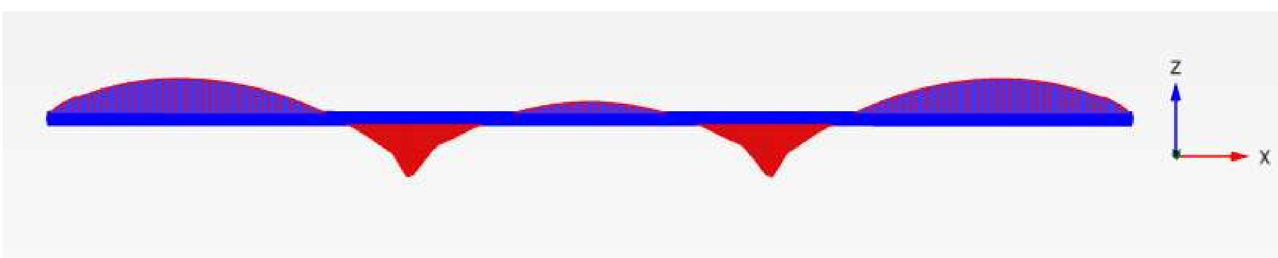
Minimum value = -0.01066 m



Shear forces Q_{13} (scaled up 2.00*10^{-3} times)

Maximum value = 612.6 kN/m

Minimum value = -614.4 kN/m

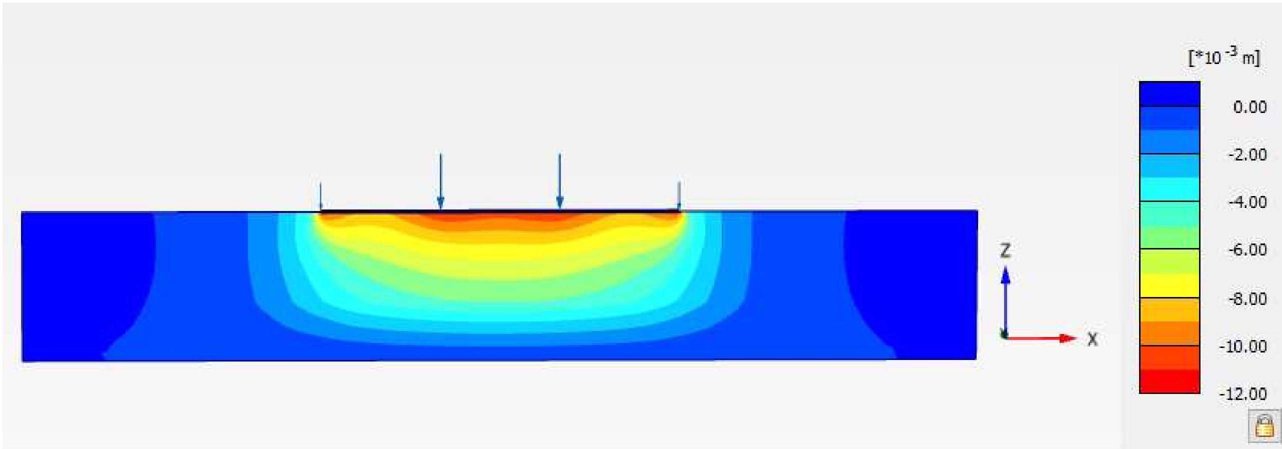


Bending moments M_{11} (scaled up 5.00*10^{-3} times)

Maximum value = 177.1 kN m/m

Minimum value = -258.0 kN m/m

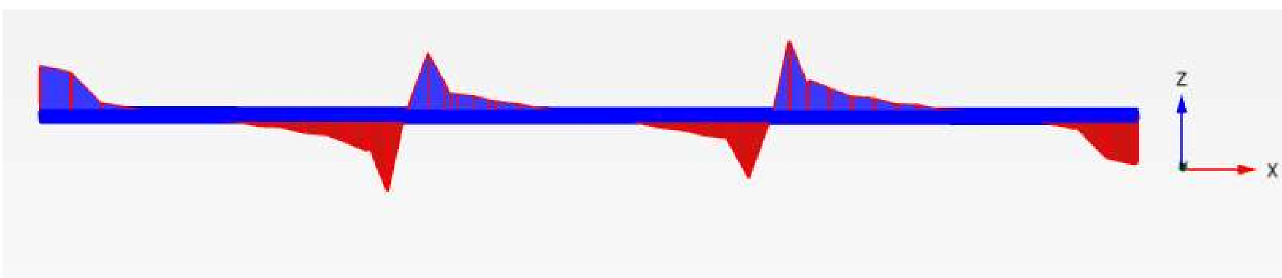
B=2.0m, t=0.6m, L=24.0m, H_1=6.0m, H_2=4.0m, F=320kN, 2F=640kN strip footing



Total displacements u_z

Maximum value = $0.1019*10^{-3}$ m

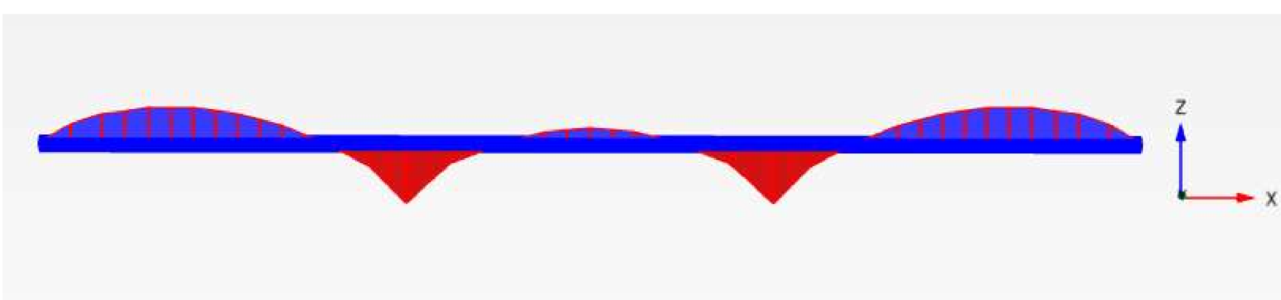
Minimum value = -0.01102 m



Shear forces Q_{13} (scaled up $5.00*10^{-3}$ times)

Maximum value = 330.2 kN/m

Minimum value = -334.8 kN/m

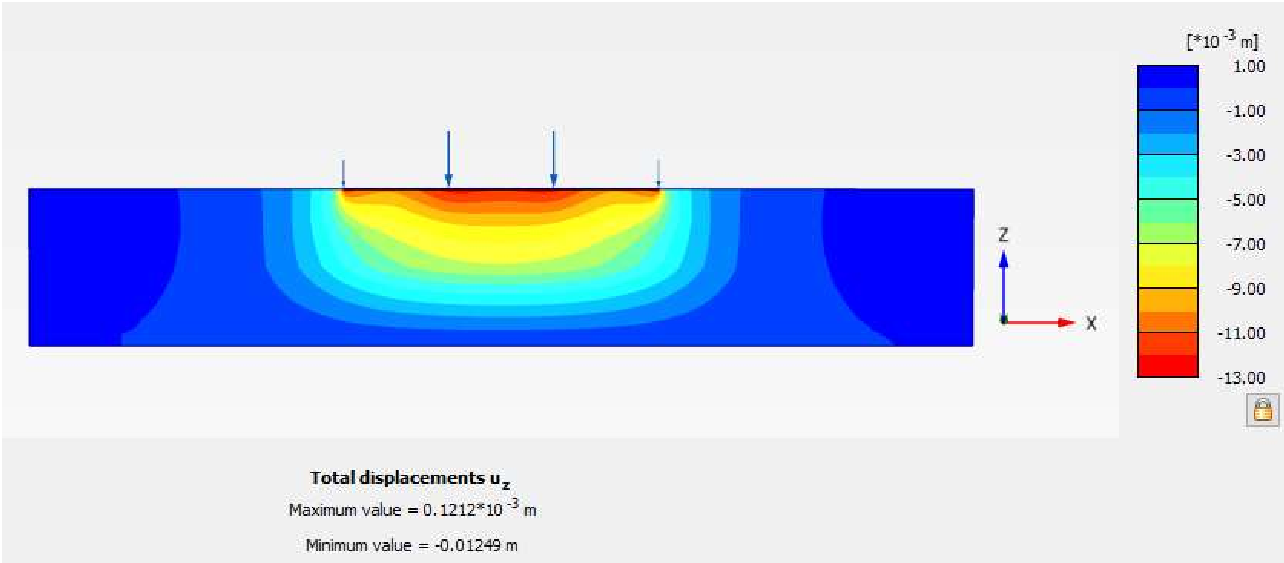


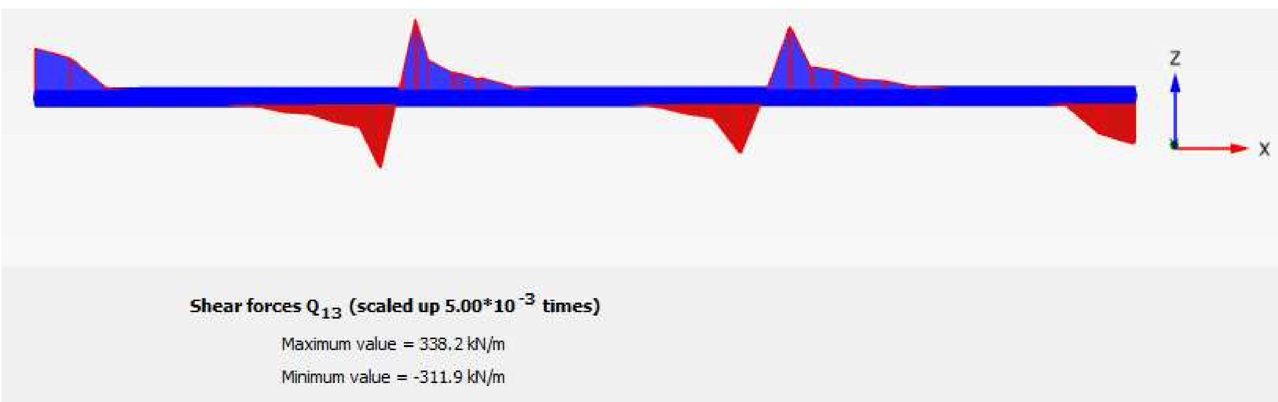
Bending moments M_{11} (scaled up $5.00*10^{-3}$ times)

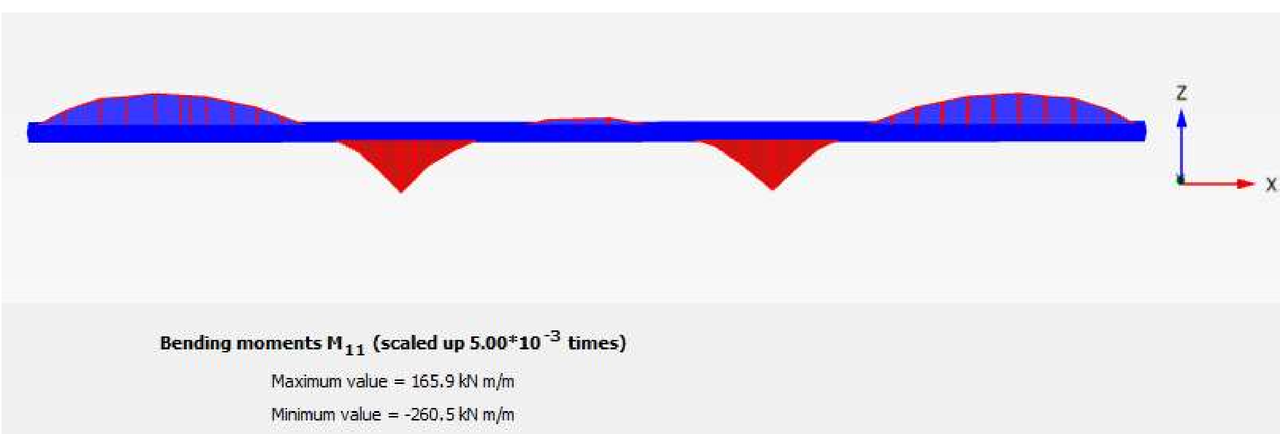
Maximum value = 162.0 kN m/m

Minimum value = -258.3 kN m/m

B=2.0m, t=0.6m, L=24.0m, H_1=6.0m, H_2=6.0m, F=320kN, 2F=640kN strip footing

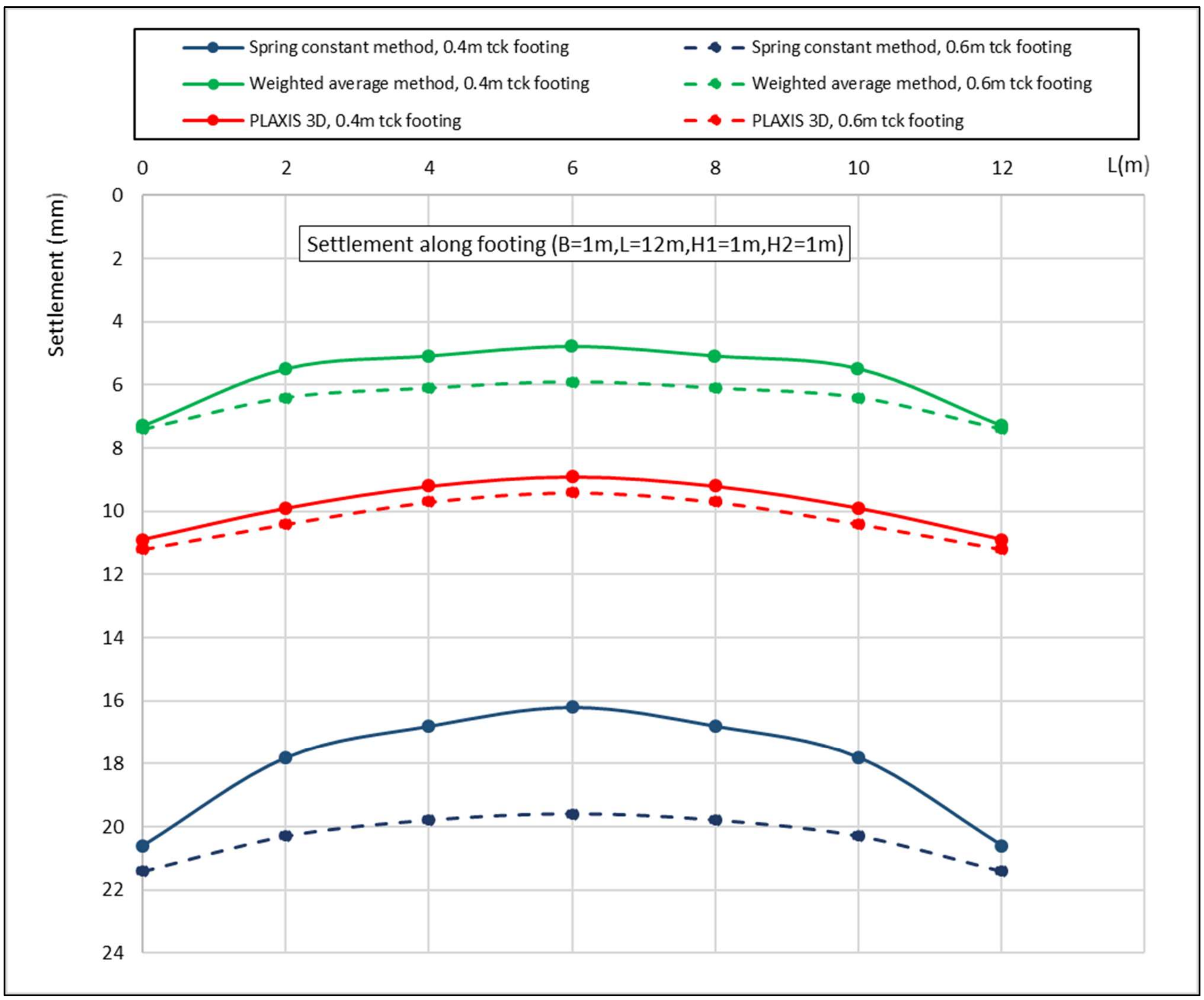




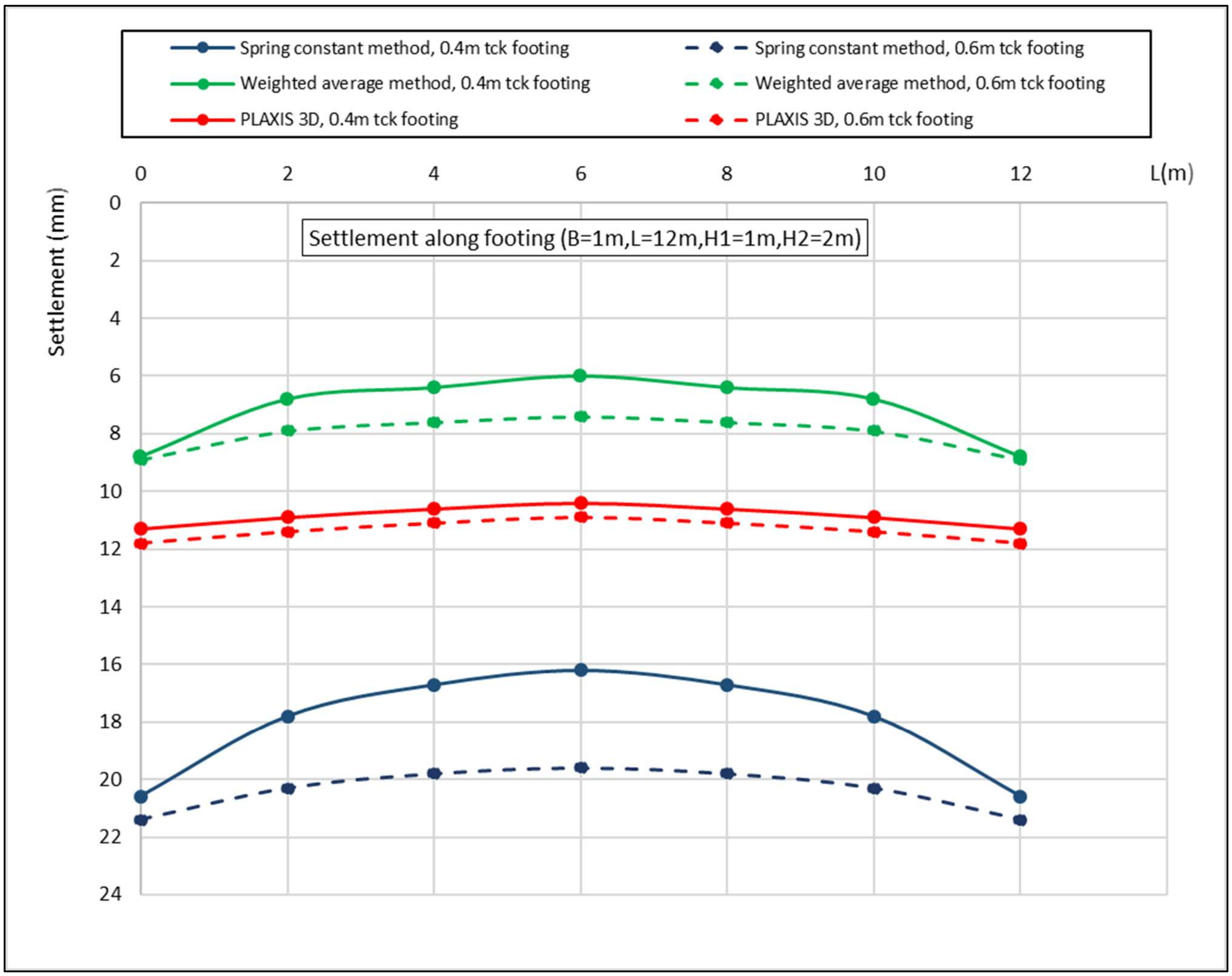


7.4 Settlement curves along footing

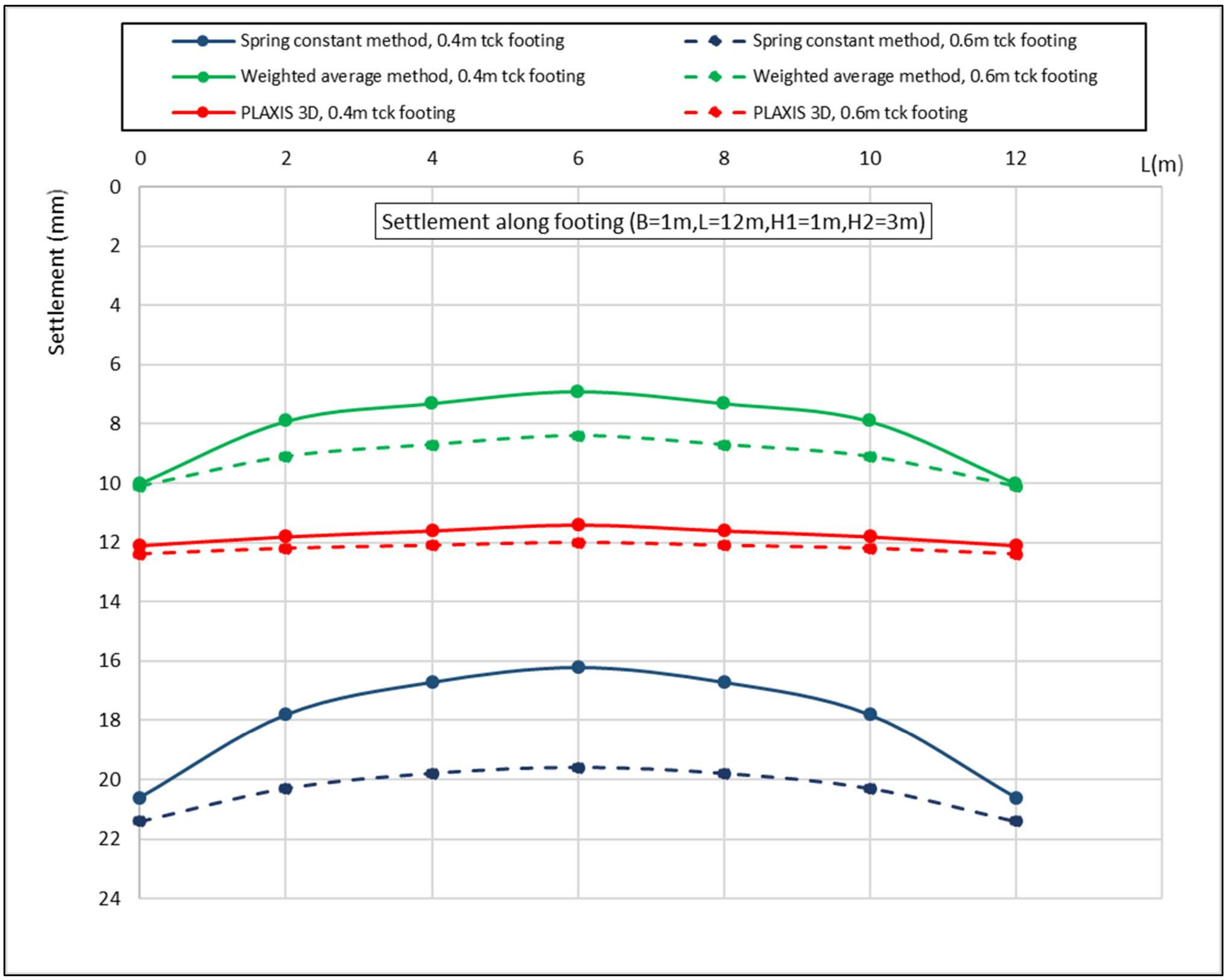
B=1.0m, L=12.0m, H_1=1.0m, H_2=1.0m, F=80kN, 2F=160kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	20.6	21.4	7.3	7.4	10.9	11.2
2	17.8	20.3	5.5	6.4	9.9	10.4
4	16.8	19.8	5.1	6.1	9.2	9.7
6	16.2	19.6	4.8	5.9	8.9	9.4
8	16.8	19.8	5.1	6.1	9.2	9.7
10	17.8	20.3	5.5	6.4	9.9	10.4
12	20.6	21.4	7.3	7.4	10.9	11.2



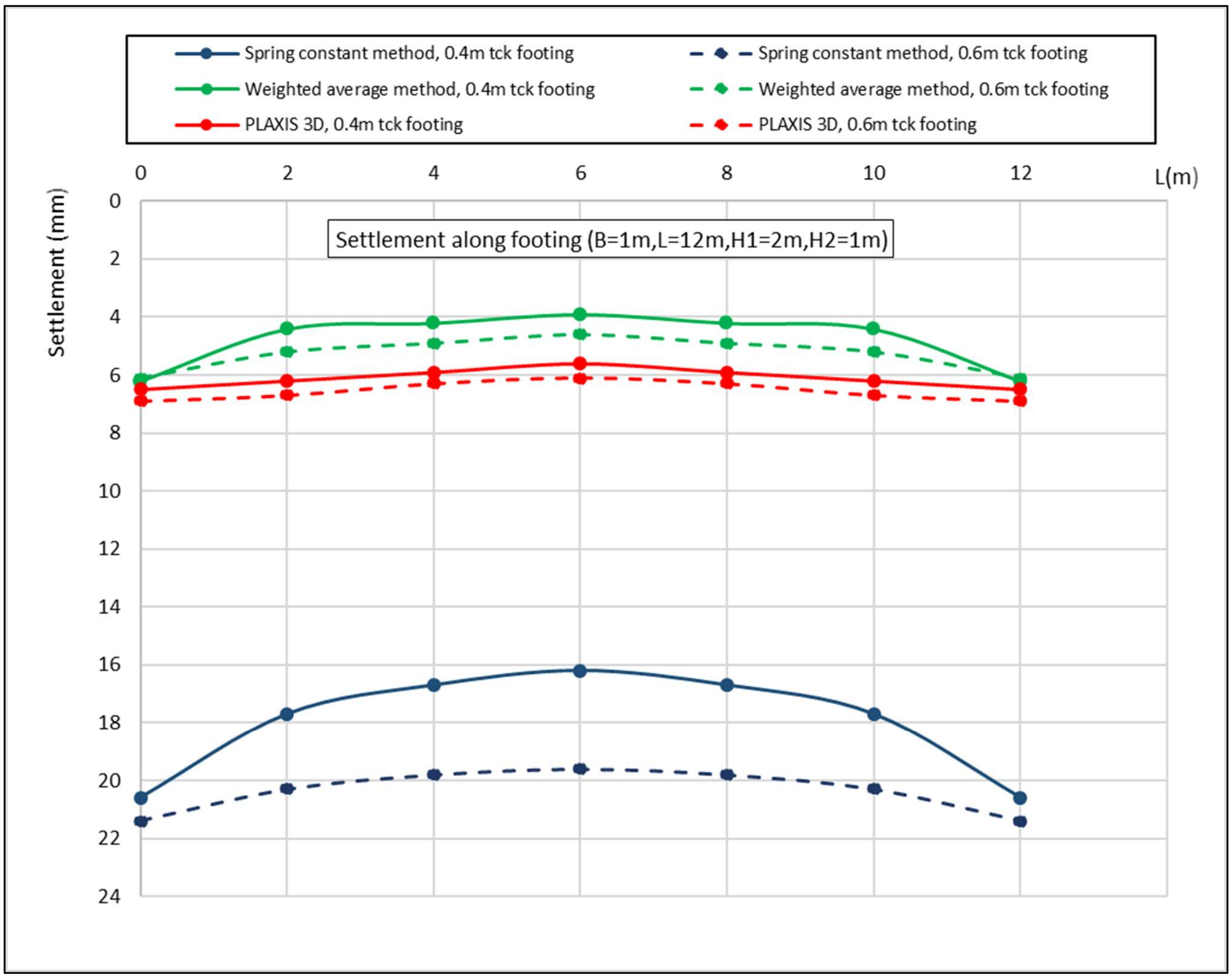
B=1.0m, L=12.0m, H_1=1.0m, H_2=2.0m, F=80kN, 2F=160kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	20.6	21.4	8.8	8.9	11.3	11.8
2	17.8	20.3	6.8	7.9	10.9	11.4
4	16.7	19.8	6.4	7.6	10.6	11.1
6	16.2	19.6	6.0	7.4	10.4	10.9
8	16.7	19.8	6.4	7.6	10.6	11.1
10	17.8	20.3	6.8	7.9	10.9	11.4
12	20.6	21.4	8.8	8.9	11.3	11.8



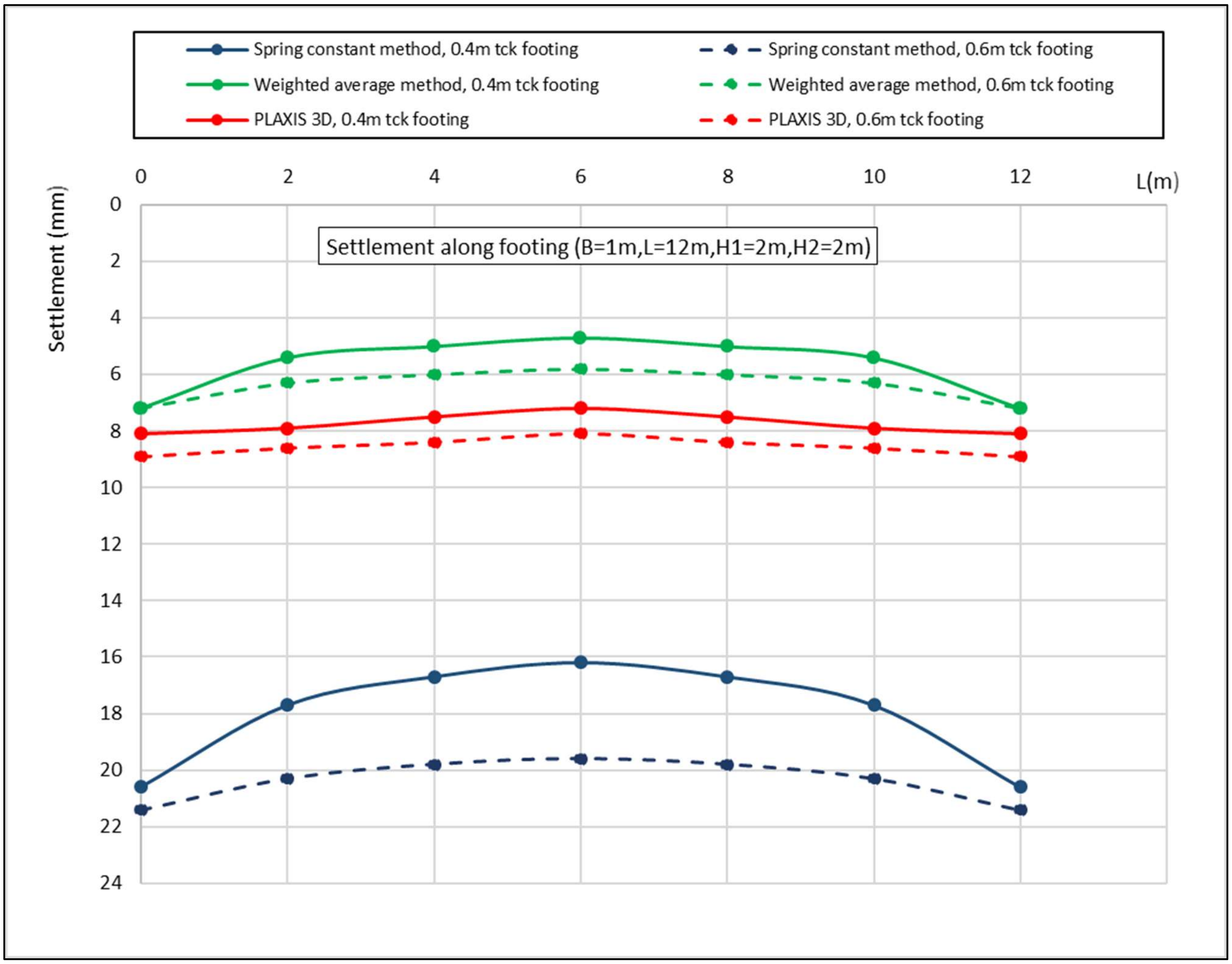
B=1.0m, L=12.0m, H_1=1.0m, H_2=3.0m, F=80kN, 2F=160kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	20.6	21.4	10	10.1	12.1	12.4
2	17.8	20.3	7.9	9.1	11.8	12.2
4	16.7	19.8	7.3	8.7	11.6	12.1
6	16.2	19.6	6.9	8.4	11.4	12.0
8	16.7	19.8	7.3	8.7	11.6	12.1
10	17.8	20.3	7.9	9.1	11.8	12.2
12	20.6	21.4	10	10.1	12.1	12.4



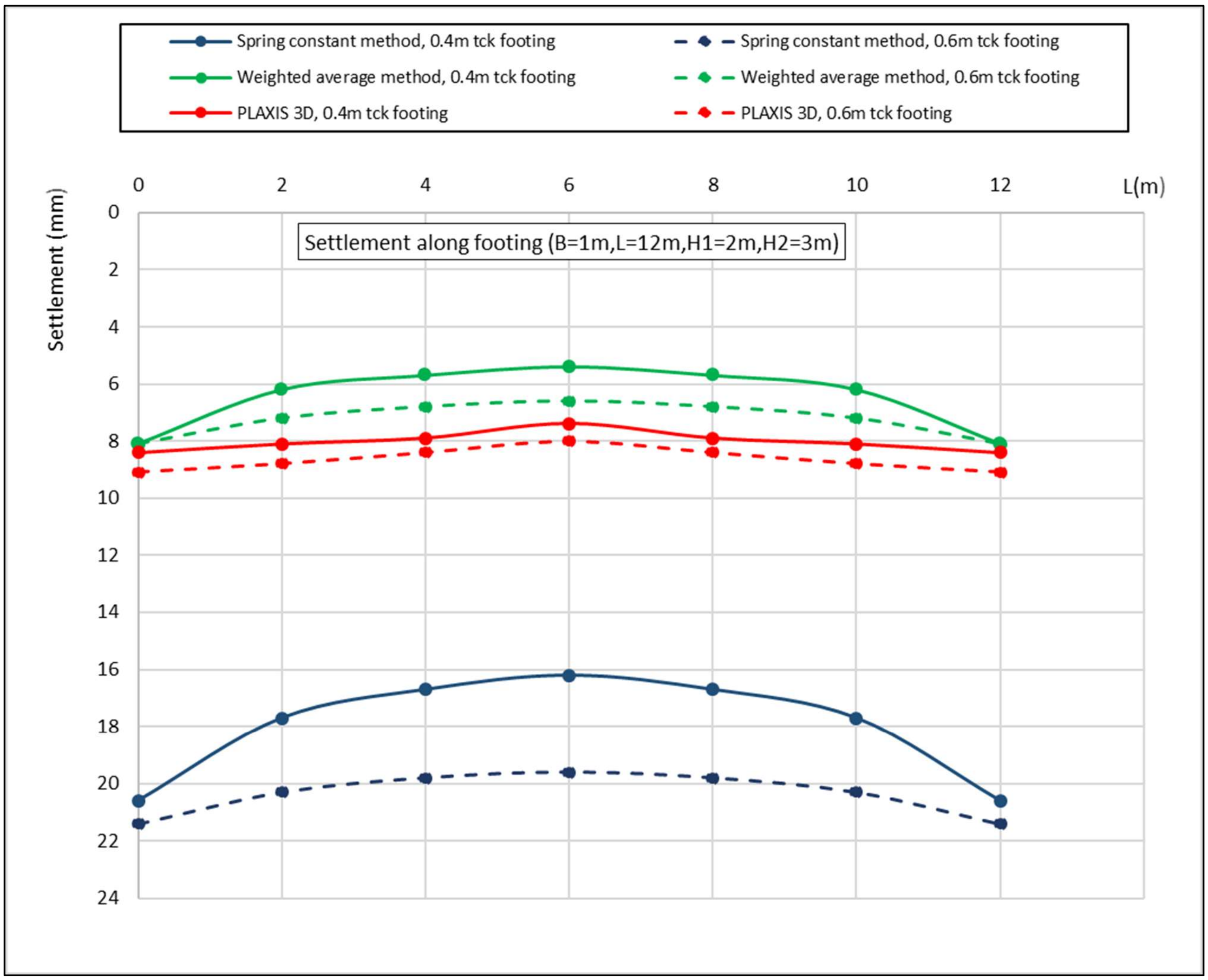
B=1.0m, L=12.0m, H_1=2.0m, H_2=1.0m, F=80kN, 2F=160kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	20.6	21.4	6.2	6.1	6.5	6.9
2	17.7	20.3	4.4	5.2	6.2	6.7
4	16.7	19.8	4.2	4.9	5.9	6.3
6	16.2	19.6	3.9	4.6	5.6	6.1
8	16.7	19.8	4.2	4.9	5.9	6.3
10	17.7	20.3	4.4	5.2	6.2	6.7
12	20.6	21.4	6.2	6.1	6.5	6.9



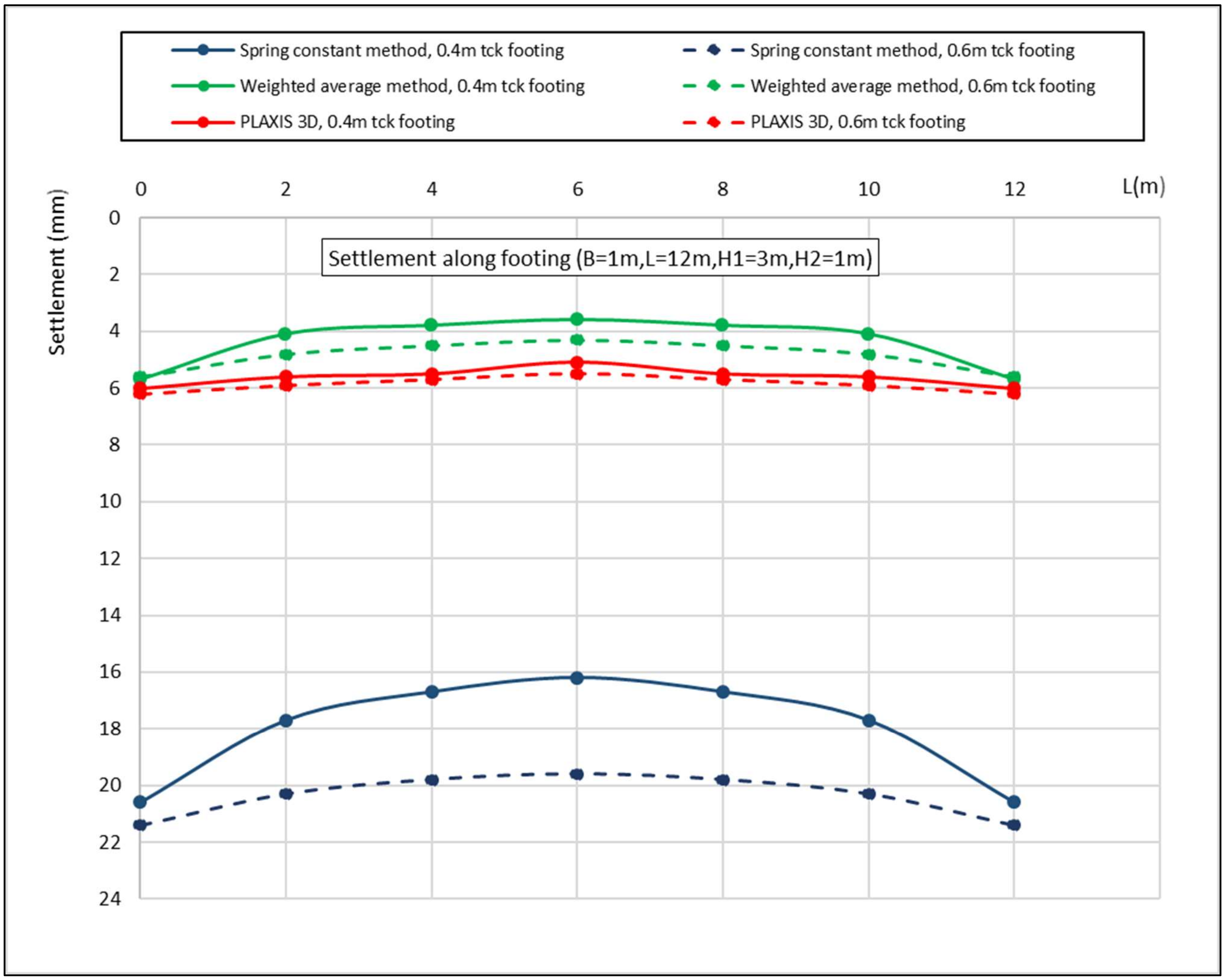
B=1.0m, L=12.0m, H_1=2.0m, H_2=2.0m, F=80kN, 2F=160kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	20.6	21.4	7.2	7.2	8.1	8.9
2	17.7	20.3	5.4	6.3	7.9	8.6
4	16.7	19.8	5.0	6.0	7.5	8.4
6	16.2	19.6	4.7	5.8	7.2	8.1
8	16.7	19.8	5.0	6.0	7.5	8.4
10	17.7	20.3	5.4	6.3	7.9	8.6
12	20.6	21.4	7.2	7.2	8.1	8.9



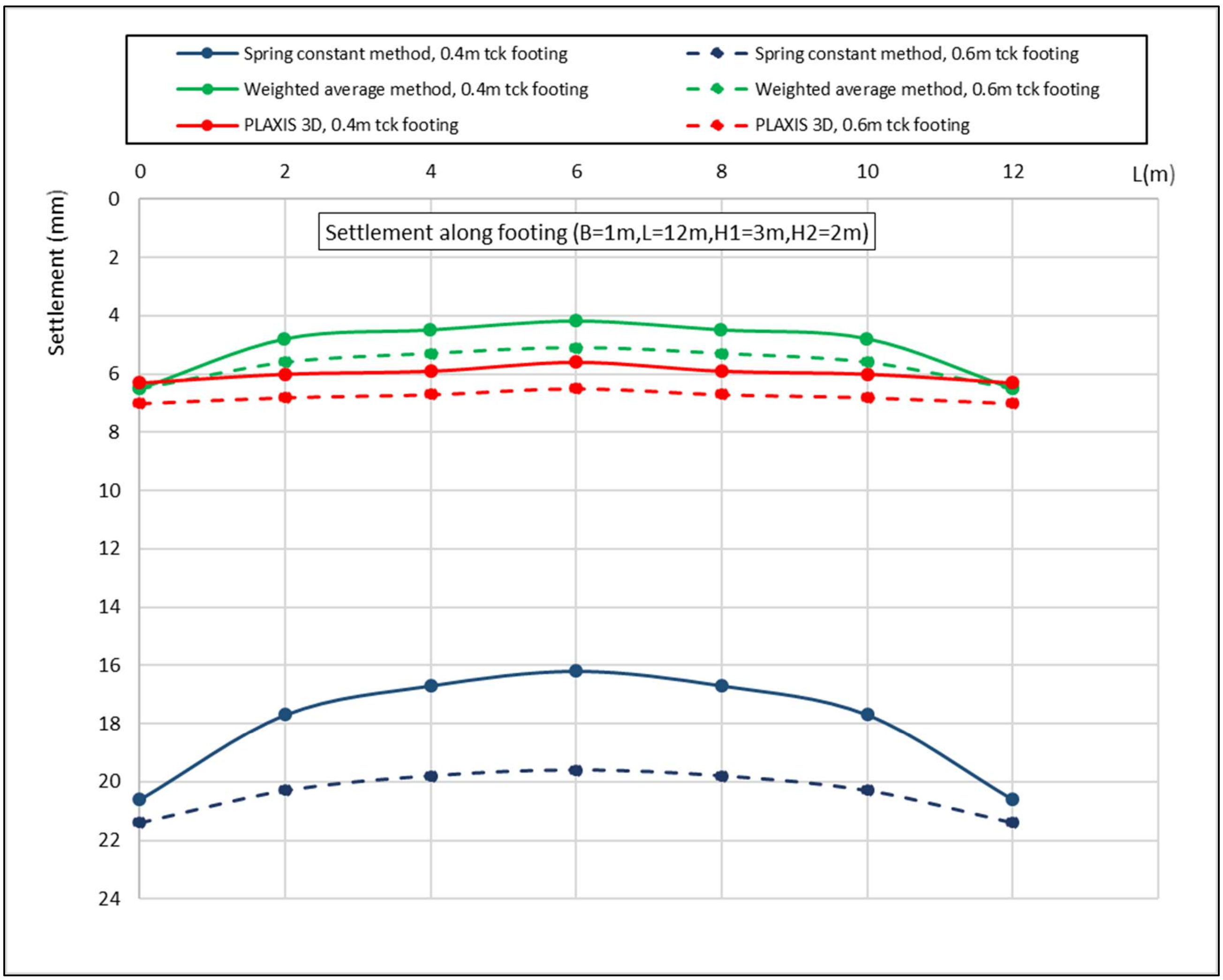
B=1.0m, L=12.0m, H_1=2.0m, H_2=3.0m, F=80kN, 2F=160kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	20.6	21.4	8.1	8.1	8.4	9.1
2	17.7	20.3	6.2	7.2	8.1	8.8
4	16.7	19.8	5.7	6.8	7.9	8.4
6	16.2	19.6	5.4	6.6	7.4	8
8	16.7	19.8	5.7	6.8	7.9	8.4
10	17.7	20.3	6.2	7.2	8.1	8.8
12	20.6	21.4	8.1	8.1	8.4	9.1



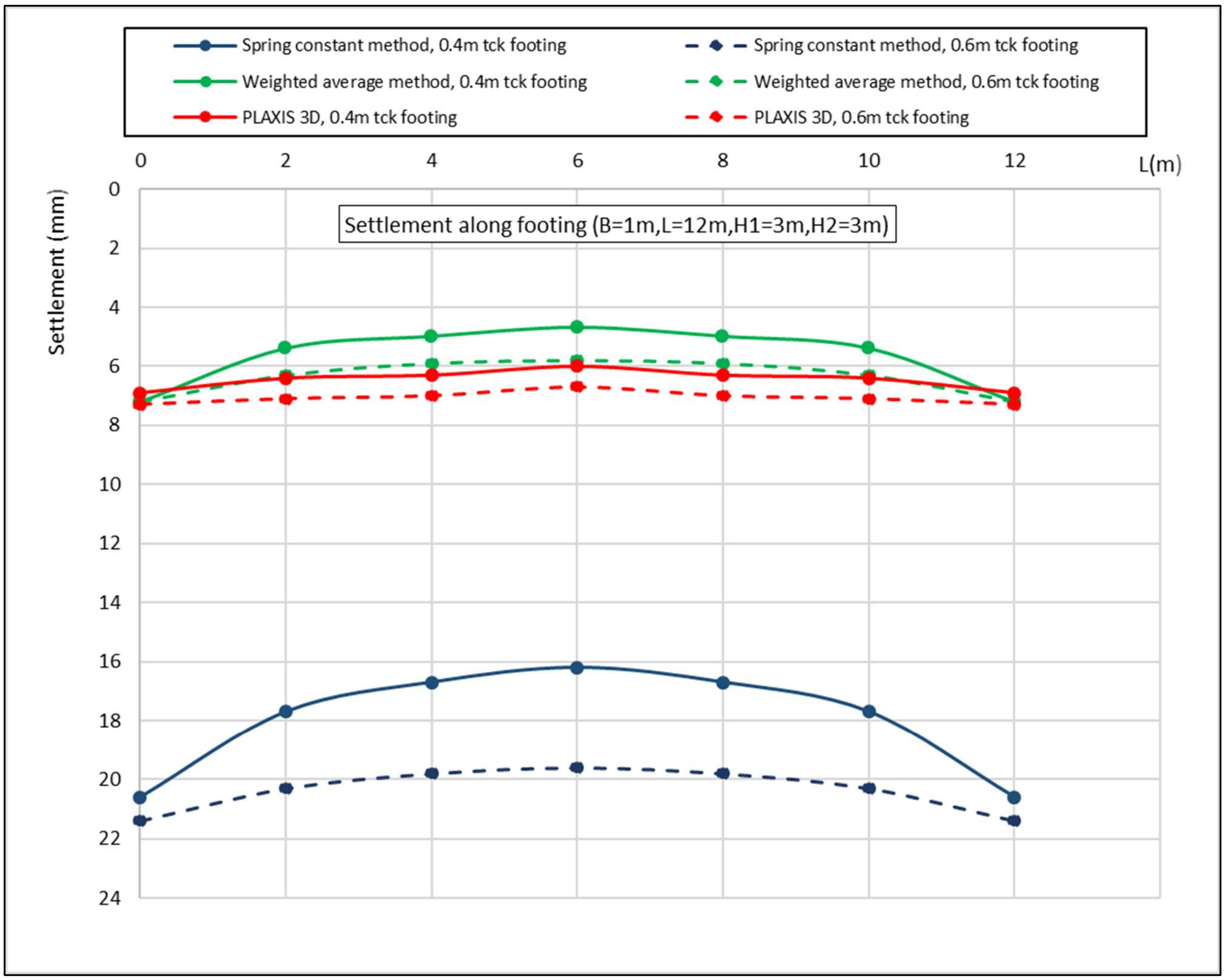
B=1.0m, L=12.0m, H_1=3.0m, H_2=1.0m, F=80kN, 2F=160kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	20.6	21.4	5.7	5.6	6.0	6.2
2	17.7	20.3	4.1	4.8	5.6	5.9
4	16.7	19.8	3.8	4.5	5.5	5.7
6	16.2	19.6	3.6	4.3	5.1	5.5
8	16.7	19.8	3.8	4.5	5.5	5.7
10	17.7	20.3	4.1	4.8	5.6	5.9
12	20.6	21.4	5.7	5.6	6.0	6.2



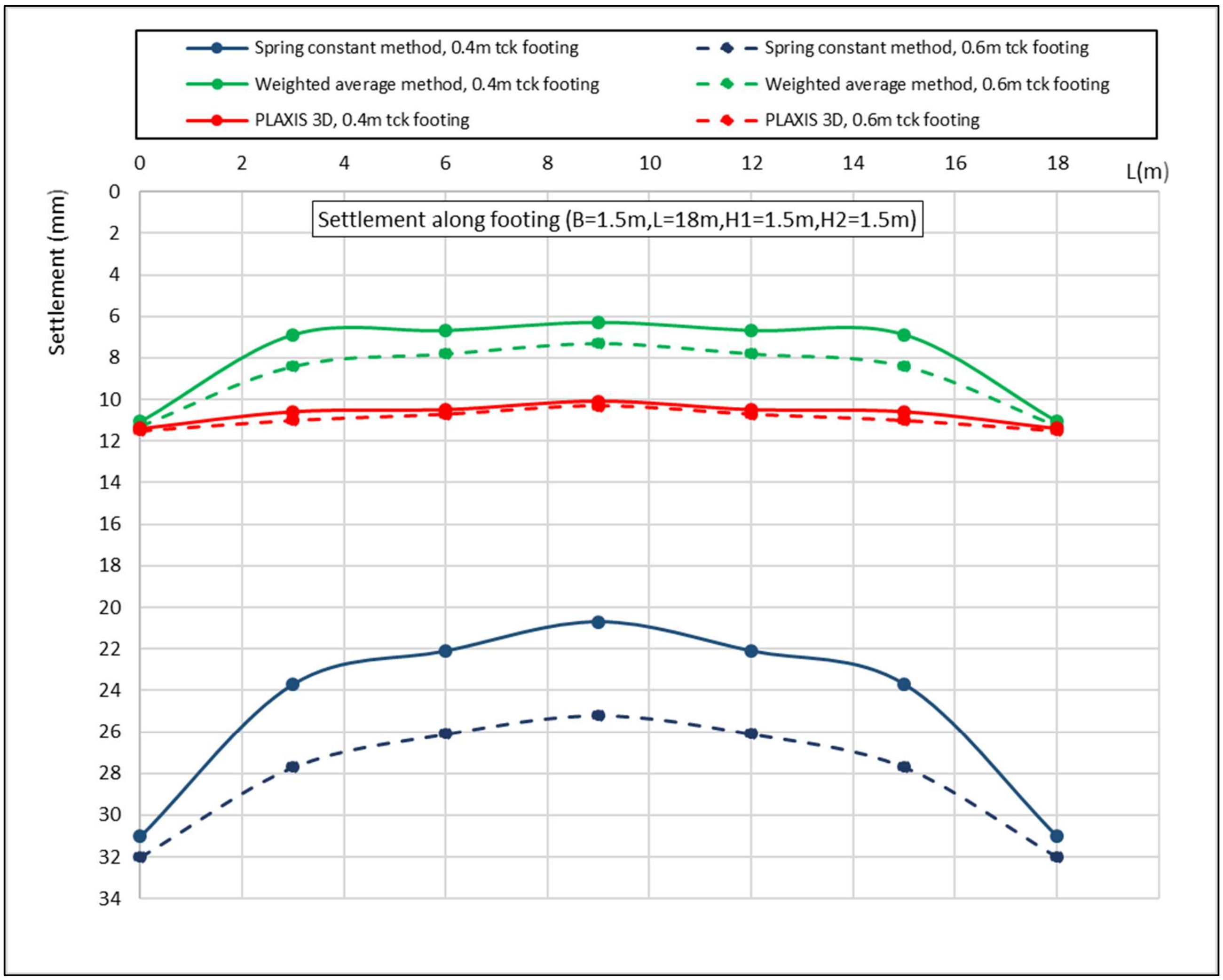
B=1.0m, L=12.0m, H_1=3.0m, H_2=2.0m, F=80kN, 2F=160kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	20.6	21.4	6.5	6.5	6.3	7.0
2	17.7	20.3	4.8	5.6	6.0	6.8
4	16.7	19.8	4.5	5.3	5.9	6.7
6	16.2	19.6	4.2	5.1	5.6	6.5
8	16.7	19.8	4.5	5.3	5.9	6.7
10	17.7	20.3	4.8	5.6	6.0	6.8
12	20.6	21.4	6.5	6.5	6.3	7.0



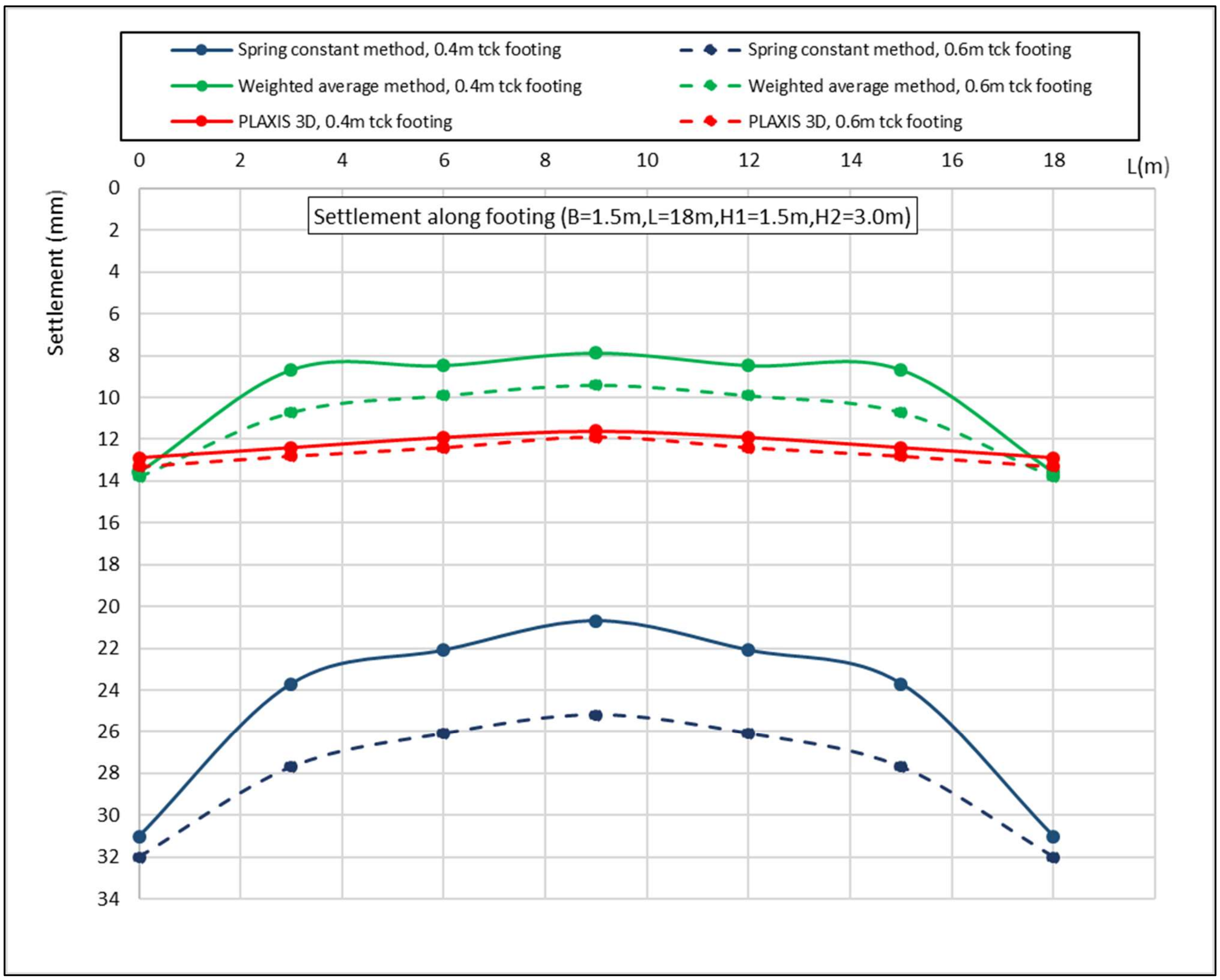
B=1.0m, L=12.0m, H_1=3.0m, H_2=3.0m, F=80kN, 2F=160kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	20.6	21.4	7.2	7.2	6.9	7.3
2	17.7	20.3	5.4	6.3	6.4	7.1
4	16.7	19.8	5.0	5.9	6.3	7.0
6	16.2	19.6	4.7	5.8	6.0	6.7
8	16.7	19.8	5.0	5.9	6.3	7.0
10	17.7	20.3	5.4	6.3	6.4	7.1
12	20.6	21.4	7.2	7.2	6.9	7.3



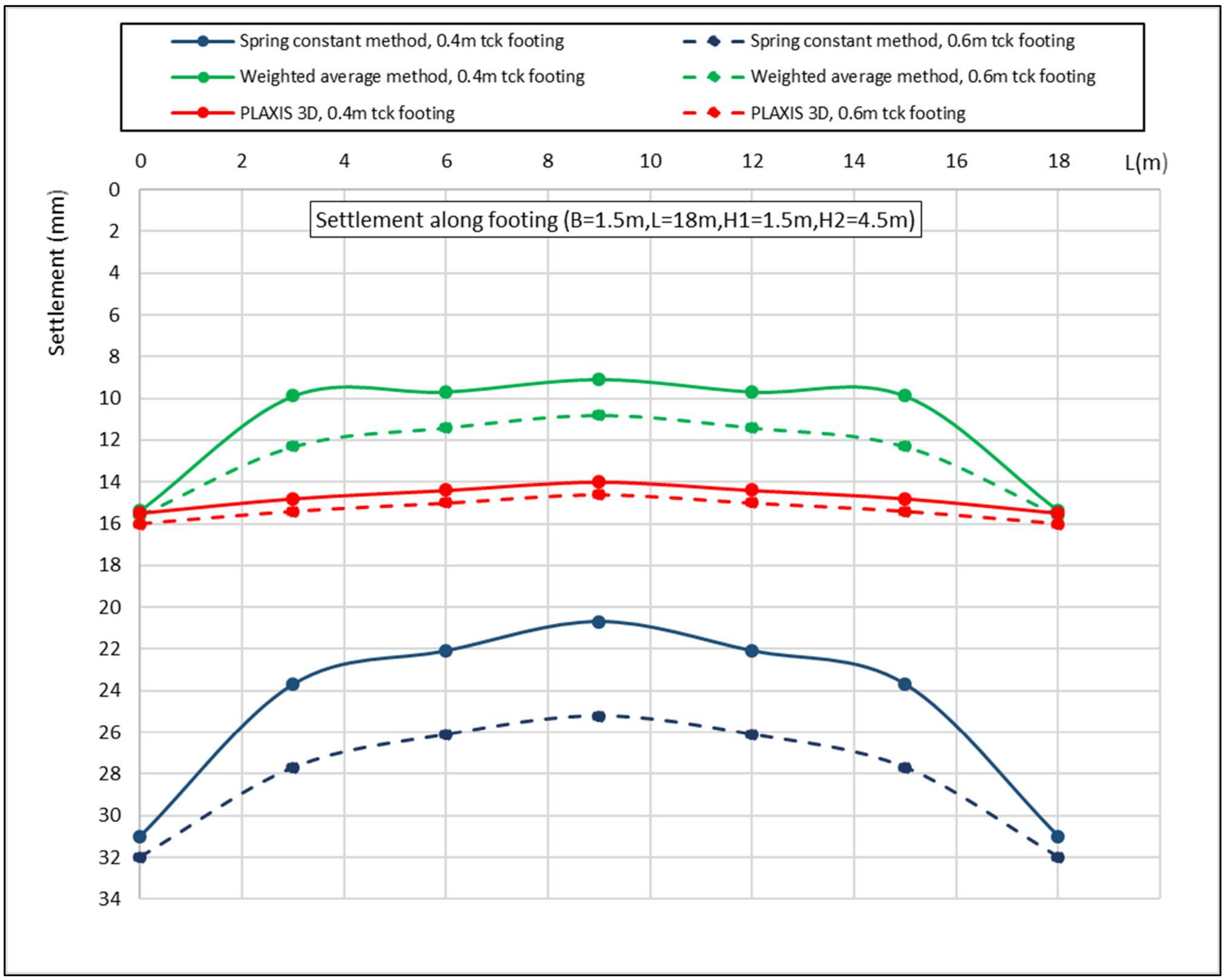
B=1.5m, L=18.0m, H_1=1.5m, H_2=1.5m, F=180kN, 2F=360kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	31.0	32.0	11.1	11.3	11.4	11.5
3	23.7	27.7	6.9	8.4	10.6	11
6	22.1	26.1	6.7	7.8	10.5	10.7
9	20.7	25.2	6.3	7.3	10.1	10.3
12	22.1	26.1	6.7	7.8	10.5	10.7
15	23.7	27.7	6.9	8.4	10.6	11
18	31.0	32.0	11.1	11.3	11.4	11.5



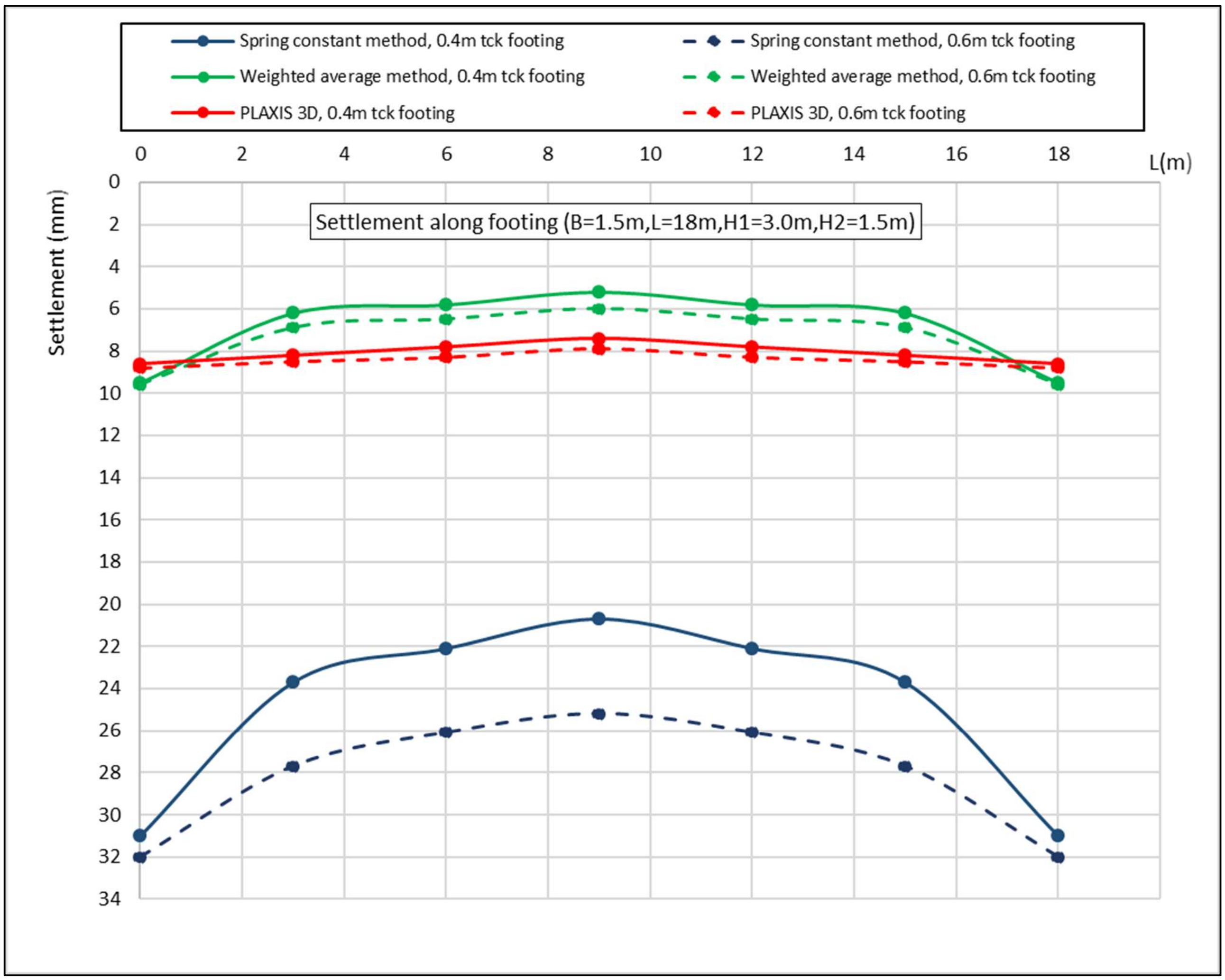
B=1.5m, L=18.0m, H_1=1.5m, H_2=3.0m, F=180kN, 2F=360kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	31.0	32.0	14.6	13.8	12.9	13.3
3	23.7	27.7	8.7	10.7	12.4	12.8
6	22.1	26.1	8.5	9.9	11.9	12.4
9	20.7	25.2	7.9	9.4	11.6	11.9
12	22.1	26.1	8.5	9.9	11.9	12.4
15	23.7	27.7	8.7	10.7	12.4	12.8
18	31.0	32.0	14.6	13.8	12.9	13.3



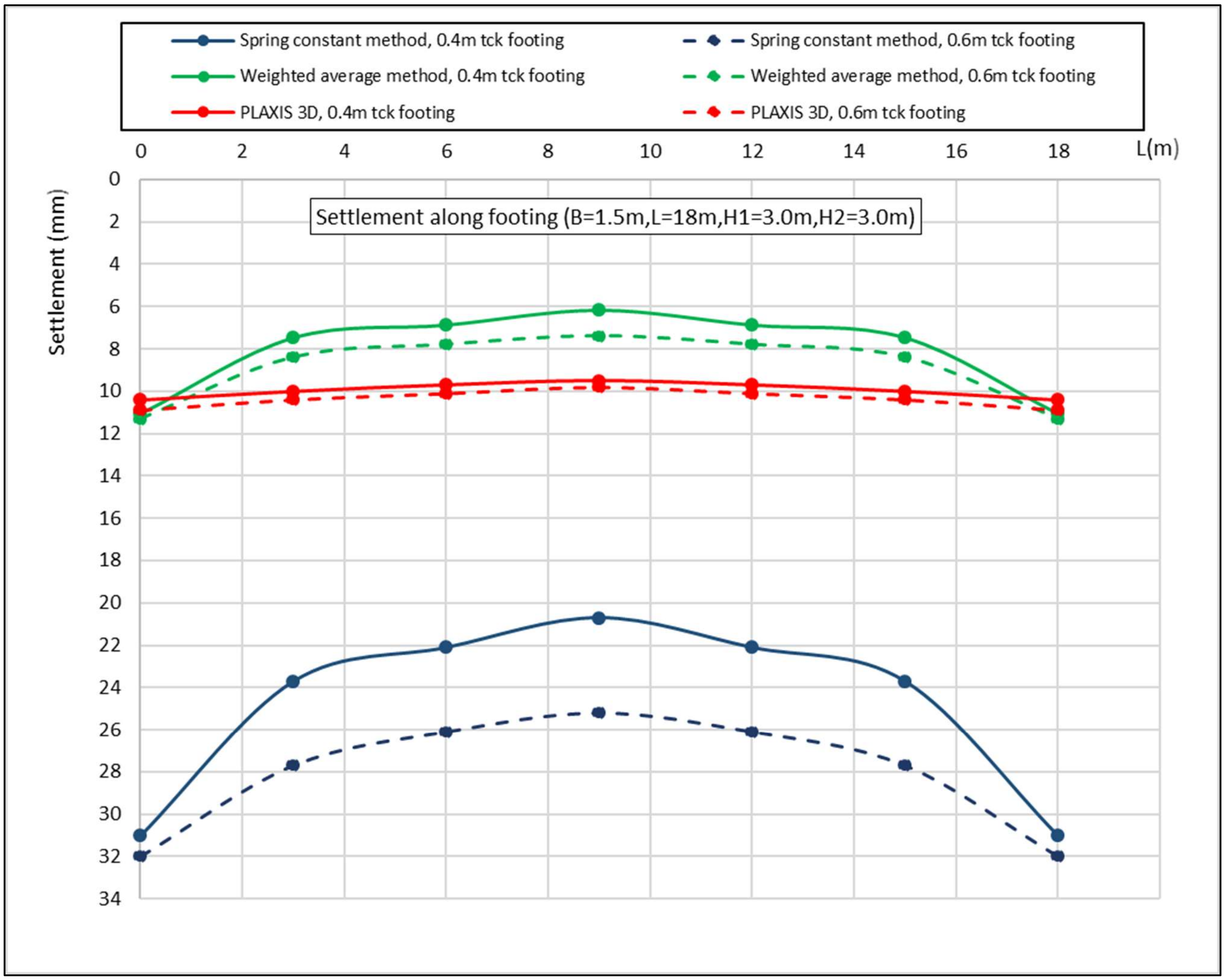
B=1.5m, L=18.0m, H_1=1.5m, H_2=4.5m, F=180kN, 2F=360kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	31	32	15.4	15.6	15.5	16
3	23.7	27.7	9.9	12.3	14.8	15.4
6	22.1	26.1	9.7	11.4	14.4	15.0
9	20.7	25.2	9.1	10.8	14.0	14.6
12	22.1	26.1	9.7	11.4	14.4	15.0
15	23.7	27.7	9.9	12.3	14.8	15.4
18	31	32	15.4	15.6	15.5	16



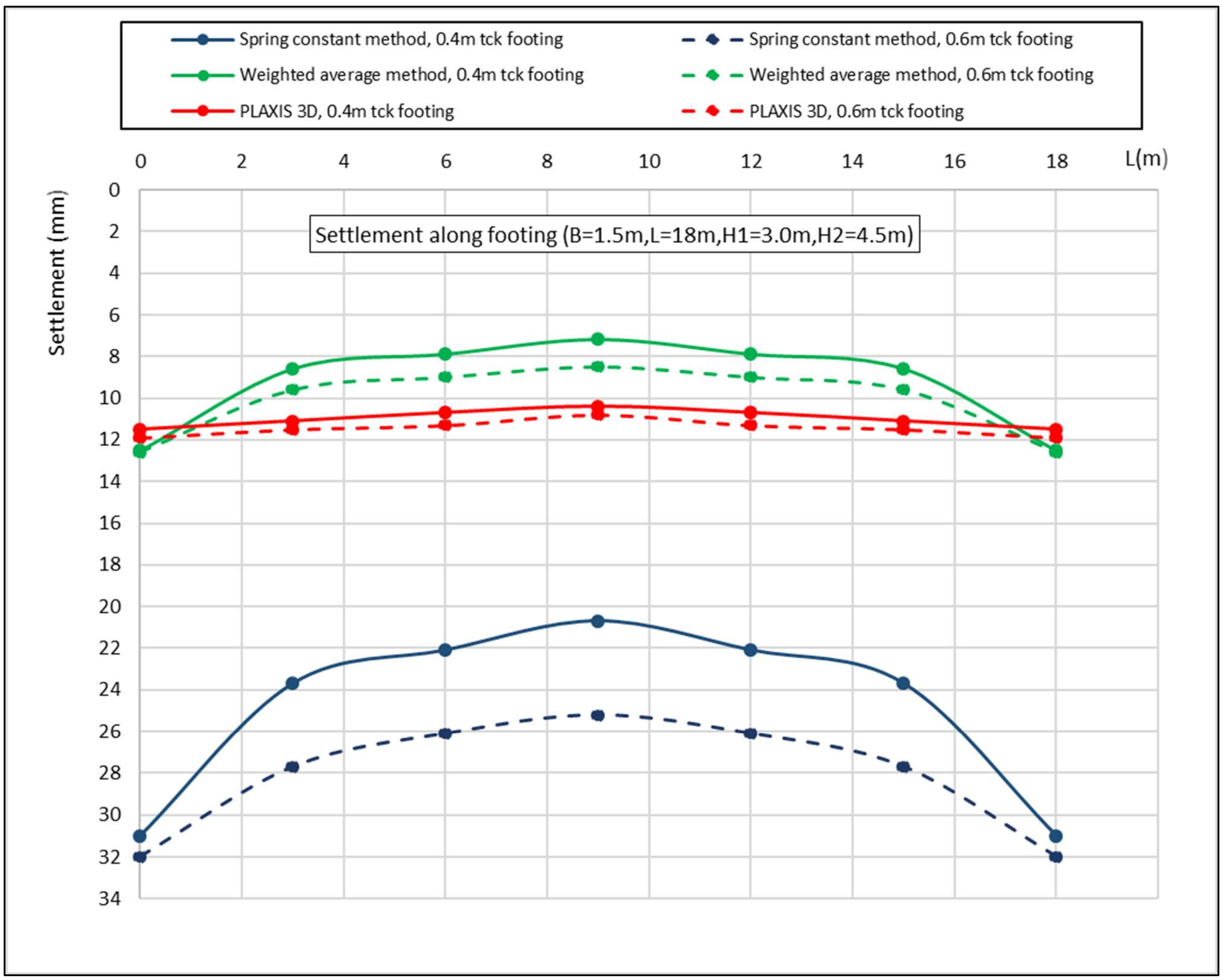
B=1.5m, L=18.0m, H_1=3.0m, H_2=1.5m, F=180kN, 2F=360kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	31	32	9.5	9.6	8.6	8.8
3	23.7	27.7	6.2	6.9	8.2	8.5
6	22.1	26.1	5.8	6.5	7.8	8.3
9	20.7	25.2	5.2	6.0	7.4	7.9
12	22.1	26.1	5.8	6.5	7.8	8.3
15	23.7	27.7	6.2	6.9	8.2	8.5
18	31	32	9.5	9.6	8.6	8.8



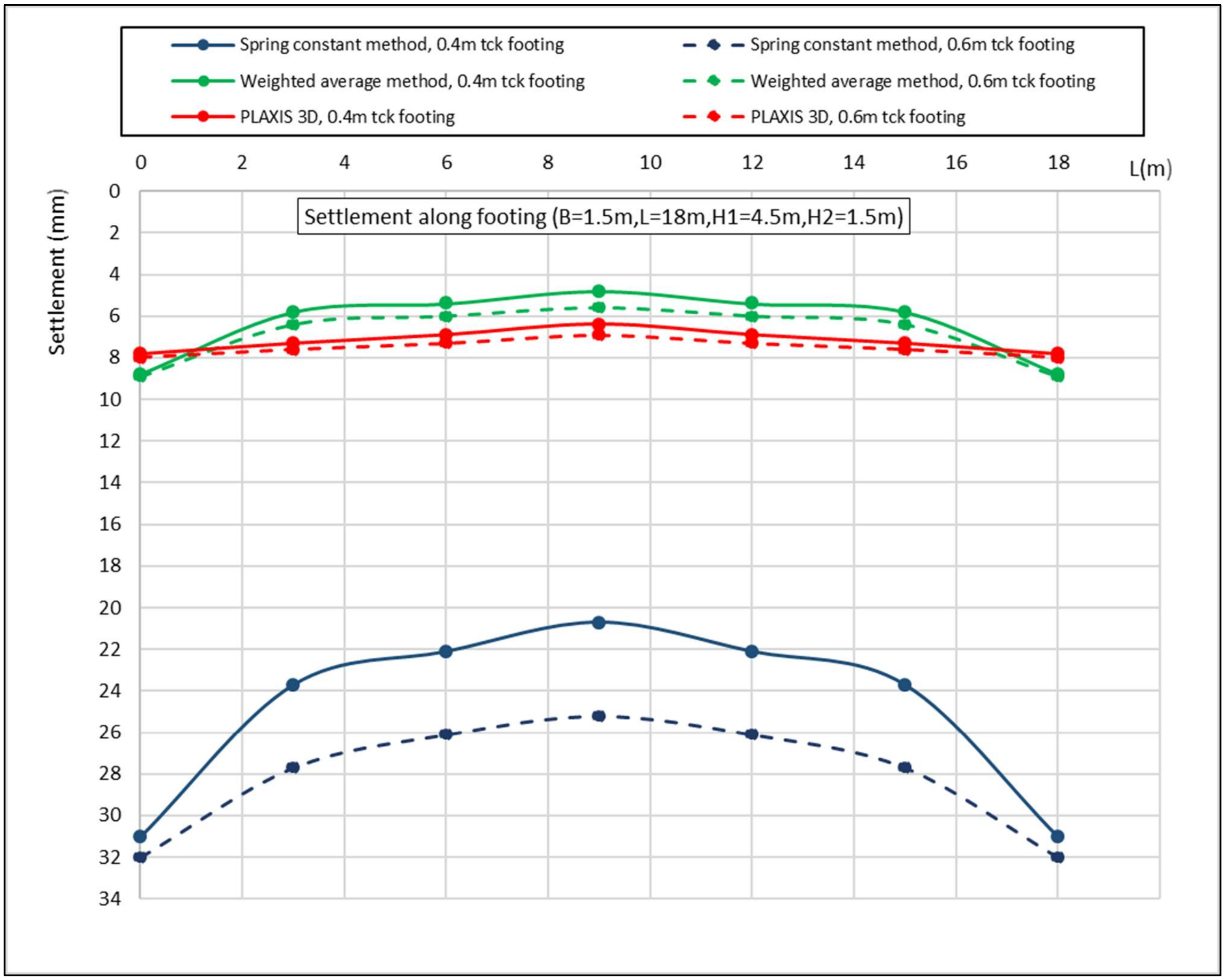
B=1.5m, L=18.0m, H_1=3.0m, H_2=3.0m, F=180kN, 2F=360kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	31.0	32.0	11.1	11.3	10.4	10.9
3	23.7	27.7	7.5	8.4	10	10.4
6	22.1	26.1	6.9	7.8	9.7	10.1
9	20.7	25.2	6.2	7.4	9.5	9.8
12	22.1	26.1	6.9	7.8	9.7	10.1
15	23.7	27.7	7.5	8.4	10	10.4
18	31.0	32.0	11.1	11.3	10.4	10.9



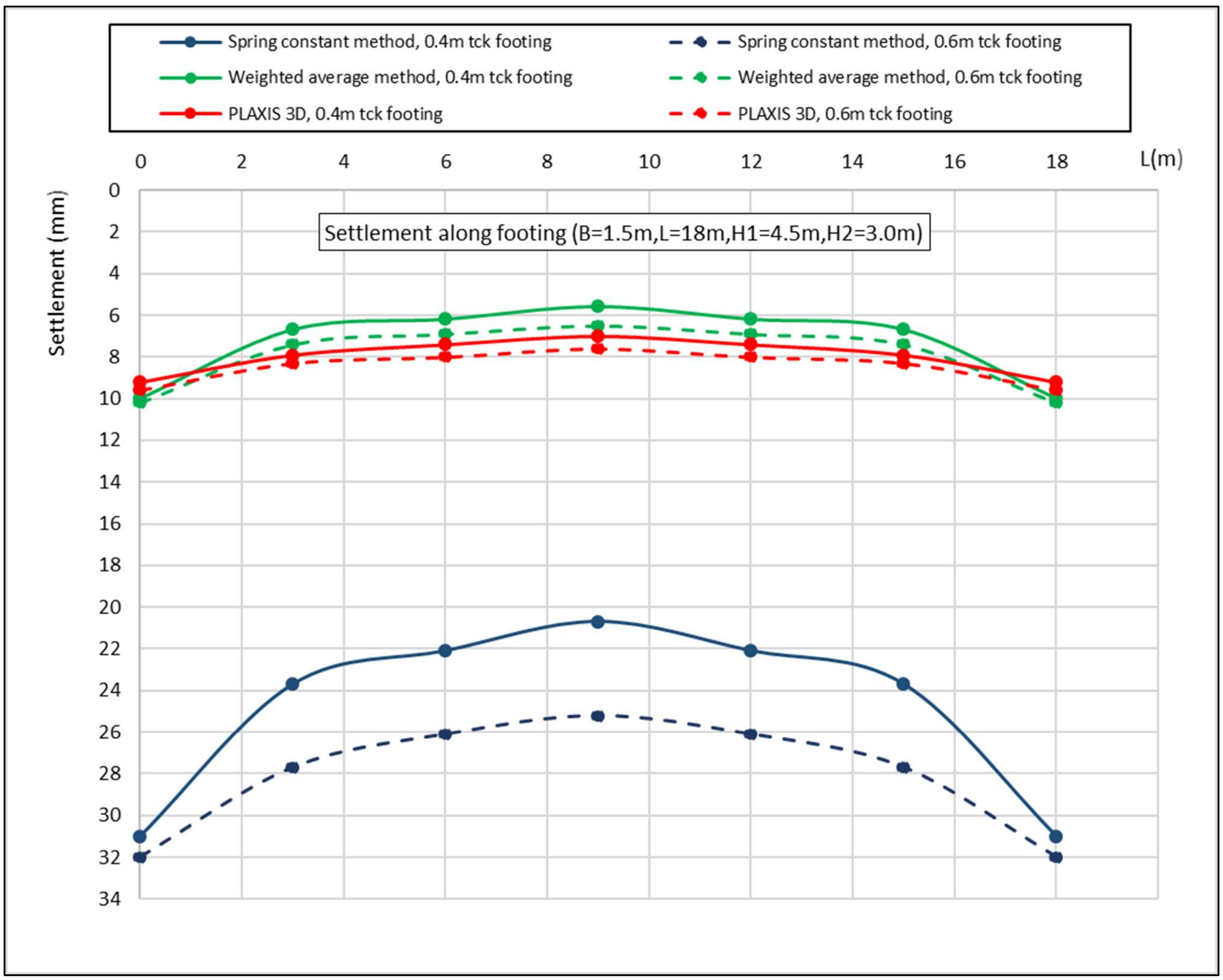
B=1.5m, L=18.0m, H_1=3.0m, H_2=4.5m, F=180kN, 2F=360kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	31.0	32.0	12.5	12.6	11.5	11.9
3	23.7	27.7	8.6	9.6	11.1	11.5
6	22.1	26.1	7.9	9.0	10.7	11.3
9	20.7	25.2	7.2	8.5	10.4	10.8
12	22.1	26.1	7.9	9.0	10.7	11.3
15	23.7	27.7	8.6	9.6	11.1	11.5
18	31.0	32.0	12.5	12.6	11.5	11.9



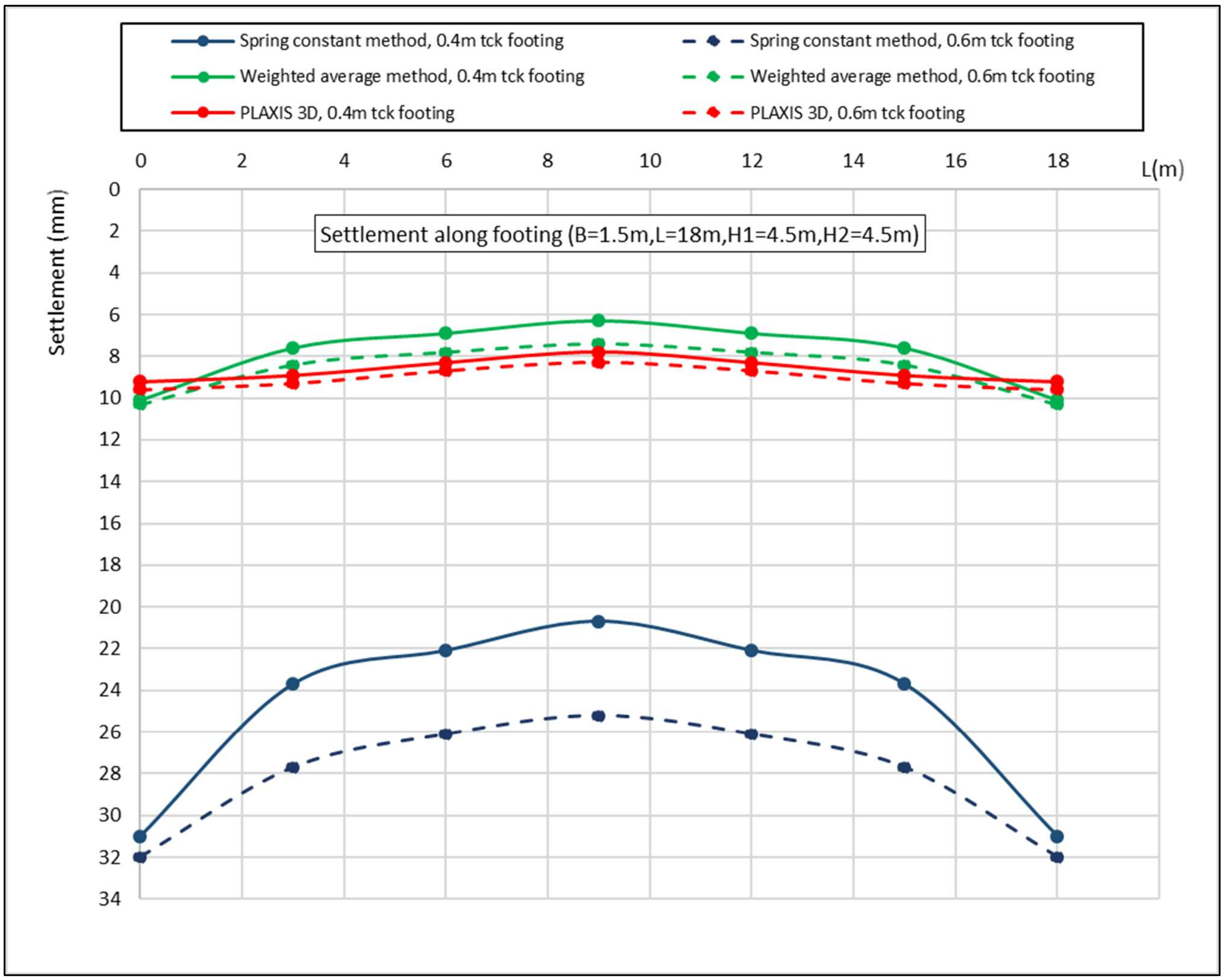
B=1.5m, L=18.0m, H_1=4.5m, H_2=1.5m, F=180kN, 2F=360kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	31.0	32.0	8.8	8.9	7.8	8
3	23.7	27.7	5.8	6.4	7.3	7.6
6	22.1	26.1	5.4	6.0	6.9	7.3
9	20.7	25.2	4.8	5.6	6.4	6.9
12	22.1	26.1	5.4	6.0	6.9	7.3
15	23.7	27.7	5.8	6.4	7.3	7.6
18	31.0	32.0	8.8	8.9	7.8	8



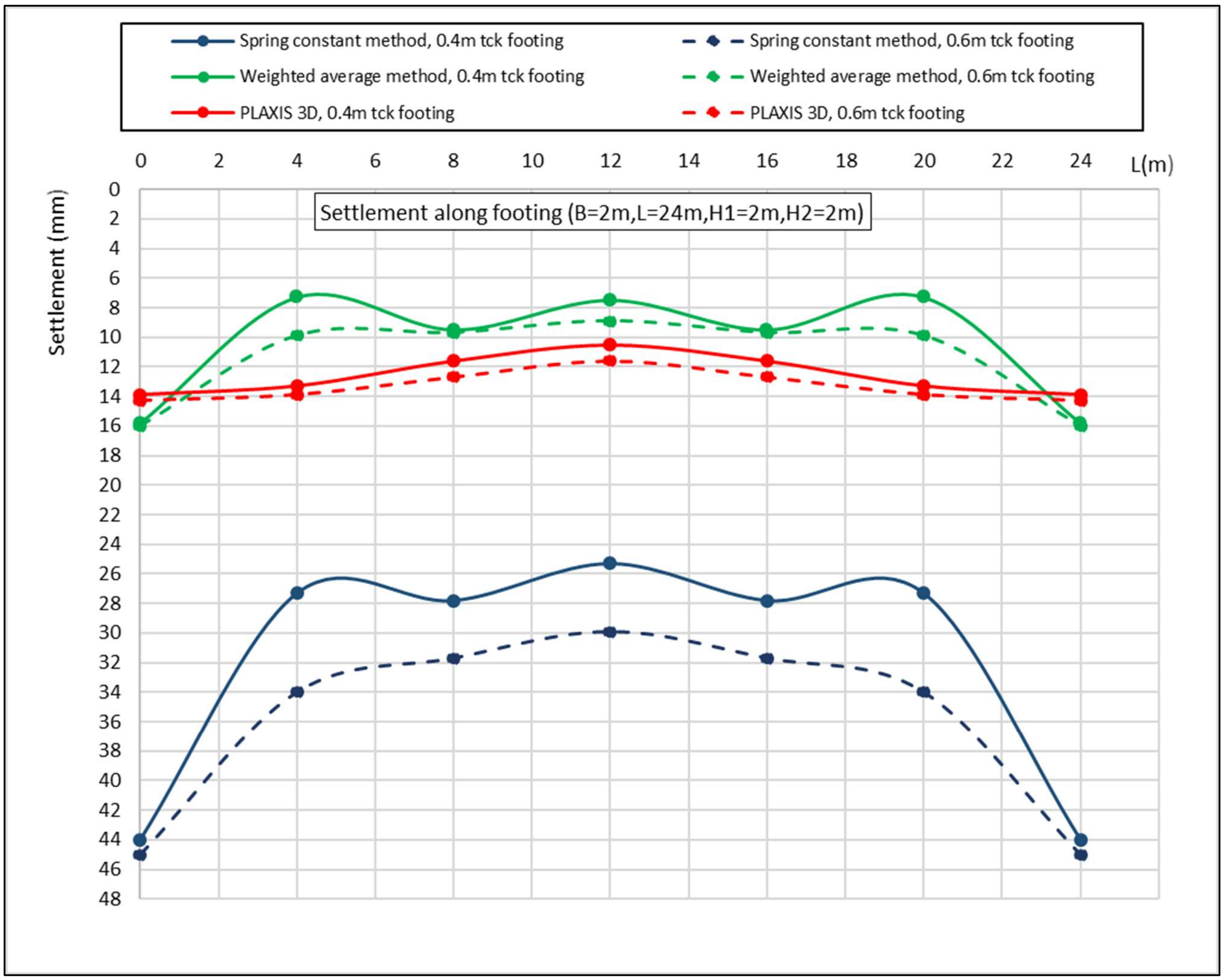
B=1.5m, L=18.0m, H_1=4.5m, H_2=3.0m, F=180kN, 2F=360kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	31.0	32.0	9.5	9.7	9.2	9.6
3	23.7	27.7	6.7	7.4	7.9	8.3
6	22.1	26.1	6.2	6.9	7.4	8
9	20.7	25.2	5.6	6.5	7.0	7.6
12	22.1	26.1	6.2	6.9	7.4	8
15	23.7	27.7	6.7	7.4	7.9	8.3
18	31.0	32.0	9.5	9.7	9.2	9.6



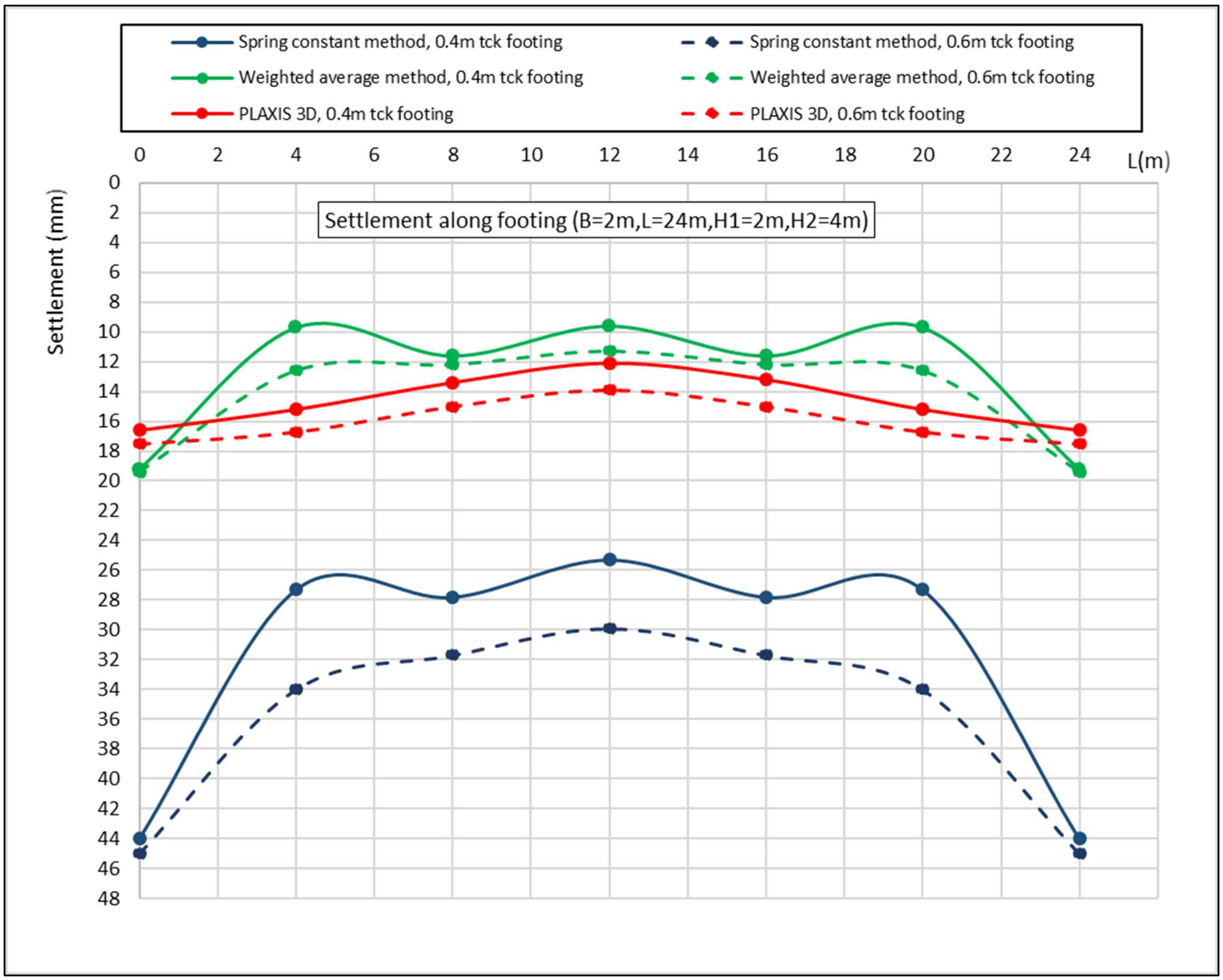
B=1.5m, L=18.0m, H_1=4.5m, H_2=4.5m, F=180kN, 2F=360kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	31.0	32.0	10.1	10.3	9.2	9.6
3	23.7	27.7	7.6	8.4	8.9	9.3
6	22.1	26.1	6.9	7.8	8.3	8.7
9	20.7	25.2	6.3	7.4	7.8	8.3
12	22.1	26.1	6.9	7.8	8.3	8.7
15	23.7	27.7	7.6	8.4	8.9	9.3
18	31.0	32.0	10.1	10.3	9.2	9.6



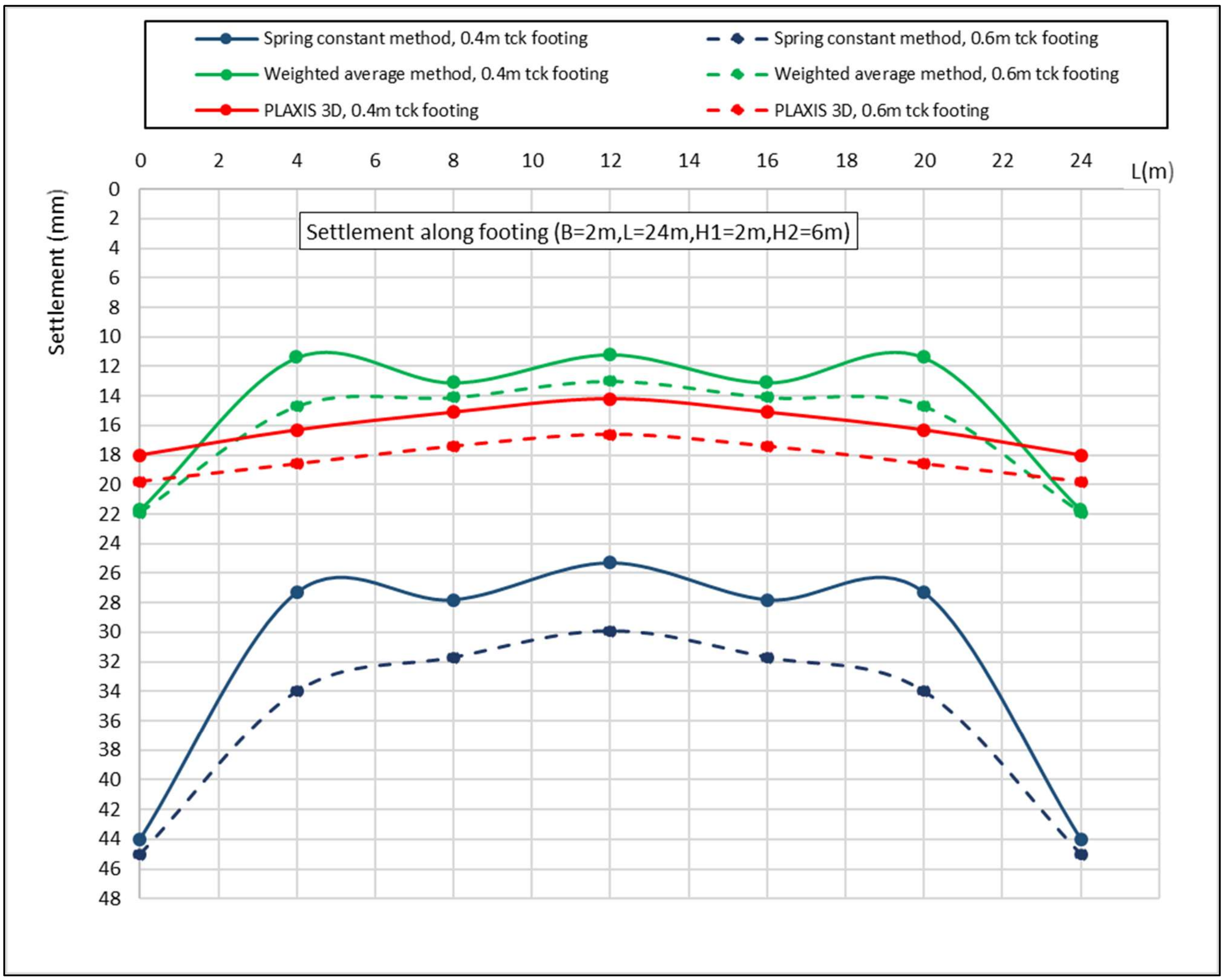
B=2.0m, L=24.0m, H_1=2.0m, H_2=2.0m, F=320kN, 2F=640kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	44	45	15.8	16	13.9	14.3
4	27.3	34	7.3	9.9	13.3	13.9
8	27.8	31.7	9.5	9.7	11.6	12.7
12	25.3	29.9	7.5	8.9	10.5	11.6
16	27.8	31.7	9.5	9.7	11.6	12.7
20	27.3	34	7.3	9.9	13.3	13.9
24	44	45	15.8	16	13.9	14.3



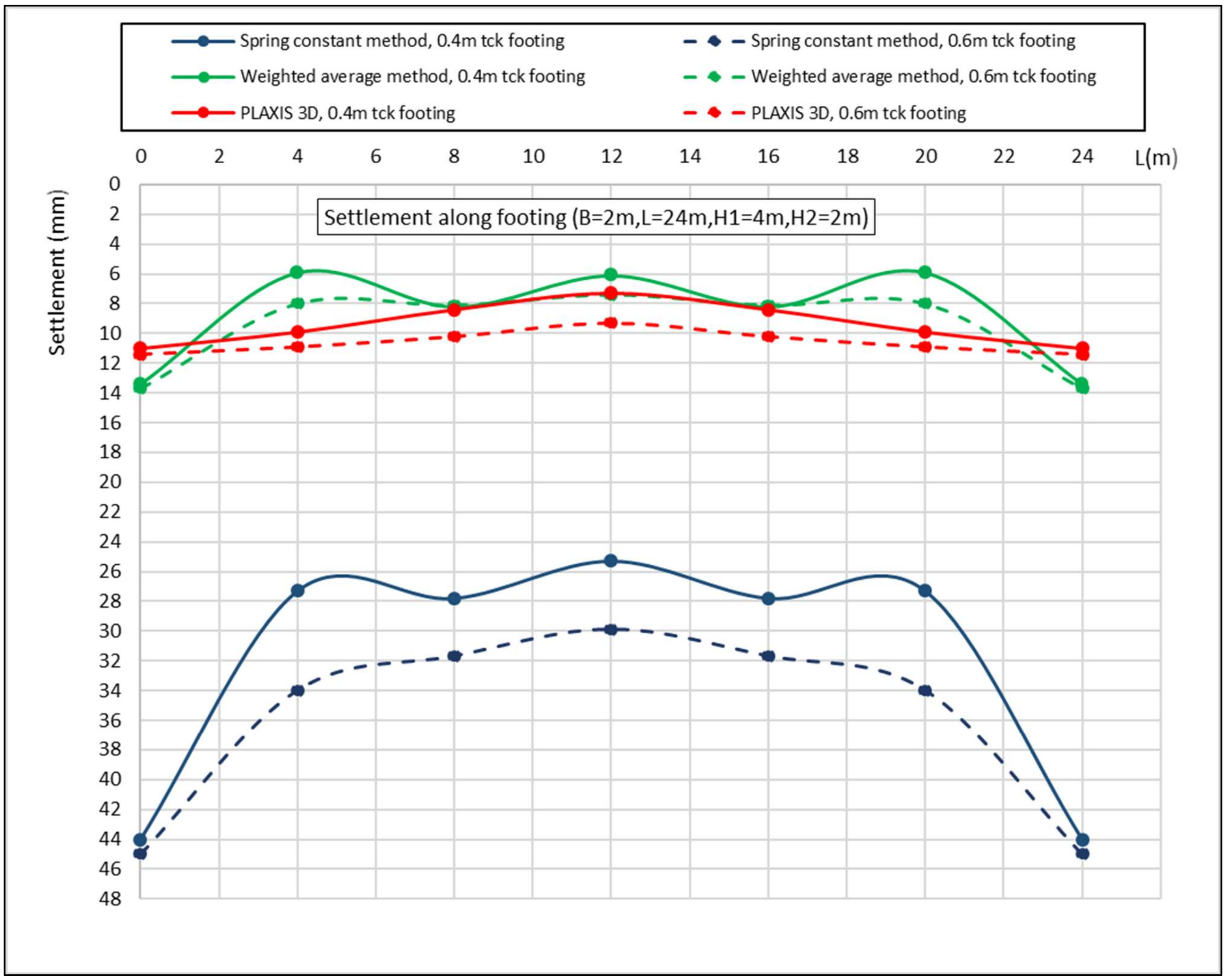
B=2.0m, L=24.0m, H_1=2.0m, H_2=4.0m, F=320kN, 2F=640kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	44	45	19.2	19.4	16.6	17.5
4	27.3	34	9.7	12.6	15.2	16.7
8	27.8	31.7	11.6	12.2	13.4	15
12	25.3	29.9	9.6	11.3	12.1	13.9
16	27.8	31.7	11.6	12.2	13.2	15
20	27.3	34	9.7	12.6	15.2	16.7
24	44	45	19.2	19.4	16.6	17.5



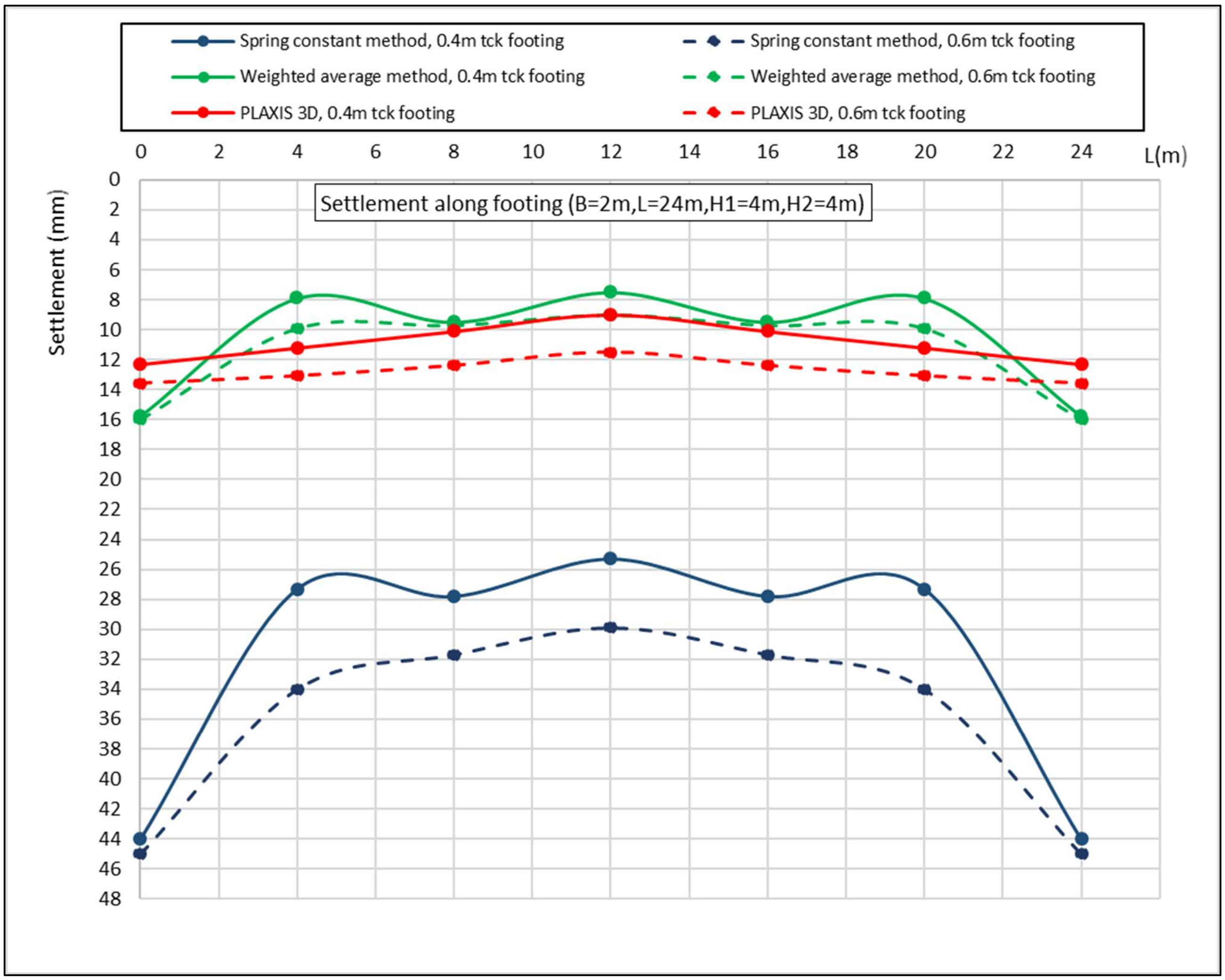
B=2.0m, L=24.0m, H_1=2.0m, H_2=6.0m, F=320kN, 2F=640kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	44	45	21.7	21.9	18	19.8
4	27.3	34	11.4	14.7	16.3	18.6
8	27.8	31.7	13.1	14.1	15.1	17.4
12	25.3	29.9	11.2	13.0	14.2	16.6
16	27.8	31.7	13.1	14.1	15.1	17.4
20	27.3	34	11.4	14.7	16.3	18.6
24	44	45	21.7	21.9	18	19.8



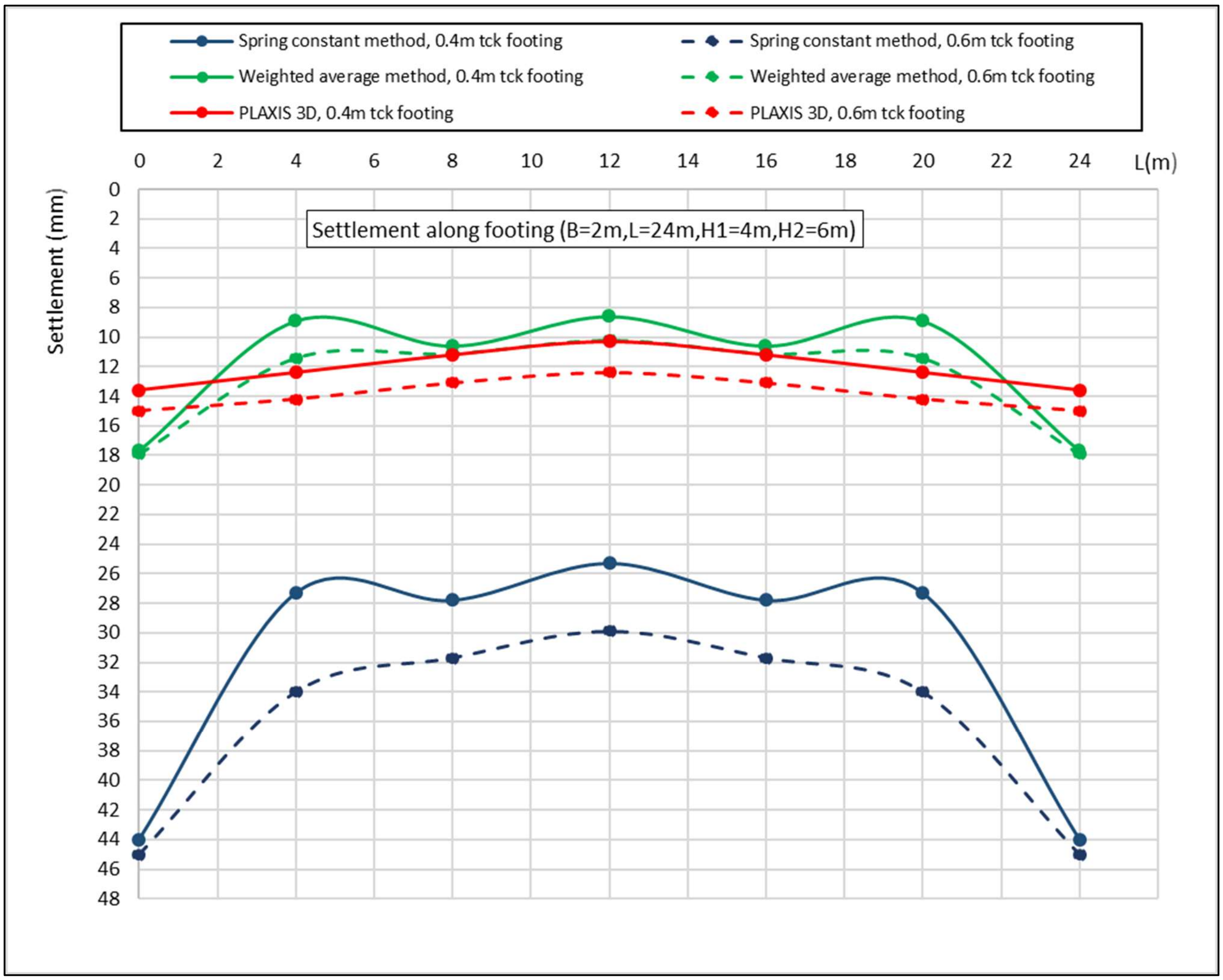
B=2.0m, L=24.0m, H_1=4.0m, H_2=2.0m, F=320kN, 2F=640kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	44	45	13.4	13.7	11	11.4
4	27.3	34	5.9	8.0	9.9	10.9
8	27.8	31.7	8.2	8.1	8.4	10.2
12	25.3	29.9	6.1	7.4	7.3	9.3
16	27.8	31.7	8.2	8.1	8.4	10.2
20	27.3	34	5.9	8.0	9.9	10.9
24	44	45	13.4	13.7	11	11.4



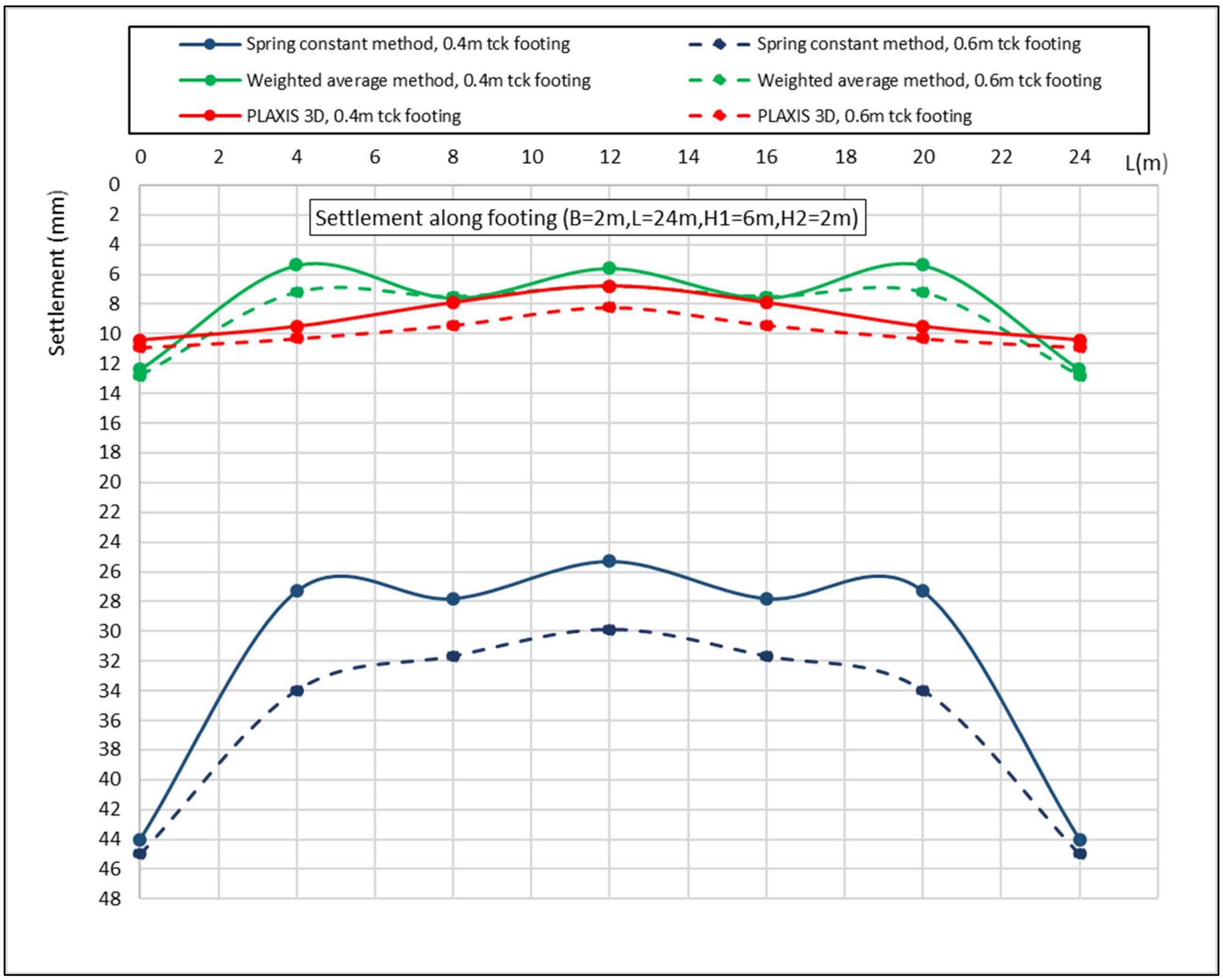
B=2.0m, L=24.0m, H_1=4.0m, H_2=4.0m, F=320kN, 2F=640kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	44	45	15.8	16.0	12.3	13.6
4	27.3	34	7.9	9.9	11.2	13.1
8	27.8	31.7	9.5	9.7	10.1	12.4
12	25.3	29.9	7.5	9.0	9	11.5
16	27.8	31.7	9.5	9.7	10.1	12.4
20	27.3	34	7.9	9.9	11.2	13.1
24	44	45	15.8	16.0	12.3	13.6



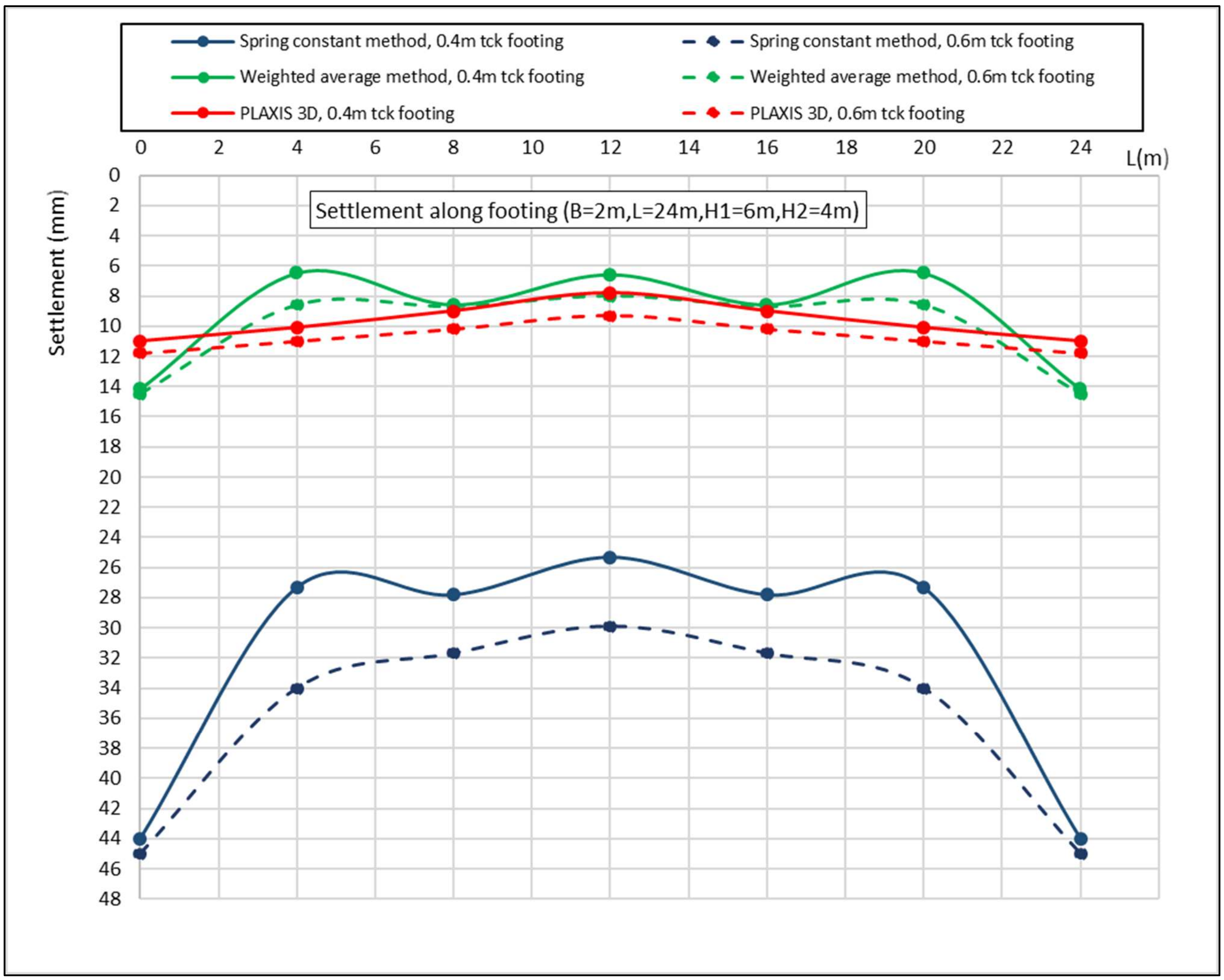
B=2.0m, L=24.0m, H_1=4.0m, H_2=6.0m, F=320kN, 2F=640kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	44	45	17.7	17.9	13.6	15
4	27.3	34	8.9	11.4	12.4	14.2
8	27.8	31.7	10.6	11.1	11.2	13.1
12	25.3	29.9	8.6	10.2	10.3	12.4
16	27.8	31.7	10.6	11.1	11.2	13.1
20	27.3	34	8.9	11.4	12.4	14.2
24	44	45	17.7	17.9	13.6	15



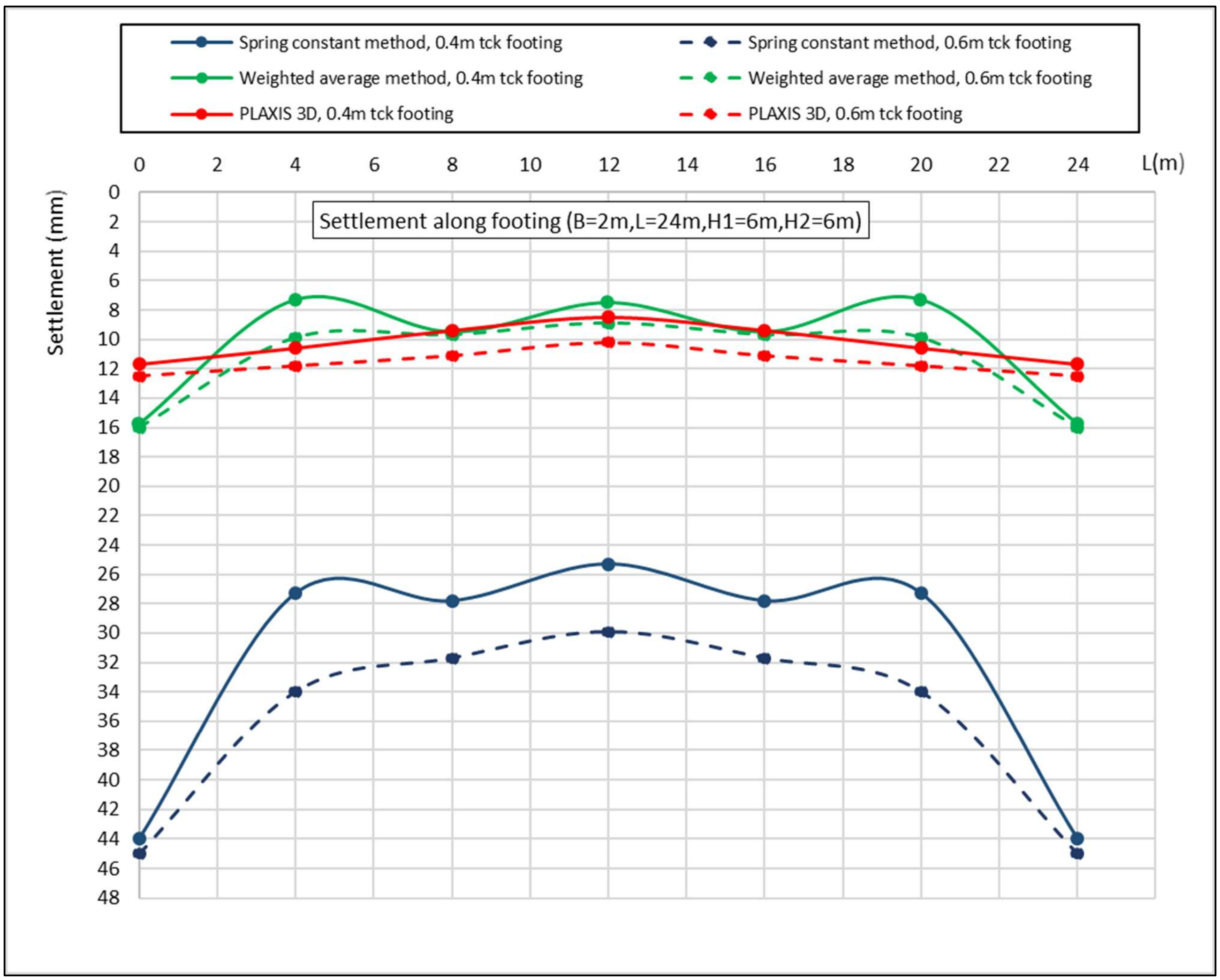
B=2.0m, L=24.0m, H_1=6.0m, H_2=2.0m, F=320kN, 2F=640kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	44	45	12.4	12.8	10.4	10.9
4	27.3	34	5.4	7.2	9.5	10.3
8	27.8	31.7	7.6	7.5	7.9	9.4
12	25.3	29.9	5.6	6.8	6.8	8.2
16	27.8	31.7	7.6	7.5	7.9	9.4
20	27.3	34	5.4	7.2	9.5	10.3
24	44	45	12.4	12.8	10.4	10.9



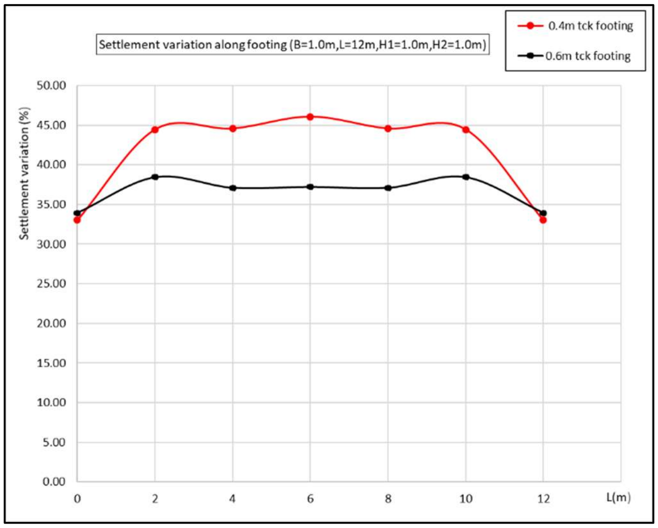
B=2.0m, L=24.0m, H_1=6.0m, H_2=4.0m, F=320kN, 2F=640kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	44	45	14.2	14.5	11	11.8
4	27.3	34	6.5	8.6	10.1	11
8	27.8	31.7	8.6	8.7	9	10.2
12	25.3	29.9	6.6	8.0	7.8	9.3
16	27.8	31.7	8.6	8.7	9	10.2
20	27.3	34	6.5	8.6	10.1	11
24	44	45	14.2	14.5	11	11.8

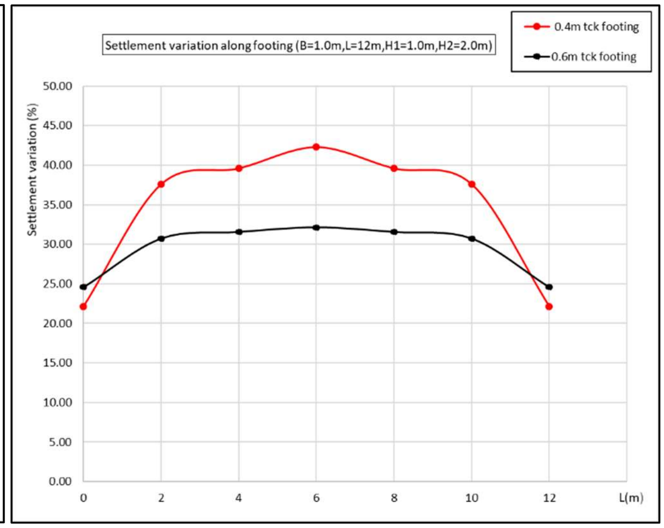


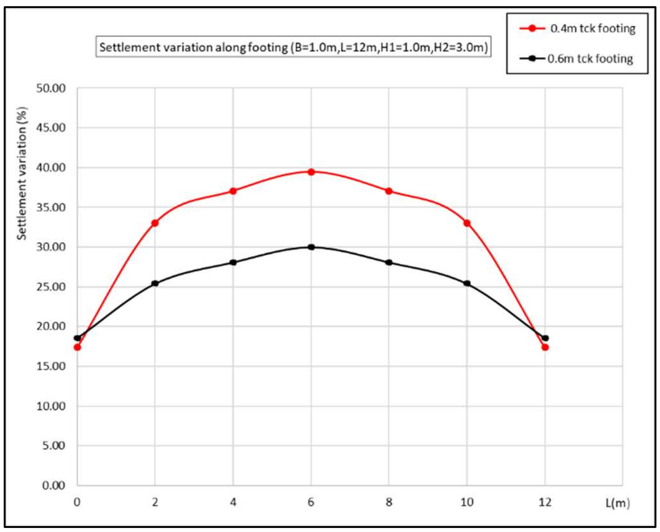
B=2.0m, L=24.0m, H_1=6.0m, H_2=6.0m, F=320kN, 2F=640kN						
Distance from edge of the footing (m)	Settlement using equivalent spring theory and Heternyi method (mm)		Settlement using weighted average of soil layers and Heternyi method (mm)		Settlement using PLAXIS 3D model (mm)	
	t=0.4m	t=0.6m	t=0.4m	t=0.6m	t=0.4m	t=0.6m
0	44	45	15.8	16.0	11.7	12.5
4	27.3	34	7.3	9.9	10.6	11.8
8	27.8	31.7	9.5	9.7	9.4	11.1
12	25.3	29.9	7.5	8.9	8.5	10.2
16	27.8	31.7	9.5	9.7	9.4	11.1
20	27.3	34	7.3	9.9	10.6	11.8
24	44	45	15.8	16.0	11.7	12.5

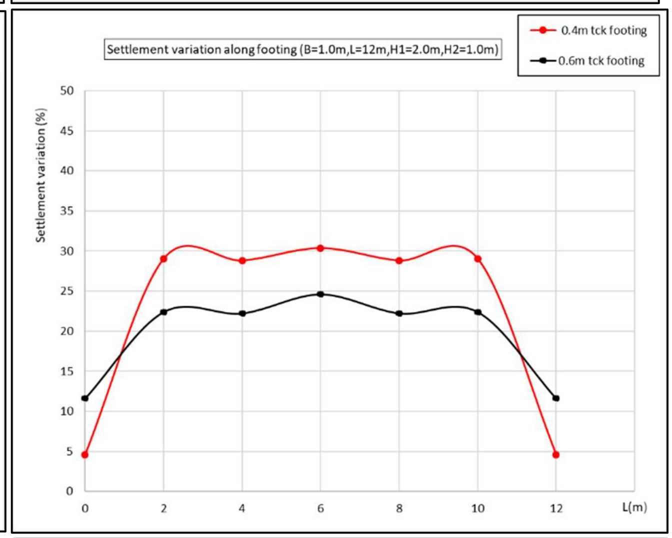


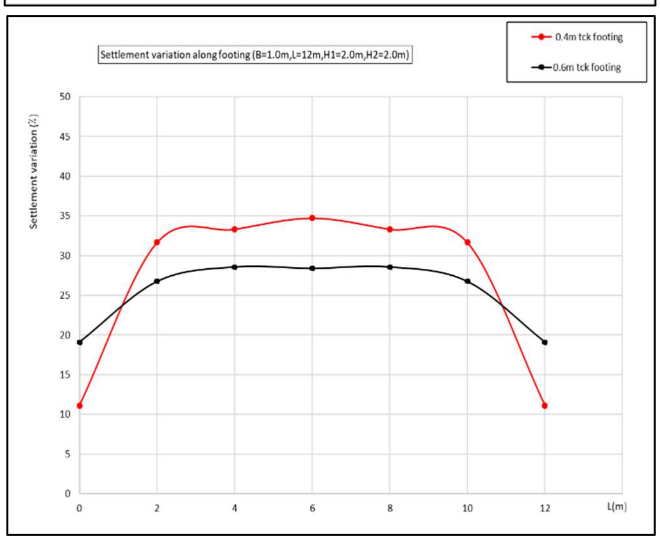
7.5 Settlement difference between PLAXIS 3D and weighted average equivalent subgrade modulus applied in Heternyi method curve

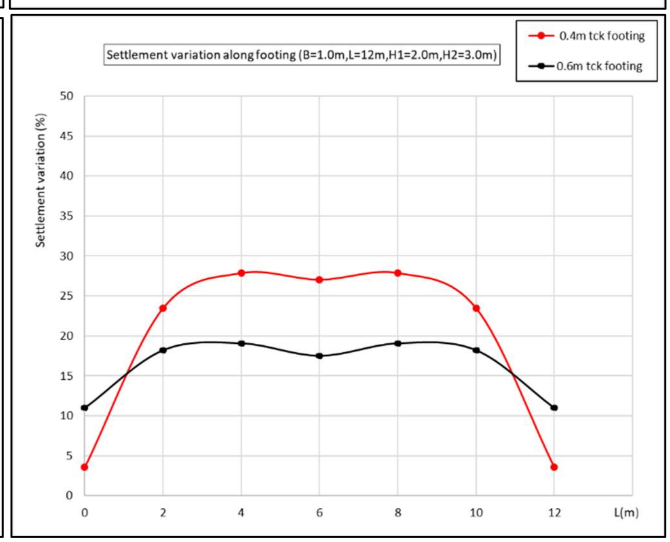


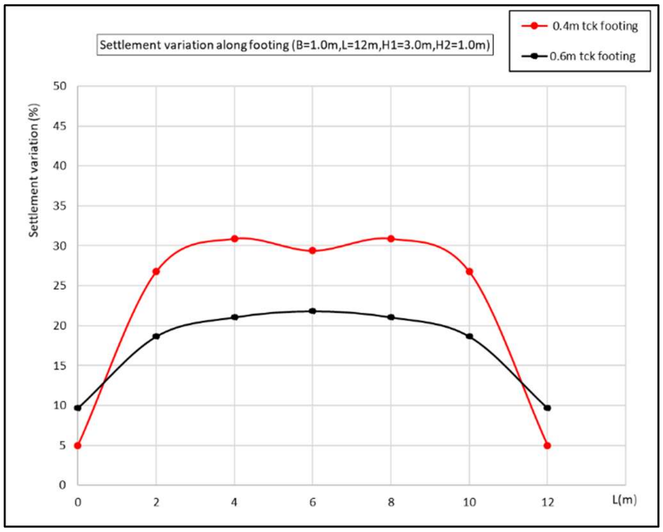


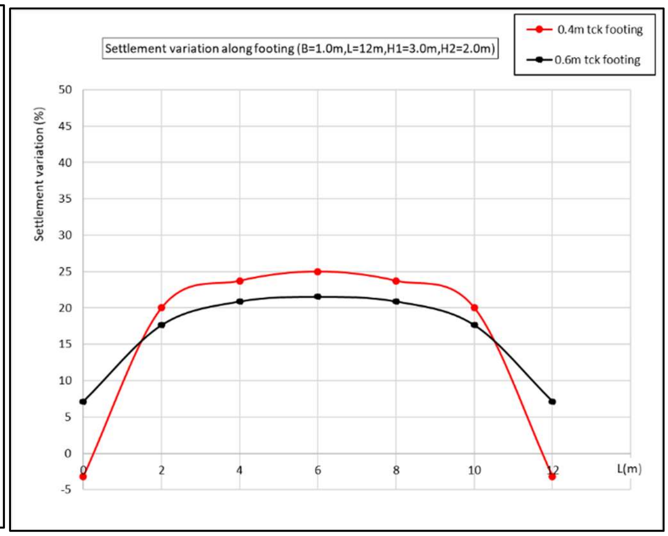


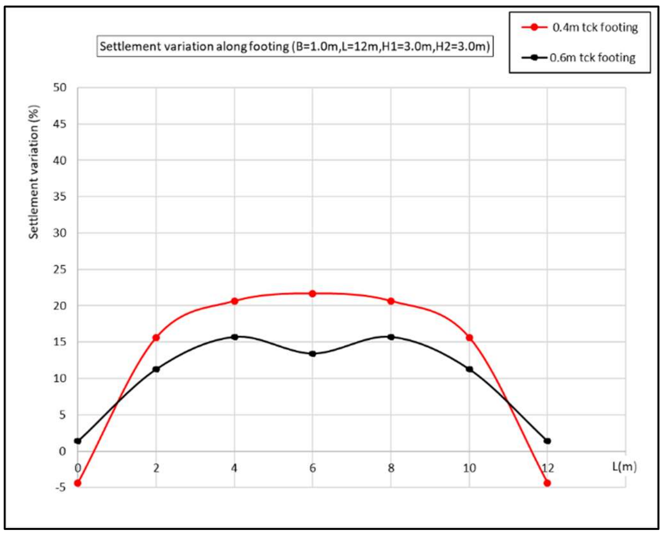


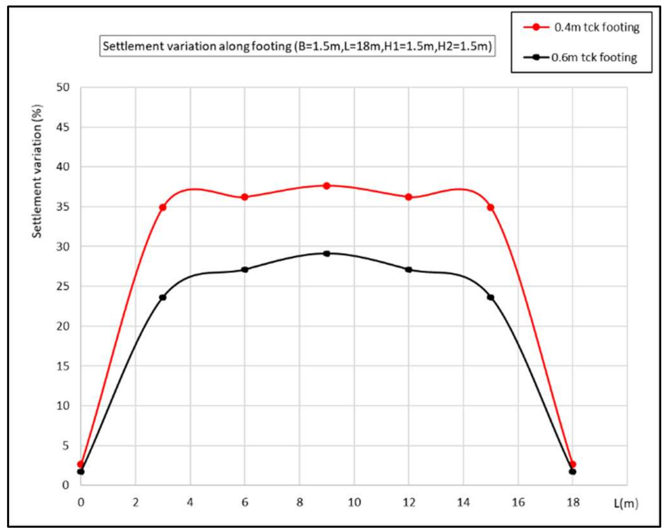


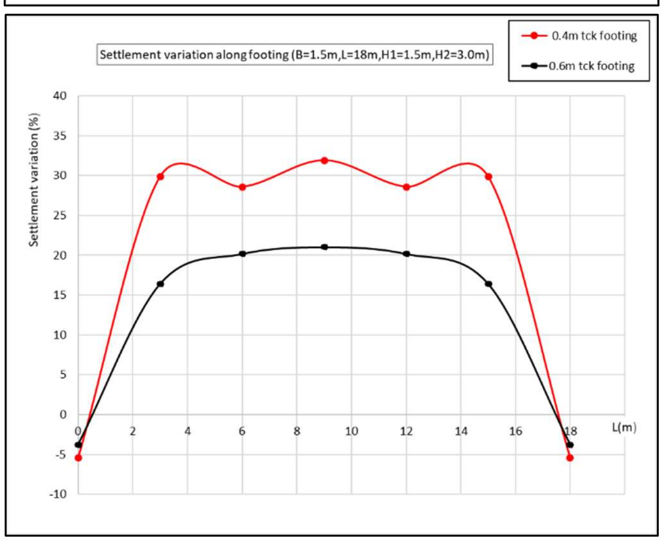


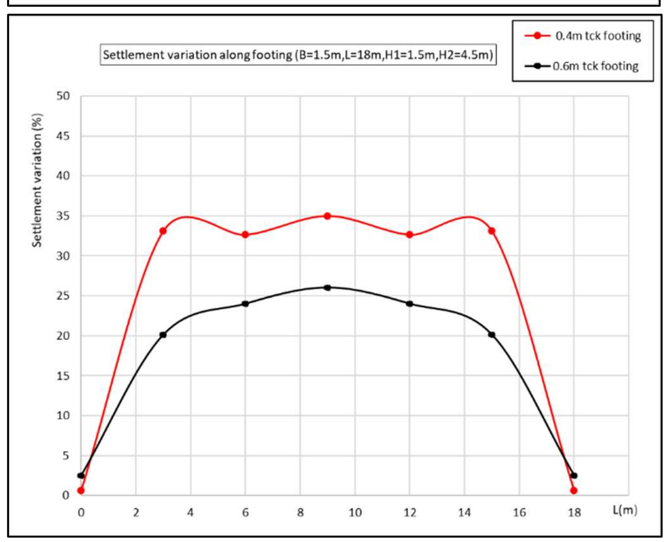

Settlement variation along footing (B=1.0m,L=12m,H1=3.0m,H2=1.0m)
0.4m tck footing
0.6m tck footing
Settlement variation (%)
L(m)

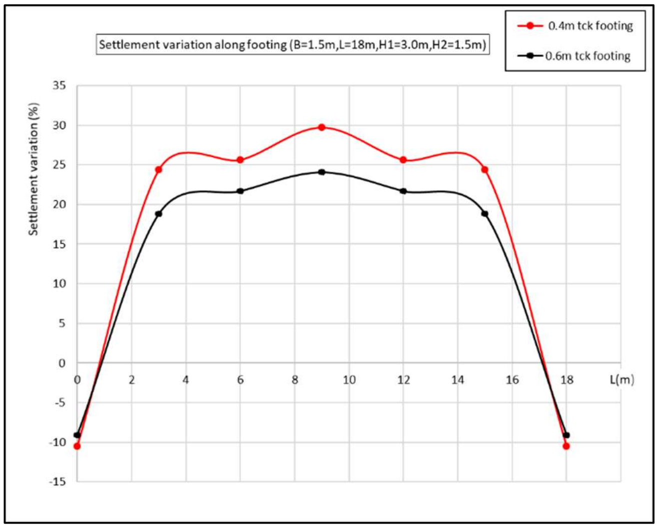

Settlement variation along footing (B=1.0m,L=12m,H1=3.0m,H2=2.0m)
0.4m tck footing
0.6m tck footing
Settlement variation (%)
L(m)

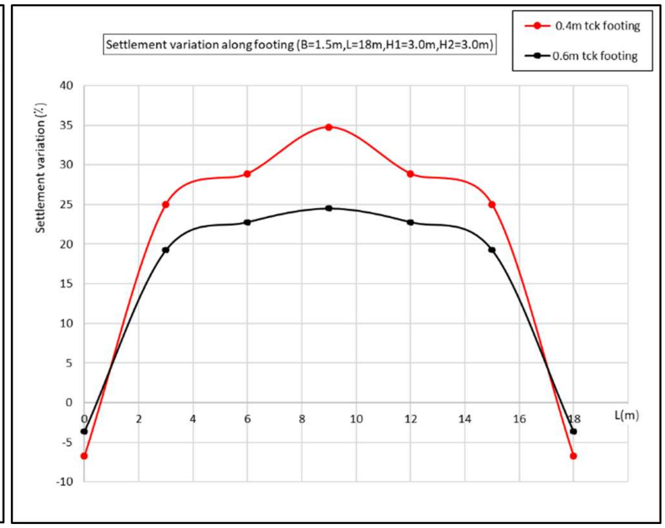

Settlement variation along footing (B=1.0m,L=12m,H1=3.0m,H2=3.0m)
0.4m tck footing
0.6m tck footing
Settlement variation (%)
L(m)

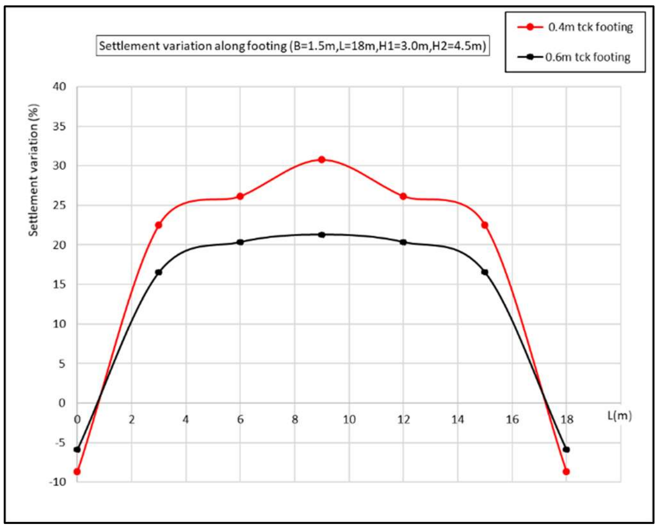

Settlement variation along footing (B=1.5m,L=18m,H1=1.5m,H2=1.5m)
0.4m tck footing
0.6m tck footing
Settlement variation (%)
L(m)

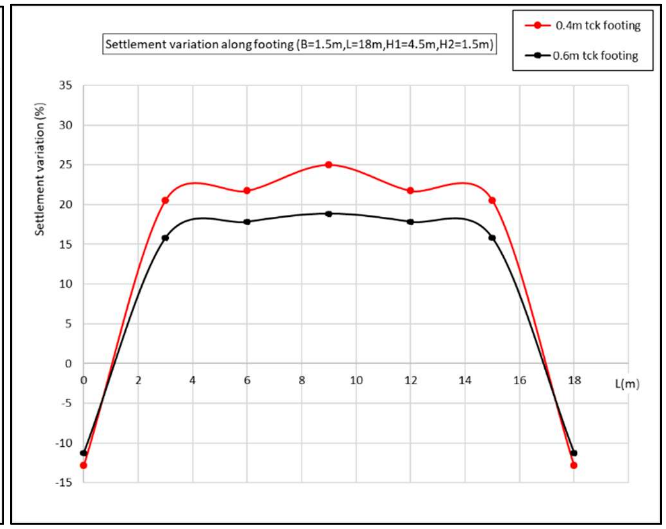

Settlement variation along footing (B=1.5m,L=18m,H1=1.5m,H2=3.0m)
0.4m tck footing
0.6m tck footing
Settlement variation (%)
L(m)

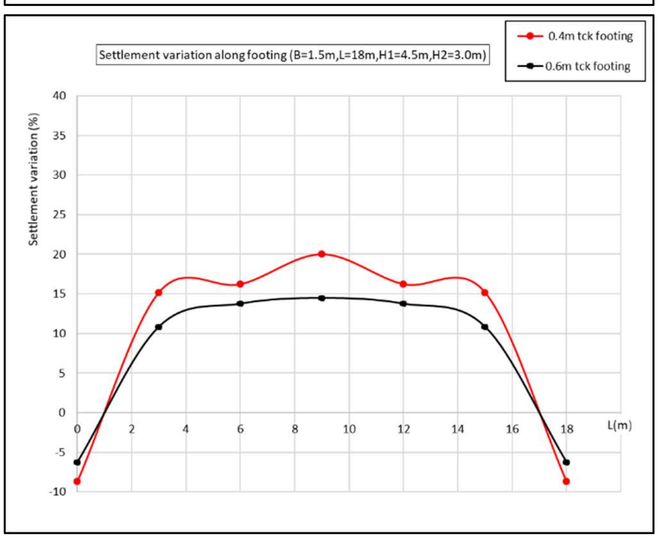

Settlement variation along footing (B=1.5m,L=18m,H1=1.5m,H2=4.5m)
0.4m tck footing
0.6m tck footing
Settlement variation (%)
L(m)

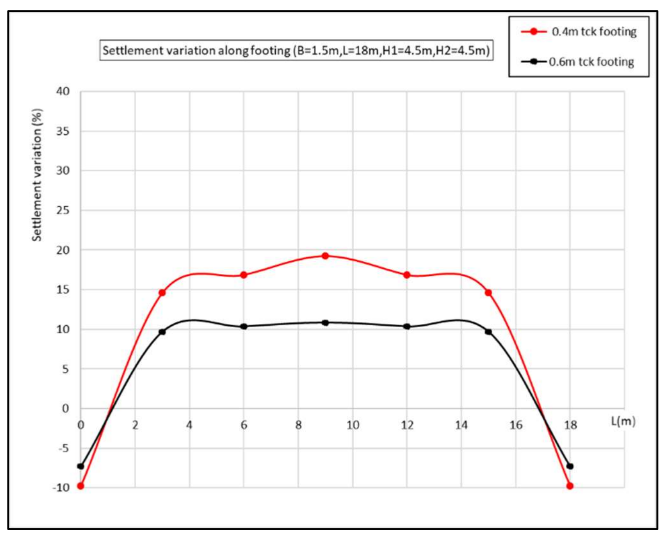

Settlement variation along footing (B=1.5m,L=18m,H1=3.0m,H2=1.5m)
0.4m tck footing
0.6m tck footing
Settlement variation (%)
L(m)

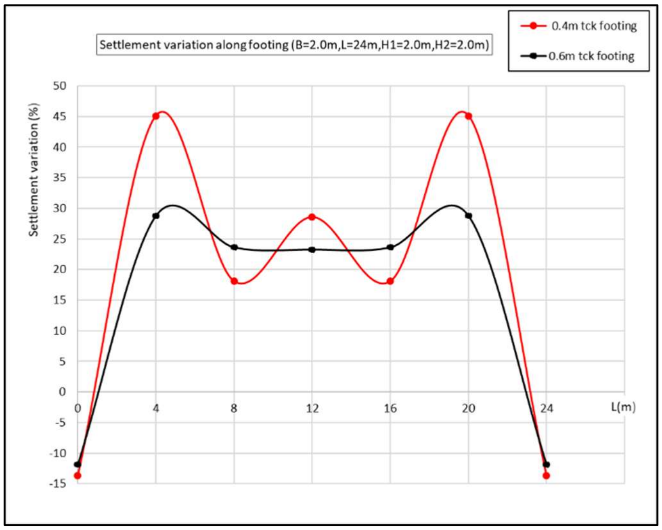

Settlement variation along footing (B=1.5m,L=18m,H1=3.0m,H2=3.0m)
0.4m tck footing
0.6m tck footing
Settlement variation (%)
L(m)

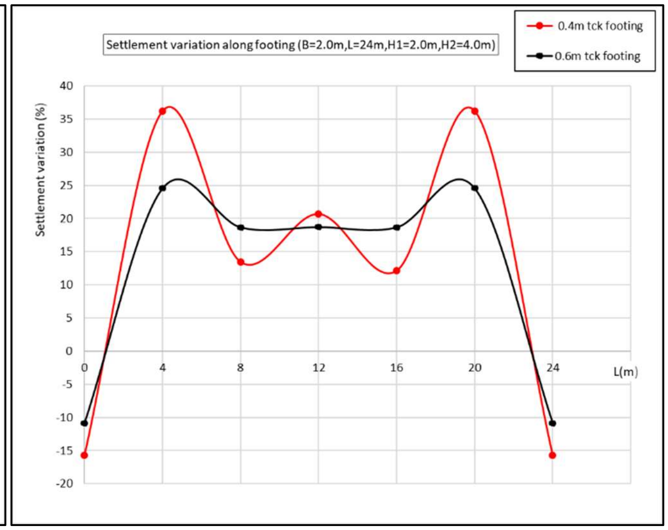

Settlement variation along footing (B=1.5m,L=18m,H1=3.0m,H2=4.5m)
0.4m tck footing
0.6m tck footing
Settlement variation (%)
L(m)

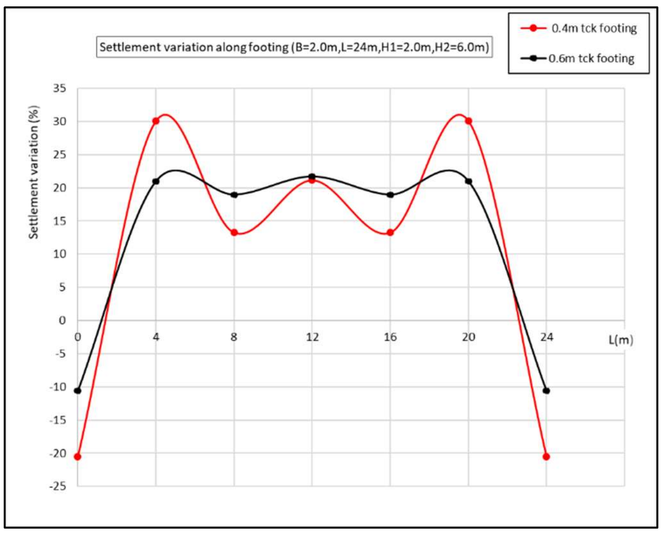

Settlement variation along footing (B=1.5m,L=18m,H1=4.5m,H2=1.5m)
0.4m tck footing
0.6m tck footing
Settlement variation (%)
L(m)

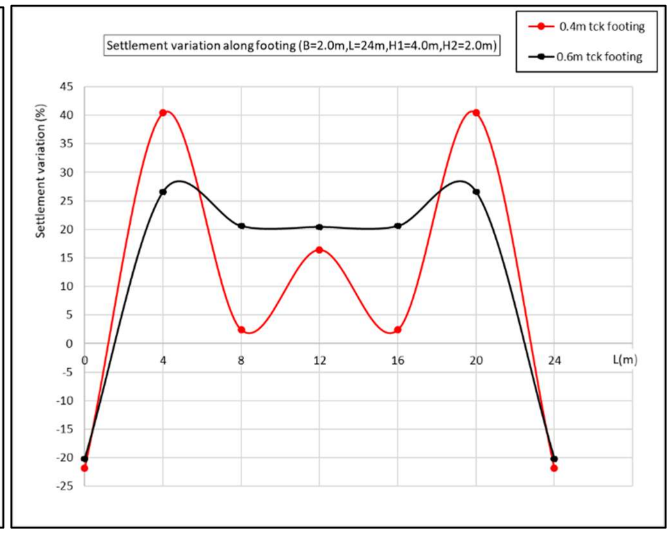

Settlement variation along footing (B=1.5m,L=18m,H1=4.5m,H2=3.0m)
0.4m tck footing
0.6m tck footing
Settlement variation (%)
L(m)

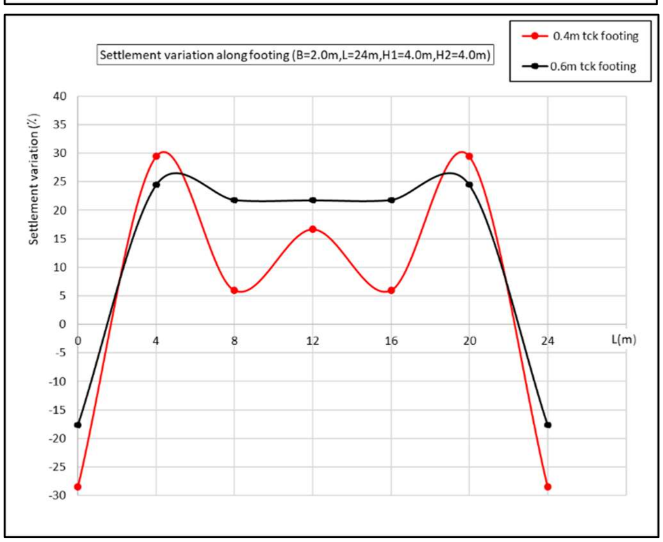

Settlement variation along footing (B=1.5m,L=18m,H1=4.5m,H2=4.5m)
0.4m tck footing
0.6m tck footing
Settlement variation (%)
L(m)

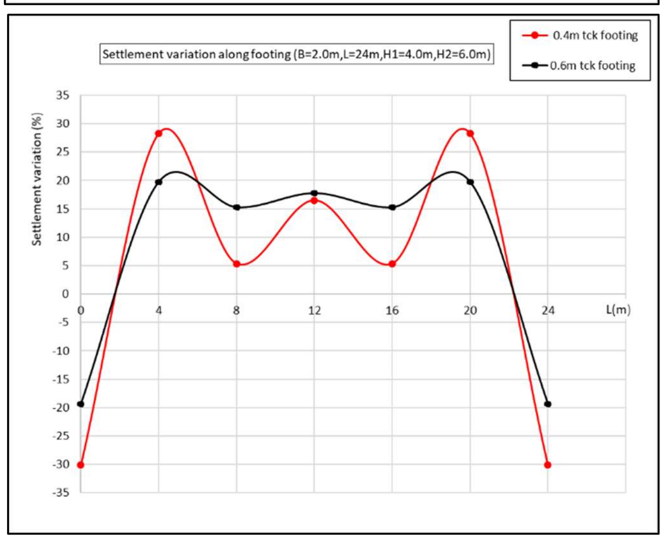

Settlement variation along footing (B=2.0m,L=24m,H1=2.0m,H2=2.0m)
0.4m tck footing
0.6m tck footing
Settlement variation (%)
L(m)

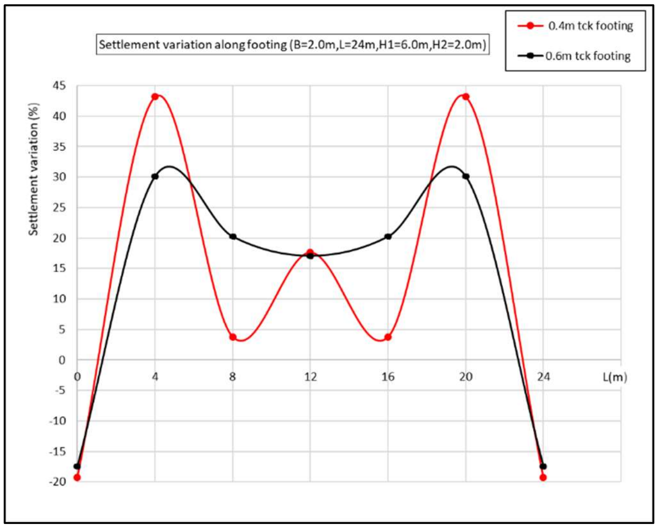

Settlement variation along footing (B=2.0m,L=24m,H1=2.0m,H2=4.0m)
0.4m tck footing
0.6m tck footing
Settlement variation (%)
L(m)

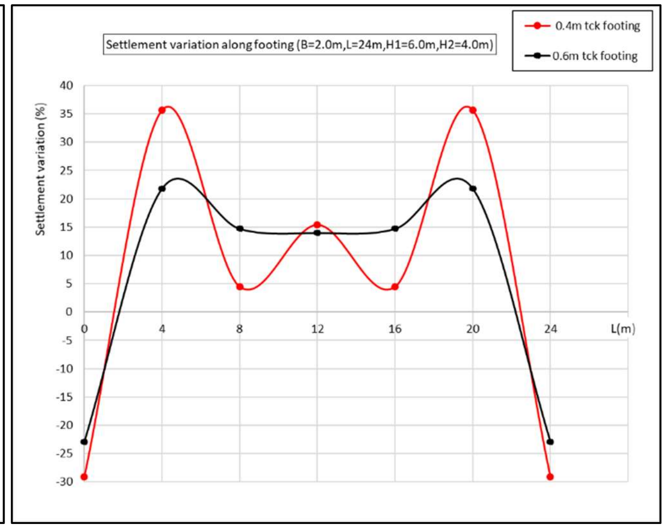

Settlement variation along footing (B=2.0m,L=24m,H1=2.0m,H2=6.0m)
0.4m tck footing
0.6m tck footing
Settlement variation (%)
L(m)

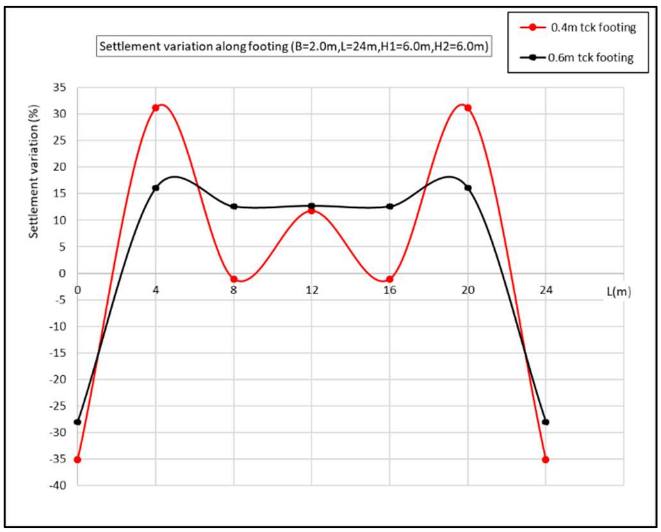

Settlement variation along footing (B=2.0m,L=24m,H1=4.0m,H2=2.0m)
0.4m tck footing
0.6m tck footing
Settlement variation (%)
L(m)


Settlement variation along footing (B=2.0m,L=24m,H1=4.0m,H2=4.0m)
0.4m tck footing
0.6m tck footing
L(m)


Settlement variation along footing (B=2.0m,L=24m,H1=4.0m,H2=6.0m)
0.4m tck footing
0.6m tck footing
Settlement variation (%)
L(m)


0.4m tck footing
0.6m tck footing
Settlement variation along footing (B=2.0m,L=24m,H1=6.0m,H2=2.0m)
Settlement variation (%)
L(m)


0.4m tck footing
0.6m tck footing
Settlement variation along footing (B=2.0m,L=24m,H1=6.0m,H2=4.0m)
Settlement variation (%)
L(m)


0.4m tck footing
0.6m tck footing
Settlement variation along footing (B=2.0m,L=24m,H1=6.0m,H2=6.0m)
Settlement variation (%)
L(m)