MODELING WEEKLY RAINFALL IN COLOMO CITY

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DECLARATION

ñI declare that this is my own work and this thesis does not incorporate without acknowledgement any material previously submitted for a Degree or Diploma in any other University or institute of higher learning and to the best of my knowledge and belief it does not contain any material previously published or written by another person except where the acknowledgement is made in the text.ò

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Name of the supervisor: Senior Professor T. S. G. Peiris

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ABSTRACT

Modeling weekly rainfall has become a demanding assignment due to the complexity of rainfall pattern. Accurate inferences on weekly rainfall prediction facilitate to fill the noticeable gap with respect to the climate monitoring to reduce the climate stress in the country. However, relatively, few measures have been taken to perform the modeling of rainfall in the context of long memory. This study therefore, provides an assessment of such a phenomenon by fitting a novel time series models to weekly rainfall. As the weekly rainfall exhibits the blend features of long memory and time dependence variance, various class of long memory models were fitted by accounting the heteroskedasticity. The best fitted model developed is ARFIMA-GARCH for deseasonalized data. The model was trained using weekly rainfall data from 1990 to 2014 and validated using data from 2015 to 2017 in Colombo city, obtained from the Department of Meteorology, Sri Lanka. The exact maximum likelihood estimation method was utilized to estimate model parameters. For the evaluation of the suitability of the method for parameter estimation, Monte Carlo simulations were carried out with various non seasonally and seasonally fractionally differenced parameter values along with the variance model parameters. The forecasting performance of the five types of long memory models developed was evaluated based on the novel index developed using absolute error for an independent data set in addition to the classical indicators. The rainfall percentiles with the 95% confidence intervals were also developed by exploring temporal variability of weekly rainfall based on parametric approach and bootstrapping approach. It was found that the high likelihood to form extreme rainfall events during beginning of South West Monson (SWM) (30th April to 10th June) and during withdrawal of SWM rainfall (17th-30th September) as well as with the time span from 8th October to 11th November during Second Inter Monsoon (SIM) rainfall. Based on the real coverage probabilities which derived using bootstrap calibration, it was found that there is a discrepancy of the nominal and calculated coverage probabilities of the 95% confidence intervals of rainfall percentiles. The deviation of the normality of the fitted distribution with the small size of sample could be a reason for the such a disparity. The novel long range dependency model is recommended to be used in forecasting weekly rainfall in Colombo city in Sri Lanka since the forecasting performance of the new model is not much diluted with the increase of the forecasting length. The study highlights various challenges for applied statisticians in modeling weekly rainfall.

Keywords: Weekly Rainfall, Long Range Dependency, ARFIMA-GARCH, Forecasting, Coverage Probability, South West Monsoon

DEDICATION

To My Mother, Father, Husband and Lovely Son

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TABLE OF CONTENTS

DECLARATION ii
ABSTRACT iii
DEDICATION iv
ACKNOWLEDGEMENTS v
TABLE OF CONTENTS vii
LIST OF TABLES xii
LIST OF FIGURES xvii
LIST OF ABBREVIATIONS xxi

1. INTRODUCTION

1.1. Background 1
1.2. Climate Change in Sri Lanka 2
1.3. Annual Rainfall Pattern in Sri Lanka 3
1.4. Impact of Unpredicted Rainfall 5
1.5. Motivation to the Study 7
1.6. Objectives of the Study 9
1.7. Chapter Outline 9

2. LITERATURE REVIEW

2.1. Prediction of Rainfall Using ARIMA/SARIMA 11
    2.1.1. Prediction of Annual Rainfall 11
    2.1.2. Prediction of Seasonal Rainfall 12
    2.1.3. Prediction of Monthly Rainfall 12
    2.1.4. Prediction of Weekly Rainfall 16
2.2. Use of Artificial Neural Network for Modeling Rainfalls 17
2.3. Use of Multiple Linear Regression for Rainfall Forecasting 21
2.4. Hybrid Models for Rainfall Forecasting 23
2.5. Impact of Other Climatic Variables on Rainfall 26
2.6. Long Memory Models 27

2.6.1. Estimating of Fractional d of ARFIMA Models 27
2.6.2. Use of Long Memory Models 28
2.6.3. Use of Gegenbauer ARMA Models 28
2.6.4. Use of Seasonal Autoregressive Fractionally Integrated Moving Average (SARFIMA) Models 28
2.6.5. Models for Capture Heteroskedasticity 29
2.6.6. GARMA Class of Models with Heteroskedasticity 30
2.7. Summary of the Chapter 2 30

3. RESEARCH METHODOLOGY

3.1. Study Site 32
3.2. Data Description 32
3.3. Analysis of the Weekly Rainfall Percentiles for SWM 34
3.4. Analysis of the Weekly Rainfall Percentiles for SIM 36
3.5. The Best Fitted Statistical Distribution for Weekly Rainfall 37
3.6. The Use of Bootstrapping Approach 37
3.7. Coverage Probability for Weekly Rainfall Percentiles 38
3.8. Modeling Weekly Rainfall Using Classical Models 38
3.8.1. Stationary Series 39
3.8.2. ARIMA Modeling 39
3.8.3. SARIMA Modeling 40
3.8.4. Concept of ARCH/GARCH Modeling 40
3.8.5. Testing for the Serial Correlation 41
3.8.6. Testing for the ARCH Effect 41
3.8.7. VAR Modeling 42
3.8.8. Granger Causality Test 42
3.9. Modeling Weekly Rainfall Using Novel Approach 42
3.9.1. The Discrepancy Between Short and Long Memory Series 44

4. EXPLANATORY DATA ANALYSIS

4.1. Descriptive Analysis of Annual Rainfall 45

4.2. Descriptive Analysis of Seasonal Rainfall 46
4.3. Descriptive Analysis of Monthly Rainfall 48
4.4. Explanatory Analysis of Weekly Rainfall 50
    4.4.1. Descriptive Analysis of Weekly Rainfall for SWM 50
    4.4.2. Descriptive Analysis of Weekly Rainfall for SIM 52
    4.4.3. Descriptive Analysis of Weekly Rainfall for FIM 53
    4.4.4. Descriptive Analysis of Weekly Rainfall for NEM 55
4.5. Descriptive Analysis of the Weekly Temperature 56
    4.5.1. Minimum Weekly Temperature 56
    4.5.2. Maximum Weekly Temperature 58
    4.5.3. Average Weekly Temperature 59
4.6. Descriptive Analysis of the Relative Humidity 60
    4.6.1. Minimum Weekly Relative Humidity 60
    4.6.2. Maximum Weekly Relative Humidity 61
    4.6.3. Average Weekly Relative Humidity 63
4.7. Descriptive Analysis of the Vapor Pressure 64
    4.7.1. Minimum Weekly Vapor Pressure 64
    4.7.2. Maximum Weekly Vapor Pressure 65
    4.7.3. Average Weekly Vapor Pressure 67
4.8. Summary of the Chapter 4 67

5. COVERAGE PROBABILITY FOR WEEKLY RAINFALL PERCENTILES CONFIDENCE LIMITS

5.1. Trend Estimation 69
5.2. Weekly Rainfall Percentiles 71
5.3. The 95% Confidence Intervals for the Weekly Rainfall Percentiles using Parametric Approach 72
    5.3.1. Distribution of Weekly Rainfall 73
    5.3.2. Randomness of the Weekly Series 74
    5.3.3. Normality of Weekly Rainfall Series 77
    5.3.4. Common Distribution for Weekly Rainfall Totals 79
        5.3.4.1. Properties of the Best Fitted Models for Weeks in SWM 81

5.3.4.2. Properties of the Best Fitted Models for Weeks in SIM 82
5.3.5. Confidence Intervals for Weekly Rainfall in SMW 83
5.3.6. Confidence Intervals for Weekly Rainfall in SIM 87
5.4. The 95% Confidence Intervals for the Weekly Rainfall Percentiles using Bootstrapping Approach 89
5.4.1. Percentile Bootstrap Method 90
5.4.1.1. CI for Weekly Percentiles in SWM 91
5.4.1.2. CI for Weekly Percentiles in SIM 93
5.5. Accurate Confidence Interval Bands 94
5.5.1. Weibull Distribution 95
5.5.2. The Coverage Probability 96
5.5.3. Data for the Simulation 97
5.5.4. The Simulation Procedure 98
5.5.5. Results Obtained from the Simulation 99
5.6. Summary of the Chapter 5 103

6. MODELING OF WEEKLY RAINFALL: CLASSICAL TIME SERIES APPROACHES

6.1. Variability of Weekly Rainfall during 1990-2014 105
6.2. Identification of ARIMA Model 106
6.3. Development of GARCH/ARCH Model 114
6.4. Modeling for Deseasonalized Data 115
6.5. Modeling Weekly Rainfall with Exogenous Variables using VAR 122
6.5.1. Modeling Deseasonalized Weekly Rainfall with Exogenous Variables 130
6.6. Summary of the Chapter 6 135

7. NOVEL APPROACH TO MODEL WEEKLY RAINFALL

7.1. Concept for New Modeling 137
7.2. ARFIMA Long Range Dependency Model 138
7.3. Results of Monte Carlo Simulation - ARFIMA (0,d,0) 141
7.4. Modeling Weekly Rainfall Using ARFIMA Model 143

7.4.1. Residual Analysis for the Model ARFIMA (4, 0.05792421, 4) 144
7.5. ARFIMA Long Range Dependency Model for Deseasonalized Data 148
7.5.1. Residual Analysis 149
7.6. ARFIMA Long Range Dependency Model with Heteroskedasticity 153
7.7. The Results of Monte Carlo Simulation - ARFIMA-GARCH 155
7.8. Modeling Weekly Rainfall Using ARFIMA-GARCH Model 158
7.8.1. Residual Analysis for the Model ARFIMA (4,0.116577,6)-GARCH(1,1) 160
7.9. ARFIMA Long Memory Model for Desesonalized data with Heteroskedasticity 162
7.9.1. Residual Analysis for the Model ARFIMA (6,0.243588,5) -GARCH(1,1) for Deseasonalized Series 164
7.10. Adjusted SARFIMA -GARCH Long Range Dependency Model 166
7.11. The Result of Monte Carlo Simulation - Adjusted SARFIMA-GARCH 168
7.12. Modeling Weekly Rainfall Using Adjusted SARFIMA-GARCH Model 172
7.13. Comparison of the Five Long Range Dependency Models 175
7.14. Summary of the Chapter 7 180

8. CONCLUSIONS, RECOMMENDATIONS AND FUTURE STUDIES

8.1. Conclusions 182
8.2. Recommendations 184
8.3. Future Studies 185

9. PUBLICATIONS BASED ON THIS STUDY

9.1. List of Publications 187
REFERENCES 247
APPENDIX 1: WEEKLY RAINFALL PATTERNS AND THEIR DISTRIBUTIONS 262
APPENDIX 2: AUTO CORRELATION FUNCTIONS OF WEEKLY RAINFALL 266
APPENDIX 3: INDICES FORMATION 270

LIST OF TABLES

Table 3.1 Standard weeks in a year 34
Table 3.2 The weeks pertaining to the SWM 35
Table 3.3 The running totals of weeks pertaining to the SWM 35
Table 3.4 The weeks pertaining to the SIM 36
Table 3.5 The running totals of weeks pertaining to the SIM 36
Table 3.6 The difference between short and long memory series 44
Table 4.1 The summary statistics of annual rainfall total (in mm) for the period of 56 years (1960-2015) 45
Table 4.2 The summary statistics of seasonal rainfall total (in mm) from 1960 to 2015 47
Table 4.3 The summary statistics of monthly rainfall total (in mm) for the period of 56 years (1960-2015) 49
Table 4.4 The summary statistics of weekly rainfall total pertaining to SWM (week18-39) from 1960 to 2015 51
Table 4.5 The summary statistics of weekly rainfall total pertaining to SIM (week 40- 48) from 1960 to 2015 52
Table 4.6 The summary statistics of weekly rainfall total pertaining to FIM (week 0-17) for the period of 56 years 54
Table 4.7 The summary statistics of weekly rainfall total pertaining to NEM (week 49-52 and week 1-9) for the period of 56 years (1960-2015) 55
Table 4.8 The summary statistics of the minimum weekly temperature 57
Table 4.9 The descriptive statistics of the maximum weekly temperature 58
Table 4.10 The summary statistics of the average weekly temperature 59
Table 4.11 The descriptive statistics of the minimum weekly relative humidity 61
Table 4.12 The descriptive statistics of the maximum weekly relative humidity 62
Table 4.13 The descriptive statistics of the average weekly relative humidity 63
Table 4.14 The descriptive statistics of the minimum weekly vapor pressure 65
Table 4.15 The descriptive statistics of the maximum weekly vapor pressure 66
Table 4.16 The descriptive statistics of the average weekly vapor pressure 67
Table 5.1 The coefficients (slope) of the linear trend along with the P-values 71

Table 5.2 The coefficients of linear and quadratic along with the P-values of the randomly selected weeks 71

Table 5.3 The Probability density functions 79

Table 5.4 The best fitted statistical models and maximum likelihood estimates for weekly rainfall during SWM 81

Table 5.5 The best fitted statistical models and maximum likelihood estimates for weekly rainfall during SIM 83

Table 5.6 The formulas used for percentiles and variance estimates 84

Table 5.7 The formulas used for confidence intervals for percentiles 84

Table 5.8 The percentiles of the weekly rainfall and the corresponding 95% confidence intervals during SWM in the city Colombo 85

Table 5.9 The percentiles of the weekly rainfall and the corresponding 95% confidence intervals during SIM in the city Colombo 88

Table 5.10 The 95% confidence intervals of weekly rainfall percentile (based on 1000 bootstrap samples) pertaining to SWM (week 18-39) 91

Table 5.11 The 95% confidence intervals of weekly rainfall percentiles (based on 1000 bootstrap samples) pertaining to SIM (week 40-48) 93

Table 5.12 Descriptive statistics of the weekly rainfall data (week 24) 97

Table 5.13 The coverage probabilities of five percentiles (P_{50}-P_{90}) based on the 300 samples derived from the Sample68 99

Table 5.14 The coverage probabilities of five percentiles (P_{50}-P_{90}) based on the 300 samples derived from the Sample 423 100

Table 5.15 The coverage probabilities of five percentiles (P_{50}-P_{90}) based on the 300 samples derived from the Sample 802 101

Table 5.16 The coverage probabilities of five percentiles (P_{50}-P_{90}) based on the 300 samples derived from the Sample 1551 102

Table 5.17 Average accurate confidence level based on the 95% confidence level for Weibull percentiles 102

Table 5.18 The confidence bands of percentiles of week 24 (nominal and actual values) 103

Table 6.1 Result of Dickey Fuller Test 107

Table 6.2 Selected models and values of the selection criteria 109

Table 6.3 Test result of the Breusch-Godfrey serial correlation LM test 111

Table 6.4 Test results of the heteroskedasticity ARCH effect 112

Table 6.5	Parameter estimation of the model SARIMA(1,0,0)(1,1,0)$_{52}$-GARCH(1,2)	114
Table 6.6	The result of ARCH effect of AR(1) for deseasonalized data	116
Table 6.7	The result of serial correlation of AR(1) for deseasonalized data	116
Table 6.8	The result of the estimated AR(1)-GARCH(1,1) model for deseasonalized series	118
Table 6.9	The result of ARCH effect of AR(1)-GARCH(1,1) for deseasonalized data	118
Table 6.10	The absolute error in mm for the weekly rainfall in 2015 [AR(1) - GARCH(1,1)] model for deseasonalized data	121
Table 6.11	Result of Augmented Dickey Fuller (ADF) test for determining the stationary of the time series	122
Table 6.12	The correlation between rainfall and exogenous climatic variables at lag 1 and lag 2	123
Table 6.13	Analysis of Variance of stepwise regression at lag 1	124
Table 6.14	Analysis of Variance of stepwise regression at lag 2	124
Table 6.15	Values of the selection criterion for selecting the optimal lag order	125
Table 6.16	VAR Model for weekly rainfall series	126
Table 6.17	Result of Granger Causality test	128
Table 6.18	Result of Augmented Dickey Fuller (ADF) test for the deseasonalized rainfall series	130
Table 6.19	Values of the selection criterion for selecting the optimal lag order	130
Table 6.20	VAR model for the deseasonalized Data	131
Table 6.21	Result of Granger Causality test	133
Table 7.1	Result for exact maximum likelihood estimator of d for a generating process of ARFIMA(0,d,0)	142
Table 7.2	The parameter estimates of the model ARFIMA (4,0.05792421, 4)	143
Table 7.3	The result of ARCH LM test of ARFIMA(4,0.0579,4)	144
Table 7.4	The analysis of absolute error (in mm) for the weekly rainfall in 2015 - [ARFIMA(4,0.0579,4)]	148
Table 7.5	The parameter estimates of the model ARFIMA (5,0.05999,5)	149
Table 7.6	The result of ARCH LM test of ARFIMA(5,0.0599,5)	152
Table 7.7	The analysis of absolute error (in mm) for the weekly rainfall in 2015 [ARFIMA(5,0.0599,5)]	153

Table 7.8 The MLE of d, $Ǔ_0$, $Ǔ_1$ and $ɓ_1$ of a generating process of ARFIMA(0,d,0)-GARCH(1,1) with $Ǔ_0 = 0.15, Ǔ_1 = 0.2, ɓ_1 = 0.6$ and d=0.1 156
Table 7.9 The MLE of d, $Ǔ_0$, $Ǔ_1$ and $ɓ_1$ of a generating process of ARFIMA(0,d,0)-GARCH(1,1) with $Ǔ_0 = 0.15, Ǔ_1 = 0.2, ɓ_1 = 0.6$ and d=0.15 157
Table 7.10 The MLE of d, $Ǔ_0$, $Ǔ_1$ and $ɓ_1$ of a generating process of ARFIMA(0,d,0)-GARCH(1,1) with $Ǔ_0 = 0.15, Ǔ_1 = 0.2, ɓ_1 = 0.6$ and d=0.3 157
Table 7.11 The MLE of d, $Ǔ_0$, $Ǔ_1$ and $ɓ_1$ of a generating process of ARFIMA(0,d,0)-GARCH(1,1) with $Ǔ_0 = 0.15, Ǔ_1 = 0.2, ɓ_1 = 0.6$ and d=0.45 158
Table 7.12 The parameter estimates of the model ARFIMA (4,0.116577,6)-GARCH (1,1) 159
Table 7.13 The result of weighted Ljung-Box test on standardized residuals of the model ARFIMA(4,0.116577,6)-GARCH(1,1) 160
Table 7.14 The result of weighted Ljung-Box test on standardized squared residuals of the model ARFIMA(4, 0.116577,6)-GARCH(1,1) 160
Table 7.15 The result of weighted ARCH LM test of the model ARFIMA(4,0.116577,6)-GARCH(1,1) 161
Table 7.16 The absolute error in mm for the weekly rainfall in 2015 $\text{ARFIMA}\left(4,\ 0.116577,\ 6\right)-\text{GARCH}(1,1)$ 162
Table 7.17 The parameter estimates of the model ARFIMA(6,0.243588,5) - GARCH(1,1) for deseasonalized series 163
Table 7.18 The result of weighted Ljung-Box test on standardized residuals of the model ARFIMA(6,0.243588,5)-GARCH(1,1) 164
Table 7.19 The result of weighted Ljung-Box test on standardized squared residuals of the model ARFIMA(6,0.243588,5)-GARCH(1,1) 164
Table 7.20 The result of weighted ARCH LM test of the model ARFIMA(6,0.243588,5)-GARCH(1,1) 165
Table 7.21 The analysis of absolute error in (mm) for the weekly rainfall in 2015 ARFIMA(6,0.243588,5)-GARCH(1,1) for deseasonalized data 166

Table 7.22 The MLE of D, d, $Ǔ_0$, $Ǔ_1$ and $ɓ_1$ of a generating process of SARFIMA(0,d,0)(0,D,0)$_{52}$-GARCH(1,1) with $Ǔ_0 = 0.15, Ǔ_1 = 0.2, ɓ_1 = 0.6$ and d=0.1 and D=0.45 169

Table 7.23 The MLE of D, d, $Ǔ_0$, $Ǔ_1$ and $ɓ_1$ of a generating process of SARFIMA (0,d,0)(0,D,0)$_{52}$-GARCH(1,1) with $Ǔ_0 = 0.15, Ǔ_1 = 0.2, ɓ_1 = 0.6$ and d=0.15 and D=0.45 170

Table 7.24 The MLE of D, d, $Ǔ_0$, $Ǔ_1$ and $ɓ_1$ of a generating process of SARFIMA(0,d,0)(0,D,0)$_{52}$-GARCH(1,1) with $Ǔ_0 = 0.15, Ǔ_1 = 0.2, ɓ_1 = 0.6$ and d=0.3 and D=0.3 171

Table 7.25 The MLE of D, d, $Ǔ_0$, $Ǔ_1$ and $ɓ_1$ of a generating process of SARFIMA(0,d,0)(0,D,0)$_{52}$-GARCH(1,1) with $Ǔ_0 = 0.15, Ǔ_1 = 0.2, ɓ_1 = 0.6$ and d=0.45 and D=0.1 172

Table 7.26 The parameter estimates of the model SARFIMA(1,0.115677,1)(1,0.170750,0)$_{52}$ with GARCH(1,1) 173

Table 7.27 The analysis of absolute error in mm for the weekly rainfall in 2015 $\text{SARFIMA}\ (1,\ 0.116,\ 1)\times(1,\ 0.171,\ 0)_{52}$ 175

Table 7.28 The weights assigned for the absolute forecasting error category 176

Table 7.29 The comparison of five long range dependency models 177

LIST OF FIGURES

Figure 3.1 The city of Colombo is in the Colombo district 33
Figure 4.1 Annual rainfall in Colombo city in mm (1960-2015) 46
Figure 4.2 Seasonal rainfall during the time span from the 1960 to 2015 47
Figure 4.3 Distribution of rainfall for the four seasons from 1960 to 2015 48
Figure 4.4 Box plot of the monthly rainfall 50
Figure 4.5 Box plot of the weekly rainfall pertaining to the SWM 51
Figure 4.6 Box plot of the weekly rainfall pertaining to the SIM 53
Figure 4.7 Box plot of the weekly rainfall pertaining to the FIM 54
Figure 4.8 Box plot of the weekly rainfall pertaining to the NEM 56
Figure 4.9 The time series plot of the minimum weekly temperature 57
Figure 4.10 Mean minimum weekly temperature 58
Figure 4.11 Mean maximum weekly temperature 59
Figure 4.12 The time series plot of the minimum weekly relative humidity 60
Figure 4.13 Mean minimum weekly relative humidity 61
Figure 4.14 Mean maximum weekly relative humidity 62
Figure 4.15 The time series plot of the average weekly relative humidity 63
Figure 4.16 The time series plot of the minimum weekly vapor pressure 64
Figure 4.17 Mean minimum weekly vapor Pressure 65
Figure 4.18 The time series plot of the maximum weekly vapor pressure 66
Figure 4.19 Mean maximum weekly vapor Pressure 67
Figure 5.1 The time series plots of the weekly rainfall of weeks 1,10,18 and 23 69
Figure 5.2 The time series plots of the weekly rainfall of weeks 32,39,46 and 51 70
Figure 5.3 Histogram of the total weekly rainfall for week numbers: week 20, 28, 32 and 34 73
Figure 5.4 Histogram of the total weekly rainfall for week numbers: week 41, 44,46 and 48 74
Figure 5.5 The auto correlation plot of the week 20 belongs to SWM 75

Figure 5.6 The auto correlation plot of the week 34 belongs to SWM 75
Figure 5.7 The auto correlation plot of the week 41 pertains to SIM 76
Figure 5.8 The auto correlation plot of the week 44 pertains to SIM 76
Figure 5.9 The normal probability plot of the week 20 in SWM 77
Figure 5.10 The normal probability plot of the week 34 in SWM 77
Figure 5.11 The normal probability plot of the week 41 in SIM 78
Figure 5.12 The normal probability plot of the week 44 in SIM 78
Figure 5.13 The 90th Percentiles of running total of weekly rainfall and 95% confidence intervals during SWM in Colombo 87
Figure 5.14 The 90th Percentiles of running total of weekly rainfall and 95% confidence intervals during SIM in Colombo 89
Figure 5.15 Density functions of Weibull distribution with different scale and shape parameters 95
Figure 5.16 Histogram of weekly rainfall data (week 24) 97
Figure 6.1 Time series plot of the weekly rainfall $\{Y_t\}$ from 1990 to 2014 105
Figure 6.2 Autocorrelation plot of the series from 1990 to 2014 106
Figure 6.3 ACF of the Z_t series from 1990 to 2014 with 52 lag 107
Figure 6.4 PACF of the series from 1990 to 2014 with 52 lag 108
Figure 6.5 The correlogram plot of the residual of the model SARIMA $(1,0,0) \times (1,1,0)_{52}$ 110
Figure 6.6 The normality test of the residuals of the model SARIMA $(1,0,0)\ (1,1,0)_{52}$ 111
Figure 6.7 The correlogram of squared residuals of the model SARIMA $(1,0,0)\ (1,1,0)_{52}$ 112
Figure 6.8 Observed and predicted weekly rainfall in 2015 using the model SARIMA $(1,0,0)(1,1,0)_{52}$ 113
Figure 6.9 Observed and predicted weekly rainfall in 2015 using the model SARIMA $(1,0,0)(1,1,0)_{52}$-GARCH(1,2) 115
Figure 6.10 The correlogram of squared residuals of the model AR(1) for deseasonalized data 117
Figure 6.11 The correlogram of residuals derived from the model AR(1)-GARCH(1,1) for the deseasonalized data 119

Figure 6.12 The correlogram of squared residuals derived from the model AR(1)-GARCH(1,1) for the deseasonalized data 120

Figure 6.13 Actual and predicted weekly rainfall in 2015 using AR(1)-GARCH(1,1) for deseasonalized data 121

Figure 6.14 Impulse response function of average relative humidity to rainfall 129

Figure 6.15 Impulse response function of average relative humidity to deseasonalized rainfall 134

Figure 6.16 Actual and predicted rainfall in 2015 using VAR model for desesonalized data 135

Figure 7.1 The periodgram of the rainfall series from 1990 to 2014 137

Figure 7.2 The correlogram of residuals of the model ARFIMA(4,0.0579,4) 145

Figure 7.3 The correlogram of squared residuals of the model ARFIMA(4,0.0579,4) 146

Figure 7.4 Observed and predicted weekly rainfall in 2015 using the ARFIMA(4,0.0579,4) 147

Figure 7.5 The correlogram of residuals of the model ARFIMA(5,0.0599,5) 150

Figure 7.6 The correlogram of squared residuals of the model ARFIMA(5,0.0599,5) 151

Figure 7.7 Observed and predicted weekly rainfall in 2015 using the ARFIMA(5,0.0599,5) for deseasonalized data 152

Figure 7.8 Observed and predicted weekly rainfall in 2015 using the model $\text{ARFIMA }(4,\ 0.116577,\ 6)-\text{GARCH}(1,1)$ 161

Figure 7.9 Observed and predicted weekly rainfall in 2015 using the model ARFIMA(6,0.243588,5)-GARCH(1,1) for deseasonalized data 165

Figure 7.10 Observed and predicted weekly rainfall in 2015 using the $\text{SARFIMA }(1,\ 0.116,\ 1)\times(1,\ 0.171,\ 0)_{52}$ 174

Figure 7.11 Observed and predicted weekly rainfall from 2015 to 2017 using the model ARFIMA (4,0.05792,4) 178

Figure 7.12 Observed and predicted weekly rainfall from 2015 to 2017 using the model ARFIMA (5,0.05999,5) for deseasonalized data 178

Figure 7.13 Observed and predicted weekly rainfall from 2015 to 2017 using the model ARFIMA(4,0.116577,6)-GARCH (1,1) 179

Figure 7.14 Observed and predicted weekly rainfall from 2015 to 2017 using the model ARFIMA(6,0.243588,5)-GARCH(1,1) for deseasonalized data 179

Figure 7.15 Observed and predicted weekly rainfall from 2015 to 2017 using the adjusted SARFIMA(1,0.115677,1) ×(1,0.17075,0) - GARCH(1,1) model 180

LIST OF ABBREVIATIONS

Abbreviation	**Description**
ACF	Auto Correlation Function
AIC	Alkina Information Criterion
ANN	Artificial Neural Network
ARCH	Autoregressive Conditional Heteroskedasticity
ARFIMA	Autoregressive Fractionally Integrated Moving Average
ARIMA	Autoregressive Integrated Moving Average
ARMA	Autoregressive Moving Average
CV	Coefficient of Variation
FIM	First Inter Monsoon
GARCH	Generalized Autoregressive Conditional Heteroskedasticity
MAE	Mean Absolute Error
MAPE	Mean Absolute Percentage Error
MAX	Maximum
MFE	Mean Forecast Error
MIN	Minimum
MLE	Maximum Likelihood Estimators
MLR	Multiple Linear Regression
MRE	Mean Relative Error
MSE	Mean Square Error
NEM	North East Monsoon
PACF	Partial Auto Correlation Function
RMSE	Root Mean Square Error
SARFIMA	Seasonal Autoregressive Fractionally Integrated Moving Average
SARIMA	Seasonal Auto Regressive Integrated Moving Average
SD	Standard Deviation
SE	Standard Error
SIM	Second Inter Monsoon

SWM	South West Monsoon
TMIN	Minimum Temperature
VAR	Vector Auto Regression