

# **MODELING WEEKLY RAINFALL IN COLOMO CITY**

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## DECLARATION

I declare that this is my own work and this thesis does not incorporate without acknowledgement any material previously submitted for a Degree or Diploma in any other University or institute of higher learning and to the best of my knowledge and belief it does not contain any material previously published or written by another person except where the acknowledgement is made in the text.

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## ABSTRACT

Modeling weekly rainfall has become a demanding assignment due to the complexity of rainfall pattern. Accurate inferences on weekly rainfall prediction facilitate to fill the noticeable gap with respect to the climate monitoring to reduce the climate stress in the country. However, relatively, few measures have been taken to perform the modeling of rainfall in the context of long memory. This study therefore, provides an assessment of such a phenomenon by fitting a novel time series models to weekly rainfall. As the weekly rainfall exhibits the blend features of long memory and time dependence variance, various class of long memory models were fitted by accounting the heteroskedasticity. The best fitted model developed is ARFIMA-GARCH for deseasonalized data. The model was trained using weekly rainfall data from 1990 to 2014 and validated using data from 2015 to 2017 in Colombo city, obtained from the Department of Meteorology, Sri Lanka. The exact maximum likelihood estimation method was utilized to estimate model parameters. For the evaluation of the suitability of the method for parameter estimation, Monte Carlo simulations were carried out with various non seasonally and seasonally fractionally differenced parameter values along with the variance model parameters. The forecasting performance of the five types of long memory models developed was evaluated based on the novel index developed using absolute error for an independent data set in addition to the classical indicators. The rainfall percentiles with the 95% confidence intervals were also developed by exploring temporal variability of weekly rainfall based on parametric approach and bootstrapping approach. It was found that the high likelihood to form extreme rainfall events during beginning of South West Monsoon (SWM) (30<sup>th</sup> April to 10<sup>th</sup> June) and during withdrawal of SWM rainfall (17<sup>th</sup>-30<sup>th</sup> September) as well as with the time span from 8<sup>th</sup> October to 11<sup>th</sup> November during Second Inter Monsoon (SIM) rainfall. Based on the real coverage probabilities which derived using bootstrap calibration, it was found that there is a discrepancy of the nominal and calculated coverage probabilities of the 95% confidence intervals of rainfall percentiles. The deviation of the normality of the fitted distribution with the small size of sample could be a reason for the such a disparity. The novel long range dependency model is recommended to be used in forecasting weekly rainfall in Colombo city in Sri Lanka since the forecasting performance of the new model is not much diluted with the increase of the forecasting length. The study highlights various challenges for applied statisticians in modeling weekly rainfall.

Keywords: Weekly Rainfall, Long Range Dependency, ARFIMA-GARCH, Forecasting, Coverage Probability, South West Monsoon

**DEDICATION**

*To My Mother, Father, Husband and Lovely Son*

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## LIST OF ABBREVIATIONS

<b>Abbreviation</b>	<b>Description</b>
ACF	Auto Correlation Function
AIC	Alkina Information Criterion
ANN	Artificial Neural Network
ARCH	Autoregressive Conditional Heteroskedasticity
ARFIMA	Autoregressive Fractionally Integrated Moving Average
ARIMA	Autoregressive Integrated Moving Average
ARMA	Autoregressive Moving Average
CV	Coefficient of Variation
FIM	First Inter Monsoon
GARCH	Generalized Autoregressive Conditional Heteroskedasticity
MAE	Mean Absolute Error
MAPE	Mean Absolute Percentage Error
MAX	Maximum
MFE	Mean Forecast Error
MIN	Minimum
MLE	Maximum Likelihood Estimators
MLR	Multiple Linear Regression
MRE	Mean Relative Error
MSE	Mean Square Error
NEM	North East Monsoon
PACF	Partial Auto Correlation Function
RMSE	Root Mean Square Error
SARFIMA	Seasonal Autoregressive Fractionally Integrated Moving Average
SARIMA	Seasonal Auto Regressive Integrated Moving Average
SD	Standard Deviation
SE	Standard Error
SIM	Second Inter Monsoon

SWM	South West Monsoon
TMIN	Minimum Temperature
VAR	Vector Auto Regression