STUDIES ON

THE STRENGTH AND DEFLECTION OF

HYPERBOLIC PARABOLOID SHELL ROOFS

A THESIS

SUBMITTED FOR THE AWARD OF THE DEGREE OF
DOCTOR OF PHILOSOPHY
IN CIVIL ENGINEERING

BY

A. C. MATHAI

DEPARTMENT OF CIVIL ENGINEERING
INDIAN INSTITUTE OF TECHNOLOGY, MADRAS

APRIL 1971

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CERTIFICATE

This is to certify that the thesis entitled "STUDIES ON THE STRENGTH AND DEFLECTION OF HYPERBOLIC PARABOLOID SHELL ROOFS", that is being submitted by Shri A.C. Mathai, to the Indian Institute of Technology, Madras, in fulfilment of the requirements for the award of the Degree of 'Doctor of Philosophy' in Civil Engineering, is a record of bonafide research work carried out by him in this Department under my supervision and guidance.

Shri Mathai has worked on the problem for about three years. The results embodied in this thesis have not been submitted to any other University or Institute for the award of any degree or diploma.

UOM Verified Signature

(Dr.P.C. Varghese)

Professor & Head of the Department
Department of Civil Engineering

Date: 22nd April 1971.

Indian Institute of Technology, Madras.

ACKNOWLEDGEMENTS

The author considers it a great privilege to have had the opportunity to carry out the work reported in this thesis under the guidance of Dr. P.C. Varghese, Professor and Head of the Department of Civil Engineering. He records his deep sense of gratitude to Prof. Varghese for his invaluable guidance and constant encouragement.

The author acknowledges the valuable discussions he had with Mr. T.P.Ganesan, Mr. H. Achyutha and other members of the Faculty of Civil Engineering. He is thankful to Messrs. M.S.Mathews, V.G. Idichandy and P.K. Aravindan of the Civil Engineering Department, Mr. V.K. Vaidyan of the Physics Department and Mr.R. Kubendran of the Photographic Section for their help in the experimental work and in the preparation of the thesis.

The facilities and finances provided by the Institute and the willing co-operation of the staff of the Civil Engineering Laboratories are gratefully acknowledged. Computer facilities were provided by the Centre of Advanced Study in Biophysics, University of Madras.

The author is thankful to the Government of Kerala and the Director of Technical Education, Kerala for sanctioning him leave from the Government College of Engineering, Trivandrum to undertake this research.

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SYNOPSIS

Published literature on experimental studies of hipped hypar shells is very limited. Design procedures for calculation of strength and deflection which take into consideration both the rigidities of edge beams and the extension of ties have not been studied in detail. Methods to calculate the ultimate strength of such shells are also not fully known. In this thesis an attempt is made to study these aspects of a hypar shell so as to evolve workable procedures for design purposes.

In Chapter I, the geometry of the shell is discussed and the need for further research is brought out. Chapter 2 is devoted to a review of existing literature relevant to the present work.

A simplified method for calculation of deflections of hipped hypar shell roofs is developed in Chapter 3. The method can take into account variations of edge beam size and extensional rigidity of ties. The validity of the simplified procedure has also been examined for one case by comparing the results obtained by this method with those obtained by the finite difference solution of the equations of bending theory.

Chapter 4 describes briefly the experimental work. Thirty one tests, done in four phases, are reported. Full particulars are included only regarding the tests of the fourth phase.

In Chapter 5, the test results (fourth phase) pertaining to the elastic studies are presented and compared with the theoretical results. The results are presented for the two cases of (a) uniformly loaded shell with corners held in position and (b) unloaded shell subjected to symmetrical horizontal displacement at the supports. Chapters 6 and 7 bring out the influence of extensional rigidity of ties and of size of edge beams respectively on the working load behaviour of the shells. The behaviour of the test models at cracking and ultimate load stages is discussed in Chapter 8.

A study of the ultimate strength of reinforced concrete hipped hypar shells is made in Chapter 9. Four possible modes of failures are identified and methods to calculate the ultimate strength are proposed. The influences of strength and extensional rigidity of the ties and the point of application of the tie force on the ultimate strength are brought out. Ultimate strength of the test models are compared with the theoretical predictions.

A brief discussion on prestressing of the ties is given in Chapter 10. The major conclusions arrived at are grouped in Chapter 11. Some design recommendations and suggestions for future research are also presented.

NOTATION

Shell geometry.

X, Y, Z	- Coordinate axes.
a, b	- Plan length of shell quadrant in X and Y directions respectively.
c	- Shell rise
h	- Shell thickness
b_r	- Width of beam
d	- Depth of beam inclusive of shell thickness
k	- $\frac{ab}{c}$ or $\frac{a^2}{c}$ for shells square in plan
k_1	- $\frac{1}{k}$
γ	- $\frac{h^2}{6c^2}$
x, y, z	- Coordinates of any point on the shell
$\bar{x}$	- $\frac{x}{a}$
$\bar{y}$	- $\frac{y}{a}$
R	- $\sqrt{k^2 + x^2 + y^2}$
S	- $\sqrt{k^2 + x^2}$
T	- $\sqrt{k^2 + y^2}$

Elastic and geometric properties of shell and effective arch

E	- Modulus of elasticity of the material of shell.
E_t	- Modulus of elasticity of tie
$E_{c,t}$	- Modulus of elasticity of concrete in the tie
G	- Modulus of rigidity
ν	- Poisson's ratio
D	- $\frac{Eh^3}{12(1-\nu^2)}$
I	- Moment of inertia

I_x - Moment of inertia of effective arch at a distance x from the crown.

I_o - Moment of inertia of effective arch at the crown.

z_x - Rise of effective arch above tie level, at a distance x from the crown

A - Area of tie

$A_{c,t}$ - Area of concrete (transformed area when untensioned steel is present) in the tie.

A_{sc} - Area of steel in compression

A_{st} - Area of steel in tension

A_b - Area of beam.

Strengths of materials and sections.

M_u - Ultimate moment capacity

$M_{u,c}$ - Ultimate moment capacity (for hogging moment) of the weakest section near the shell corner.

P_u - Ultimate axial load capacity

H_u - Ultimate horizontal reaction at supports in the directions of coordinate axes (yield strength of tie)

C_c - Ultimate load capacity in compression of unit width of shell panel.

C_t - Ultimate load capacity in tension of unit width of shell panel

σ_y - Yield strength of steel

σ_{cyl} - Cruching strength of standard concrete cylinder.

Loads and forces acting.

q - Uniformly distributed load per unit area.

q_u	- Ultimate load capacity per unit area
q_{cr}	- Buckling load (per unit area)
p_o	- Dead load of shell per unit area
N_{xx}, N_{yy}, N_{xy}	- In-plane stress resultants.
M	- Bending moment due to external load or applied displacement.
M_x	- M at a distance x from the crown of effective arch.
M_e	- Moment developed at the shell corner due to eccentricity of tie reaction.
m	- Bending moment due to unit force.
Q	- Transverse shear
H	- Horizontal reaction at each support in the direction of coordinate axes - Tie force in one tie.
H'	- Tie force when the tie is placed with eccentricity e.
H^*	- Reduction in tie force below the value given by membrane theory.
2H	- Tie force for the effective arch.
P	- (i) Axial force (ii) prestressing force to be applied at each support in the X and Y directions to obtain zero central deflection.
P_a	- Additional prestressing force in excess of P.
$P_{a,s}$	- Portion of P_a resisted by the shell
$P_{a,t}$	- Portion of P_a resisted by the tie.

<u>Displacements</u>

u,v	- Tangential displacements
w	- Normal displacement

u_x, v_y, w_z	- Displacements in the X, Y and Z directions.
$\bar{u}$	- $\frac{u}{a}$
$\bar{v}$	- $\frac{v}{a}$
$\bar{w}$	- $\frac{w}{k}$
δ_H	- Horizontal displacement at each support of the shell in the X and Y directions.
$\delta_{H,u}$	- $\delta_{H(\text{unit force})}$ - horizontal displacement at each support of the effective arch due to unit horizontal force applied at the support
δ_{H,P_a}	- δ_H due to prestressing force P_a.
δ_v	- Vertical displacement of failure mechanism due to δ_H of unity
δ_x	- Vertical displacement of effective arch at a distance x from crown
δ_c	- Vertical deflection of effective arch at the centre.
$\triangle_c$	- Central deflection of shell.

Other notations

$(\ )'$	- $\frac{\partial(\ )}{\partial x}$
$(\ )^{\cdot}$	- $\frac{\partial(\ )}{\partial y}$
$\nabla^2(\ )$	- $(\ )'' + (\ )^{\cdot\cdot}$
$\nabla^4(\ )$	- $\nabla^2\nabla^2(\ )$
$\nabla^8(\ )$	- $\nabla^4\nabla^4(\ )$
F	- Airy stress function
ϕ	- Vlasov stress-displacement function
ψ	- $w + \frac{1}{\sqrt{DEh}} F$
λ	- Spacing of square finite-difference grid.

α, β, K, K_n - Non-dimensional parameters

α_u - Reduction factor for compressive strength of concrete

e - Eccentricity of tie.

Additional symbols are defined in the text as and when they occur.